FUNDAMENTAL THEORY OF ALGEBRA

Every complex polynomial P(x)

of degree n>=1 has exactly **n** zeros

(provided a zero of multiplicity **m** is counted as **m** zeros),

and can be factored into **n** linear factors of the form

CONJUGATE PAIRS THEOREM 1)

If P(x) is a polynomial with **real** coefficients,

and a+bi is an imaginary root of the equation P(x) = 0,

a-bi then is also a root.

CONJUGATE PAIRS THEOREM Variations For Example:

If P(x) is a polynomial with **real** coefficients,

and a+b(c)^.5 is a root of the equation P(x) = 0,

a-b(c)^.5 then is also a root.

Complex polynomials
(allows for complex coefficients)

All our real rules are included within these.

Can be written as the product of <u>complex</u> linear factors.
(x-(a+bi))
which are real if b=0

When a+bi is a zero
(x-(a+bi)) is a factor

When (x-(a+bi)) is a factor
(x-(a-bi)) is a factor.

When a+bi is a zero
a-bi is a zero.

When the degree is odd,
then there is at least one real zero.

Give me 2 reasons why.

$x^2-2ax+(a^2+b^2)$ is a factor
(this is an irreducible quadratic)

When $(x-(a+bi))$ is a factor $(x-(a-bi))$ is a factor.*

$x^2-2ax+(a^2+b^2)$ is a factor
(this is an irreducible quadratic)

The sum of the squares of the real and imaginary parts

$x^2-2ax+(a^2+b^2)$

The opposite of' double the real part.

3-4i=>
3+4i

$(x-(3-4i))(x-(3+4i))$

$= x^2-6x+25$

*As long as b<>0

$$(x - (a-bi))(x - (a+bi))=$$

$$x^2-2ax+(a^2+b^2)$$

2-4i is a root.

Then the polynomial is divisible by $x^2-4x+20$

The next two pages are titled either:

"Some Neat Relationships with Quadratic Forms"
or
"Mr. Benjamin Needs to Get a Life"

$(x - (a-bi))(x - (a+bi))=$

$x^2-2ax+(a^2+b^2)=$

$(x-a)^2+b^2$

$(x - (2-3i))(x - (2+3i))=$

$-2*2$ $\quad 2^2+3^2$

$x^2-4x+13=$

$2^{1/2}$ $\quad 13-2^2$

$(x-2)^2+9$

$(x - (2-3i))(x - (2+3i))=$

3^2

$(x-2)^2+9$

$(x-a)^2+b^2=$

$x^2-2ax+(a^2+b^2)=$

$(x - (a-bi))(x - (a+bi))=$

$(x-3)^2+16=$

$2*3$ $\quad 3^2+16$

$x^2-6x+25=$

$6/-2$ $\quad ((6/-2)^2-25)^{1/2}$

$(x-(3+4i))(x-(2-4i))$

$(x-3)^2+16=$

$(-16)^{1/2}$

$(x-(3+4i))(x-(3-4i))$

$$(x-a)(x-m)=$$
$$x^2-(a+m)x+am=$$
$$\left(x-\frac{(a+m)}{2}\right)^2+\left(am-\left(\frac{(a+m)}{2}\right)^2\right)$$

(x - 1)(x +5)=

-1+5 -1*5

x^2+4x-5=

4/2 -5-$(4/2)^2$

$(x+2)^2$-9

(x - 1)(x +5)=

(-1+5)/2 -1*5-$((-1+5)/2))^2$

$(x+2)^2$-9

$$(x-b)^2+n=$$
$$x^2-2bx+(n+b^2)=$$
$$\left(x-\left(b+\sqrt{-n}\right)\right)\left(x-\left(b-\sqrt{-n}\right)\right)$$

$(x+2)^2$-9=

2*2 -9+2^2

x^2+4x-5=

Guess

(x-1)(x+5)

$(x+2)^2$-9=

-2-$9^{1/2}$ -2+$9^{1/2}$

(x-1)(x+5)

$(x-3)^2+16=$

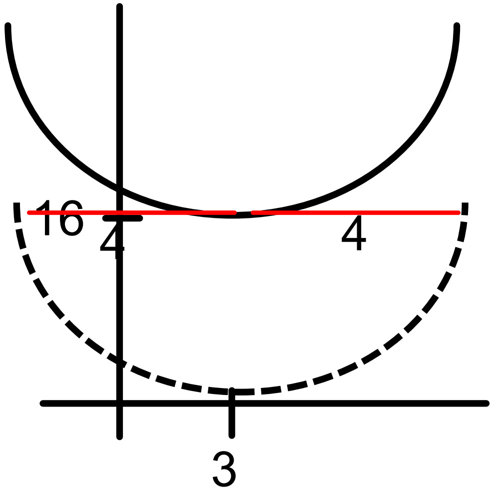
$(-16)^{1/2}$

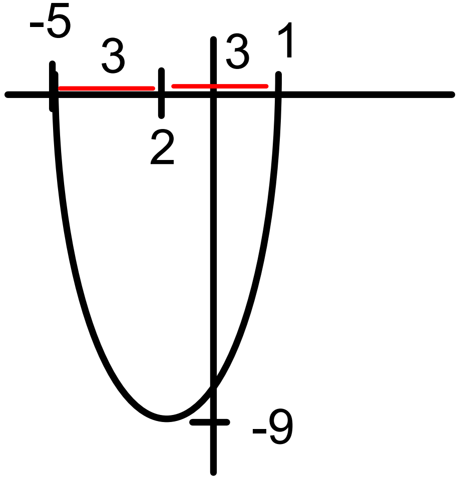
$(x-(3+4i))(x-(3-4i))$

$(x+2)^2-9=$

$-2+9^{1/2}$ $-2-9^{1/2}$

$(x-1)(x+5)$





You are responsible for the theorems in the following way:

- You must be able to apply them
- You must be able to name which one your using

$x^4-7x^3+14x^2-38x-60$. One root is $1+3i$

Then $1-3i$ is a root and $x^2-2x+10$ is a factor

$$\begin{array}{r} x^2-5x-6 \\ x^2-2x+10 \overline{)\, x^4-7x^3+14x^2-38x-60} \\ x^4-2x^3+10x^2 \\ \hline -5x^3+4x^2-38x-60 \\ -5x^3+10x^2-50x \\ \hline -6x^2+12x-60 \\ -6x^2+12x-60 \\ \hline 0 \end{array}$$

Degree 4: Known zeros of $4+i$ and $-i$

Remaining zeros?

Form the polynomial (in expanded form)

Degree 6: Known zeros: i, $4-i$, $2+i$

$(x^2+1)(x^2-8x+17)(x^2-4x+5)=$

$(x^2+1)(x^4-4x^3+5x^2-8x^3+32x^2-40x+17x^2-68x+85)=$

$(x^2+1)(x^4-12x^3+54x^2-108x+85)=$

$x^6-12x^5+54x^4-108x^3+85x^2+x^4-12x^3+54x^2-108x+85=$

$x^6-12x^5+55x^4-120x^3+139x^2-108x+85$

$f(x)= 2x^4+x^3-35x^2-113x+65$

+/-65, +/-13, +/-5, +/-1, +/-65/2, +/-13/2, +/-5/2, +/-1/2

5|	2	1	-35	-113	-65
		10	55	100	-65
	2	11	20	-13	

$(x-5)(x-1/2)(2x^2+12x+26)=$

$(x-5)(2x-1)(x^2+6x+13)=$

$(x-5)(2x-1)(x-(-3+2i))(x-(-3-2i))$

$f(x)= 2x^3+11x^2+20x-13$

1/2|	2	11	20	-13
		1	6	13
	2	12	26	