PROPERTIES OF FUNCTIONS (PC41)

- <u>***Average Rate of Change in Context***</u>

Case 1: The Average Rate of Change is Constant

You were visiting a college 275 miles away from Fairfield and start driving home at the constant rate of 55 mph. Make a sketch of the situation and write a function that relates **D** (your distance from home in miles) and **t** (the time in hours).

a) __$(275 - 55t) = d$

b) The average rate of change of **D** with respect to **t**, written as $\frac{\Delta D}{\Delta t}$, is referred to as the car's _speed_____.

c) When the average rate of change is **constant**, the function is _a linear relation__.

Case 2: The Average Rate of Change is Not Constant

An object is thrown upwards from an initial height of 10 feet and an initial velocity of 80 feet per second.

a) Write a function that describes the height **h** (in feet) of the object, as a function of time **t**, in seconds.

__$h(t) = -16t^2 + 80t + 10$_______

b) Use your algebraic skills to find the coordinates of the vertex and intercepts. Make a sketch that reflects this important information.

{By completing the square}
Vertex = (2.5,110) (0,10), (-.1220,0) {nonsense}, (5.1220,0)

c) Find the average rate of change $\frac{\Delta h}{\Delta t}$ in the interval $[0, 2]$ and then in the interval $[2, 4]$.

48 , -16

d) Explain the meaning of your results in item (c). _______________ _____The ball goes up 48 feet per second from 0 to 2 seconds and descends at 16 feet per second from 2 to 4 seconds._____

Average Rate of Change

If $(x\ , f(x))$ and $(c, f(c))$ are two points on the graph of a function $y = f(x)$, the average rate of change from x to c is defined as

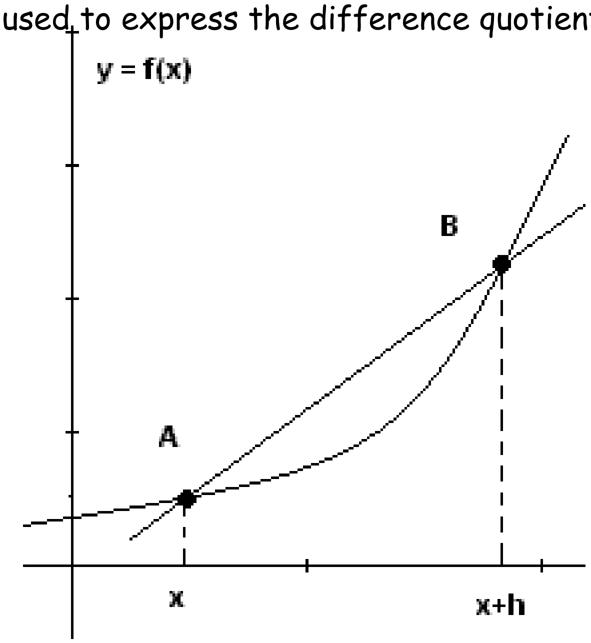
$$\boxed{\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}}$$

This expression is also called the **difference quotient**.

- **Geometric Interpretation:** The average rate of change of a function equals the slope of the _secant______ line.

Below is another formula commonly used to express the difference quotient:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$



Examples:

1) Consider $f(x) = x^3 - 9x$.

   a) Find a formula that gives the slope of the secant line (difference quotient) containing any 2 points $(x, f(x))$ and $(x+h, f(x+h))$.

$$\frac{((x+h)^3-9(x+h))-(x^3-9x)}{h} = \frac{x^3+3x^2h+3xh^2+h^3-9x-9h-x^3+9x}{h} = \frac{3x^2h+3xh^2+h^3-9h}{h}$$

$=3x^2+3xh+h^2-9$

   b) Use the formula above to find m_{sec} for h = 0.5 at x = 1.

-4.25

- **<u>Increasing and Decreasing Functions</u>**

Definitions:

(i) A function f is **increasing** on an open interval I if, for any choice $x_1 < x_2$, $f(x_1) < f(x_2)$.

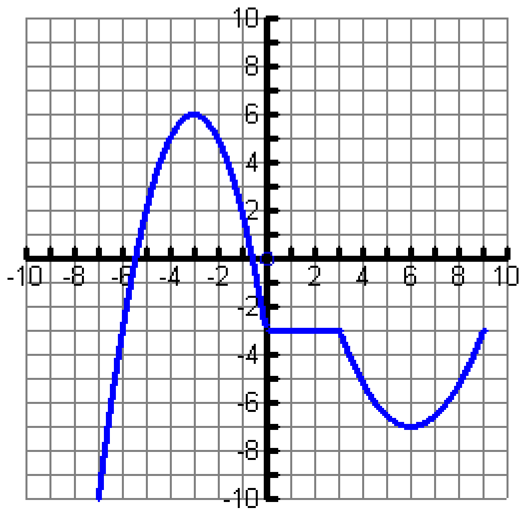
(ii) A function f is **decreasing** on an open interval I if, for any choice $x_1 < x_2$, $f(x_1) > f(x_2)$.

(iii) A function f is **constant** on an open interval I if, for all choices x in I, the values f(x) are equal.

(iv) A function f has a **local maximum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) < f(c)$.

(v) A function f has a **local minimum** at c if there is an open interval I containing c so that, for all $x \neq c$ in I, $f(x) > f(c)$.

2) Use interval notation to answer the questions below.



a) Intervals where f is increasing ___(-7,-3),(6,9)_____________

b) Intervals of decrease _____(-3,0),(3,6)_______________

c) Intervals where f is constant ______(0,3)________

d) What are the local maximum values of f. Specify at what x-values these local max occur.

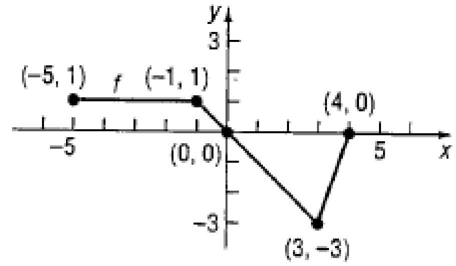
________f(-3)=6 ________________________________

e) What are the local minimum values? _______f(6)=-7_______

f) Domain of f: ______[-7,9]_________________________________

g) Range of f: _______[-10,6]____________________________

3) Use the graph of the function f below to find: (use interval notation where applicable)



a) The domain of f __[-5,4]________________

b) The range of f __[-3,1]_________________

c) What is f(-1)? ____1_______________

d) The x- and y-intercepts of f ____(0,0),(4,0)_______

e) For what values of x does f(x) = -3? _3_______

f) For what values of x is $f(x) < 0$? _(0,4)___________

g) For what values of x is $f(x) \geq 0$? [-5,0]u[4,4]

4) Using your calculator, graph the function. Locate intervals of increase/decrease and all local extrema.

$$f(x)=-0.4x^3 + 0.6x^2 + 3x - 2$$

Max (2.16,3.25) Min (-1.16,-4.05) Decreasing from (-∞,-1.16) and (2.16,+∞)
Increasing from (-1.16,2.16)

EVEN AND ODD FUNCTIONS

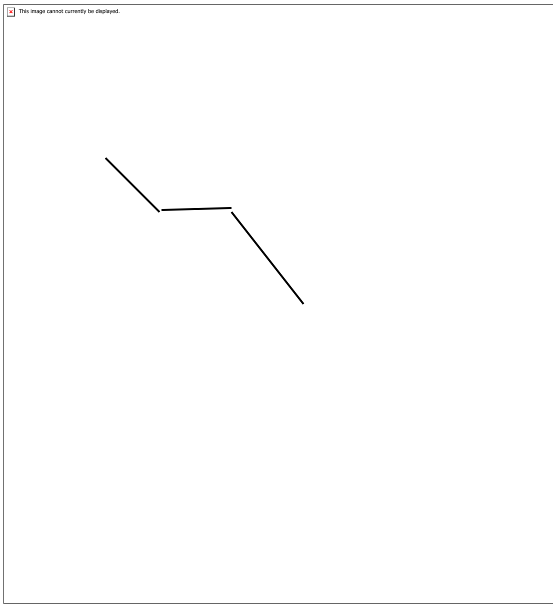
- A function is **even** if and only if whenever the point (x, y) is on the graph, the point $(-x, y)$ is also on the graph.
  Algebraically: f is even $\leftrightarrow f(-x) = f(x)$

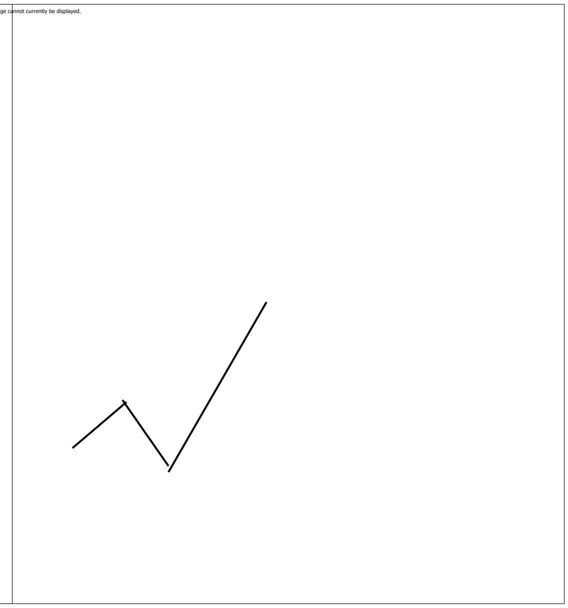
  Note: Even functions have **reflectional symmetry** about the y-axis.

- A function is **odd** if and only if whenever the point (x, y) is on the graph, the point $(-x, -y)$ is also on the graph.
  Algebraically: f is odd $\leftrightarrow f(-x) = -f(x)$

  Note: Odd functions have **rotational symmetry** about the origin.

5) Complete the graph on the left to make it into the graph of a function that is even, and the graph on the right to make it into the graph of an odd function.





7) Verify algebraically whether the functions below are even, odd or none.

a) $f(x) = \dfrac{5}{x^2 + 3}$

Even

b) $f(x) = 2x^3 + x + 1$

niether

8) A ball is thrown vertically up from an initial height of 6 ft with an initial velocity of 80 feet per second.

a) Find a function h* that relates time **t**, in seconds, to the height from the ground.

h(t) =_$-16t^2 + 80t + 6$____________________

** The general model for an object that is thrown up from an initial height of* h_o *feet at an initial velocity of* v_o *ft/sec is*

$$h(t) = -16t^2 + v_o t + h_o$$

b) Find the average rate of change of the height of the ball with respect to time (*which is the velocity of the ball…)* between t=0 and t=**2** seconds.

48 feet per second

c) Find the equation of this secant line. Graph f and this secant line. Suggestion for window: use for X [0, 6] and for Y [-10,120].

Y=48x-6

d) What is the velocity of the ball between x = 3 and x = 4. Compare this with your answer to item b.

-32 feet per second

f) What was the maximum height attained by the ball? How would you find this information algebraically?

(2.5,106) Average the x intercepts and plug the value in or find vertex form

g) Explain what happens to the velocity of the ball after the ball reaches its maximum height.
The velocity goes from positive to negative

h) When did the ball hit the ground? Explain (algebraically) how you reached your conclusion here.
After 5.07 seconds, I found the x intercept.