Learning Targets

Apply vocab: Domain, range, image, independent variable, dependent variable, implict form, explicit form

Understand and apply the definition of a function as an association between sets

Use function notation properly

Describe the domain of a function

Identify the features of a function from a graph

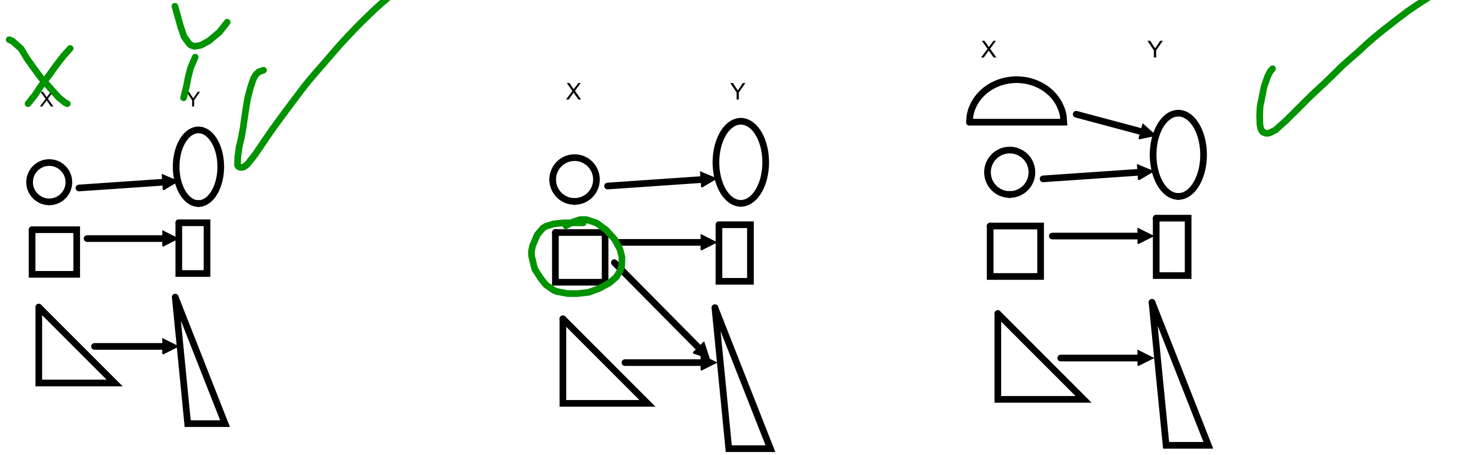
Identify the graphical features (intercepts) of function from the equation

Basic from packet

A FUNCTION is a relation that associates each element of the set X (called the Domain) with exactly one element of the set Y (called the Range).

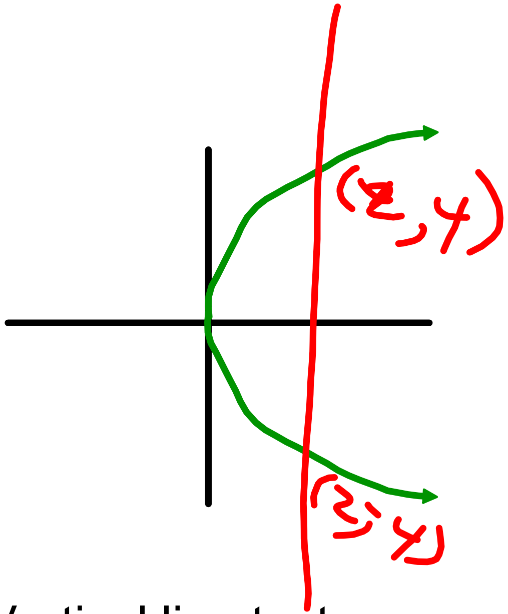
- A relation (or mapping) does not need to be mathematical in nature.
  For example it can be a list of individual rules for every item in the set X.
- X is the independent variable, Y the dependent.

Which is a function?



Packet: Page 2 number 3

Make a table and graph to verify whether or not $x=y^2$ is a function

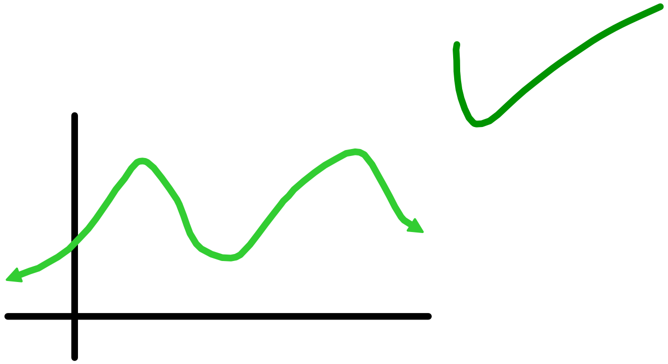


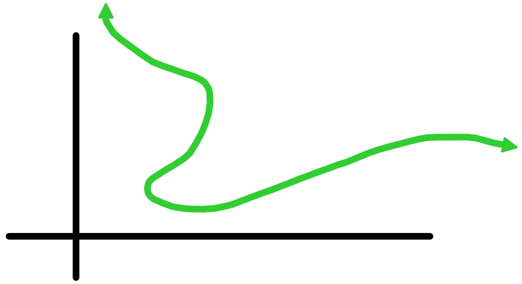
Vertical line test

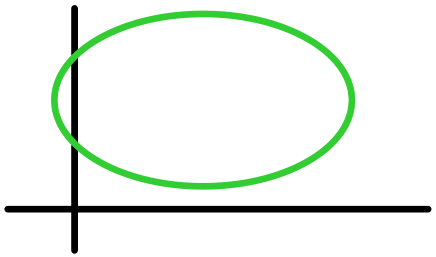
x	y
1	-1
1	1
4	-2
4	2

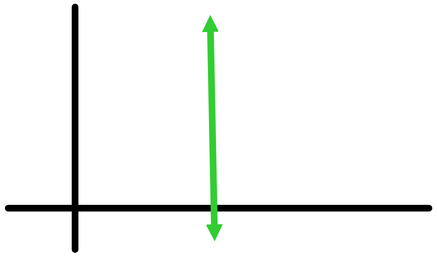
The same x associated with multiple y's

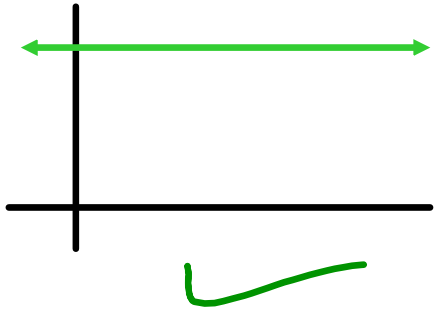
Basic from packet



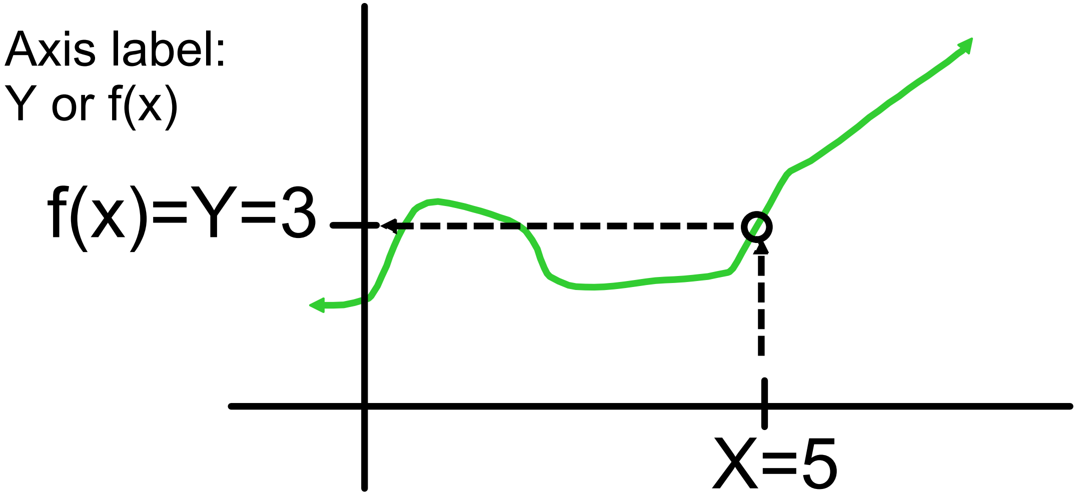








Functions applied to a curve and a graph



$(x, f(x))$

(x, y)

$(5,3)$

Page 3: Do the parts of 4

$y = x + 2$

$f(x) = x + 2$

Recap : The members of each pair should be interchangeable in your thinking

$f(x) = 3x+2$, function notation
$y = 3x +2$

$(x, f(x))$, function notation
(x,y)

$(5,17)$
$f(5) = 17$

If they tell you

$f(x) = 2x-1$

and that

$f(x) = 11$

What do you do to solve the problem?

Page 4: Independent vs. dependent, familiar

Page 5: Implicit vs. Explicit, new for many
Do the two on this page.

Implicit form: many forms, common as O(x,y) = c $2x + 4y - 14 = 0$

This is a relation of x and y.
The independent/dependent variable relation is not explict.

Explicit form: y=f(x)

$$\frac{-1}{2}x + 3\frac{1}{2} = y$$

This relation explicitly shows y to be a function of x.

Bottom line: To make explicit form => Solve for the dependent variable (y)

Range and domain.

What are the range and domain of $f(x) = x^2$?

We will focus on domain in depth.

Our examples on range will always be basic.

Domain -
Popular exclusions from "all reals"
Those arising because:

- Taking the even root of negatives.
- Dividing by 0.
- Logs of negative numbers

Answer in interval notation

Domain?

$10 - 2x = 0$

$10 - 2x < 0$

$10 < 2x$

$5 < x$

$$f(x) = \frac{1}{10-2x}$$

$$f(x) = \sqrt{10-2x}$$

$$f(x) = \log(10-2x)$$

$a^b = c$: "a" can't be 0 if b<0

$c^{(1/b)} = a$: "c" can't be "-" if b is even

$\log_a c = b$: "c" can't be "-"

1) Set up the equality or inequality to find the exclusion
2) Solve for the exclusions
3) Write your domain to exclude the exclusions
4) Plug in critical value

$$f(x)=\sqrt{10-2x}$$

10-2x<0
10<2x
5<x
This is the exclusion

This is the domain

$$(-\infty,5]$$

$$h(x)=\frac{2x}{x^2-4}$$

$x^2 - 4 = 0$
$x^2 = 4$
x=+/-2
This is the exclusion

This is the domain

$$(-\infty,-2)\cup(-2,2)\cup(2,+\infty)$$

$$g(x)=\frac{x+4}{x^3-4x}$$

$$x^3-4x=0$$

$$x(x^2-4)=0$$

$$x(x-2)(x+2)=0$$

These are the exclusions

$$x=0, x=2, x=-2$$

This is the domain

$$(-\infty,-2)\cup(-2,0)\cup(0,2)\cup(2,+\infty)$$

$$k(x)=\frac{2x}{\sqrt{-4x+10}}$$

What happens if the exclusion can be eliminated?

Are $$f(x)=\frac{x^2+6x-27}{2(x+9)}=\frac{(x+9)(x-3)}{2(x+9)}$$

$$f(x)=\frac{x-3}{2}$$ the same function? NO

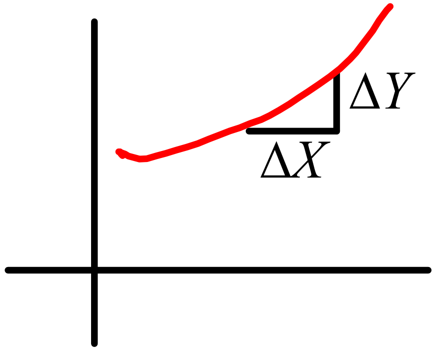
The domain is set before simplification.

$$2x=\sqrt{-11x+20}$$

$$4x^2=-11x+20$$

"If I wanted THAT solution, I would have written it that way!"

Let's say I had a complex function and I wanted to approximate it's slope at a certain interval. How could I do it?



$$\frac{\Delta Y}{\Delta X} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$ ← Clever trick #1

Clever trick #2 → $x_2 = x_1 + \Delta x \text{ if } \Delta x = h$

$$x_2 = x_1 + h$$

SO=> $$\frac{\Delta Y}{\Delta X} = \frac{f(x_1 + \Delta x) - f(x_1)}{(x_1 + \Delta x) - x_1} = \frac{f(x_1 + h) - f(x_1)}{(x_1 + h) - x_1} = \frac{f(x_1 + h) - f(x_1)}{h}$$

A tool to approximate slope over an interval.

Incremental change/slope/
average rate of change/slope of tangent line

$$c(x,h)=\frac{f(x+h)-f(x)}{h}$$

Plug a certain function into this formula, you get a new function that approximates the slope for the original function.

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f(x) = 3x-4

$$C(x,h)=$$

$$\frac{(3(x+h)-4)-(3x-4)}{h}=$$

$$\frac{3x+3h-4-3x+4}{h}=$$

$$\frac{3h}{h}=3$$

The slope of f(x)=3x-4 is 3, always

f(x) = x^2+2

$$c(x,h)=\frac{((x+h)^2+2)-(x^2+2)}{h}$$

$$=\frac{x^2+2xh+h^2+2-x^2-2}{h}=$$

$$\frac{h^2+2xh}{h}=2x+h$$

The avg. rate of chg. of g(x) = x^2 + 2 is 2x+h.

1) It changes with x
2) Depend on the interval

What is the slope of x^2+2 at $x=4$?

$f(x) = x^2+2$

$$c(x,h) = \frac{((x+h)^2+2)-(x^2+2)}{h}$$

$$= \frac{x^2+2xh+h^2+2-x^2-2}{h} =$$

$$\frac{h^2+2xh}{h} = 2x+h$$

The avg. rate of chg. of
$g(x) = x^2 + 2$ is $2x+h$.

1) It changes with x
2) Depend on the interval

Clever trick #3, ASSUME h is tiny

For all examples $f(x) = 2x^2 - 2$

A) Two ways to find the slope of the secant line.

1) For the interval [1,4]
Plug in to the ARC formula

$$ARC(1,4) = \frac{f(4) - f(1)}{4 - 1} =$$

$$ARC(1,4) = \frac{(2*4^2 - 2) - (2*1^2 - 2)}{4 - 1} = 10$$

2) Find the general equation:
For the interval 1 to x, [1,x]
then plug in.
(where "1" can be any number)

$$ARC(1,x) = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

$$ARC(1,x) = \frac{(2x^2 - 2) - (2(1)^2 - 2)}{x - 1}$$

$$= \frac{2x^2 - 2}{x - 1} = \frac{2(x+1)(x-1)}{x - 1} = 2(x+1)$$

$$ARC(1,4) = 2(4+1) = 10$$

(I would never do it this way unless I was asked to find the formula for the slope of the secant line from [1,x] first)

B) Finding the equation of the secant line:

We know find slope of the secant line one of two ways.

$M_{sec} = 10$
or
$M_{sec} = 2(x+1) = 2(4+1) = 10$

$f(1) = 2(1)^2 - 2 = 0$ so (1,0) is a point on f
$f(4) = 2(4)^2 - 2 = 30$ so (4,30) is a point on f

y=mx+b
y=10x+b

Plug in (1,0) *{you could use (4,30)}*
0=10(1)+b
-10=b

Equation of the secan line: Y=10x-10

Attachments

curve for 23 x2.ggb