We have discussed the following properties of functions"
- Range
- Domain
- Intercepts

Now we are getting more sophisticated in describing the form of the function.

Question one: Is the function going up or down
and, if so, how fast?

What concept do we have to measure that already?

The book calls this the "difference quotient"

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

The book calls the ARC

$$ARC = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

Similar, right?

Targets:

- Use the ARC formula to calculate the average rate of change over an interval.

- Use the ARC formula to calculate the equation of the slope of the secant line.

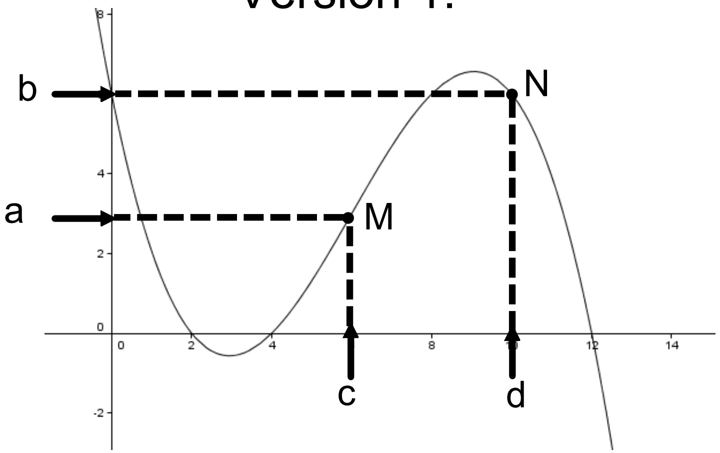
- Use the ARC formula to calculate the equation of the secant line.

The book makes this harder by changing notation.

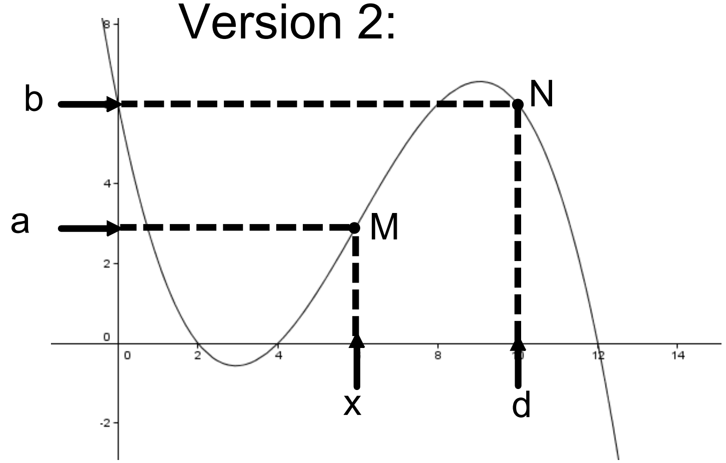
BUT... the point is you should be getting know the notation well enough to have no issue with the change.

The graph of function f(x) with Point M and N highlighted

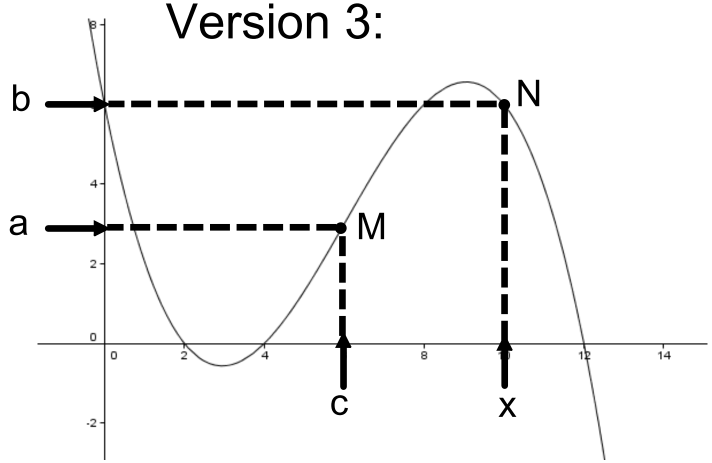
Version 1:



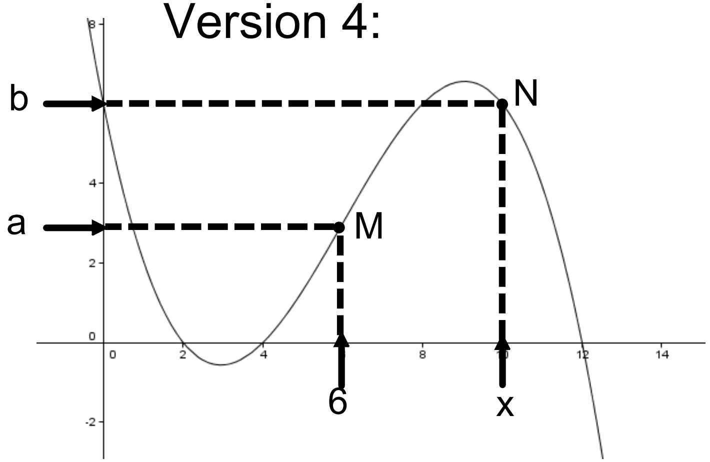
Version 2:



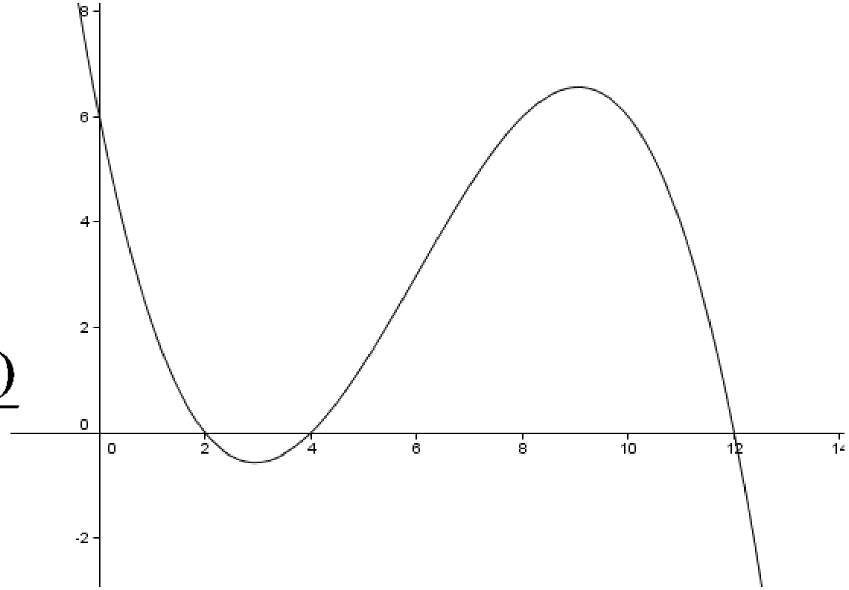
Version 3:



Version 4:



Does this function have one slope?



So, we can describe the slope in local areas.
For certain intervals.

$$Average\_rate\_of\_change = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

c implies a constant like, 7.

$$Average\_rate\_of\_change = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(7)}{x - 7}$$

curve for 23.ggb

This version is investigating
"from c to anywhere"

$$Average\_rate\_of\_change = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

Say f(x) =$2x^2$ - 2

Three uses:

1) Calculate the slope of the secant line for an interval, like [1,4]

$$ARC(1,4) = \frac{(2*4^2 - 2) - (2*1^2 - 2)}{4 - 1} = 10$$

2) Writing the equation for the slope of the secant line.

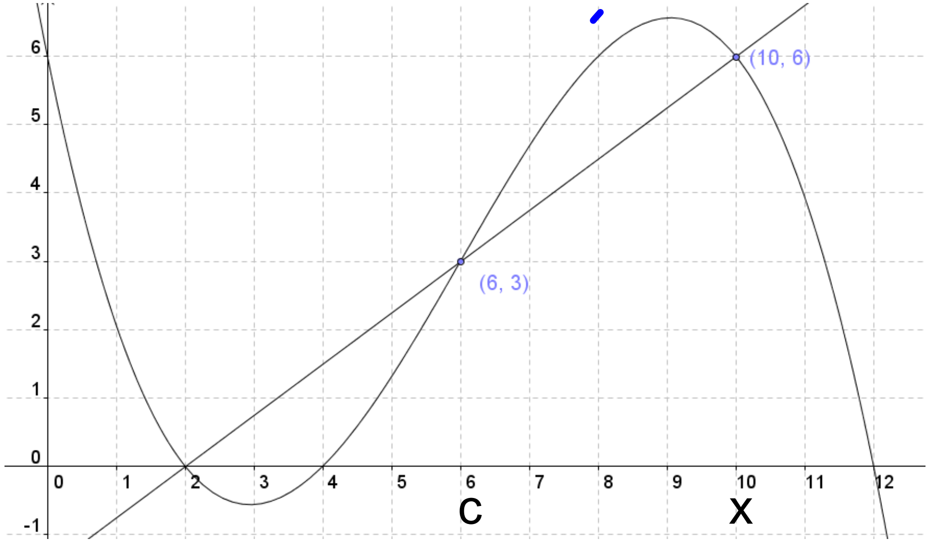
3) Write the secant line.

What is a secant line?

The secant line is a line connecting any two points on a curve.

The average rate of change formula calculates the slope of the line for a certain interval.

$$ARC = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$



The ARC formula let's you find the slope a particular secant line.

For all examples $f(x) = 2x^2 - 2$

Three different questions

1) For the interval [1,4] find the **slope** of the secant line for f(x).
Plug in to the ARC formula

$$RC(1,4) = \frac{f(4) - f(1)}{4 - 1} =$$

$$ARC(1,4) = \frac{(2*4^2 - 2) - (2*1^2 - 2)}{4 - 1} = 10$$

2) For an interval starting at c, [c,x], find the **equation for the slope of the secant line**.

(where "c" can be any number let's use 1)

Apply ARC for the interval from [1,x]

$$ARC(1,x) = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

$$ARC(1,x) = \frac{(2x^2 - 2) - (2(1)^2 - 2)}{x - 1}$$

$$= \frac{2x^2 - 2}{x - 1} = \frac{2(x+1)(x-1)}{x - 1} = 2(x+1)$$

So for [1,4] Slope/ARC = 2(4+1)=10

B) Finding **the equation of the secant line** for f(x) over [1,4]

- Find the slope (see first question)

y=mx+b
y=10x+b

Plug in a point (1,f(1)) = (1,0) or
(4,f(4))=30

0=10(1)+b
-10=b

Equation of the secan line: Y=10x-10

For all examples $f(x) = 2x^2 - 2$

1) For the interval [1,4] find the **slope** of the secant line for f(x).
Plug in to the ARC formula

$$RC(1,4) = \frac{f(4) - f(1)}{4-1} =$$

$$ARC(1,4) = \frac{(2*4^2-2)-(2*1^2-2)}{4-1} = 10$$

2) For an interval starting at c, [c,x], find the **equation for the slope of the secant line**.

(where "c" can be any number let's use 1)

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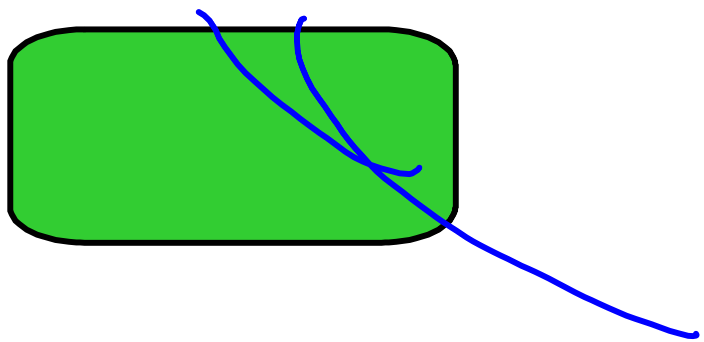


For $f(x) = 2x^2 - 2$

1) For the interval [1,4] find the **slope** of the secant line for f(x).
Plug in to the ARC formula

$$ARC = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

$$ARC(1,4) = \frac{f(4) - f(1)}{4 - 1} =$$

$$ARC(1,4) = \frac{(2*4^2 - 2) - (2*1^2 - 2)}{4 - 1} = 10$$

$M(x) = 4x^2 - 22x$

$[3, x]$

$$\frac{f(x) - f(3)}{x - 3}$$

$$\frac{4x^2 - 22x - (4 \cdot 3^2 - 22 \cdot 3)}{x - 3}$$

$$\frac{4x^2 - 22x + 30}{x - 3}$$

$4x^2 - 12x - 10x + 30$

$4x(x - 3) - 10(x - 3)$

$$\frac{(4x - 10)(x - 3)}{x - 3}$$

$4x - 10$

The slope of the secant line answers the question:

Over the interval in question is the curve, on average, going up or down and at what rate.

$$f(x) = x^3 - 9x$$

Find the equation for the slope of the secant line for the interval [3,x]

Find the slope of the secant line [3,6]

$$Arc[3,x] = \frac{(x^3 - 9x) - (3^3 - 9*3)}{x-3} =$$

$$\frac{x^3 - 9x - 27 + 27}{x-3} = \frac{x^3 - 9x}{x-3}$$

$$Arc[3,6] = \frac{x^3 - 9x}{x-3} = \frac{6^3 - 9*6}{6-3} = \frac{216-54}{3} = 54$$

Vocab note:

$f(x)=mx$

$b = 0$ can be interpreted four equivalent way.

- $f(x) = mx$

- $f(x)$ is a linear relation where both the y intercept and the x intercept are (0,0)

- The y is directly proportional to x.

- Y varies directly with x.

$f(x) = 3x$

x	y
0	0
1	3
4	12
-2	-6

Question #2) When does it go up, down, stay the same, and turn around?

- **Increasing and Decreasing Functions**

Definitions:

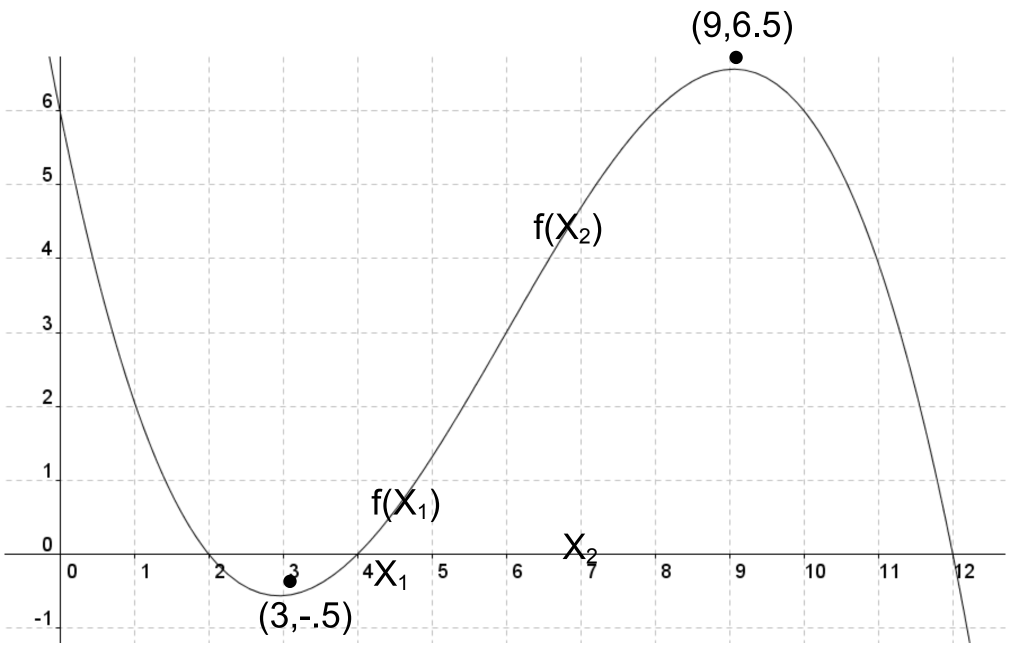
(i) A function f is **increasing** on an open interval I if, for any choice $x_1 < x_2$, $f(x_1) < f(x_2)$.

(ii) A function f is **decreasing** on an open interval I if, for any choice $x_1 < x_2$, $f(x_1) > f(x_2)$.

(iii) A function f is **constant** on an open interval I if, for all choices x in I, the values f(x) are equal.

(iv) A function f has a **local maximum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) < f(c)$.

(v) A function f has a **local minimum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) > f(c)$.



Over what intervals
is this increasing or decreasing?

Increasing

$x_1 < x_2$, $f(x_1) < f(x_2)$.

Decreasing

$x_1 < x_2$, $f(x_1) > f(x_2)$.

Local maxima?
Local minima?

curve fo

Do page 4, page 5 do #4 on top

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curve for 23 x2.ggb

2-3 arc examples.ggb