Objectives:

- Learn proper terminology for elements of polynomial division
- Learn a theorem that gives a new way to calculate f(#)

When we divide, life is easy if the divisor goes into the dividend nicely.
But remainders happen.
Lets look at division a relationship of 4 quantities.

## Different ways to express the same relationship:

Dividend

Divisor

$$\frac{13}{3} = \frac{4*3+1}{3} = 4 + \frac{1}{3}$$

Remainder

Quotient

OR

$$\frac{13}{3} = 4 + \frac{1}{3}$$

or

$$13 = 4*3+1$$

Remainder

Dividend

Quotient

Divisor

Represent dividing 21 and 4 in this way

When we divide, life is easy if the divisor goes into the dividend nicely.
But remainders happen.
Lets look at division a relationship of 4 quantities.

If we were as familiar with polynomials as we are integers, we could see rational functions in this way:

$$\frac{3x^2-8x}{x-2}=\frac{3x^2-6x-2x}{x-2}=\frac{3x(x-2)-2x}{x-2}=3x-\frac{2x}{x-2}$$

$$\frac{3x^2-8x}{x-2}=3x-\frac{2x}{x-2}$$ OR (rewrite this one)

$$\frac{f(x)}{g(x)}=q(x)+\frac{r(x)}{g(x)} \quad \text{or} \quad \underset{\text{dividend}}{f(x)}=\underset{\text{quotient}}{q(x)}\,\underset{\text{divisor}}{g(x)}+\underset{\text{remainder}}{r(x)}$$

$$\frac{3x^2-8x}{x-2}=3x-\frac{2x}{x-2}$$

$$3x^2-8x=3x\left(x-2\right)-2x$$

What is the remainder?

### Division Algorithm for Polynomials

If $f(x)$ and $g(x)$ denote polynomial functions and if $g(x)$ is not the zero polynomial, then there are unique polynomial functions $q(x)$ and $r(x)$ such that

$$\boxed{\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad \underset{\text{dividend}}{f(x)} = \underset{\text{quotient}}{q(x)}\,\underset{\text{divisor}}{g(x)} + \underset{\text{remainder}}{r(x)} \qquad (1)}$$

where $r(x)$ is either the zero polynomial or a polynomial of degree less than that of $g(x)$.

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad \underset{\text{dividend}}{f(x)} = \underset{\text{quotient}}{q(x)}\underset{\text{divisor}}{g(x)} + \underset{\text{remainder}}{r(x)}$$

If f(x)=4x2 - 12x

What is f(3)?

$$\frac{4x^2-12x}{x-3} = \frac{4x(x-3)}{x-3} = 4x$$

$$4x^2-12x = 4x(x-3)$$

So, if c is a zero of f(x), then f(c) = 0. Well, yeah.
If c is a zero, then (x-c) is a factor of f(x). Well, yeah.
If c is a zero, then dividing by (x-c) has no remainder.

What if c is not a zero?

New Problem: If $f(x)=4x^2 - 12x + 4$

What is $f(3)$?

Go ahead.

$$\frac{4x^2-12x+4}{x-3}=4x+\frac{4}{x-3}$$

$$4x^2-12x+4=4x(x-3)+4$$

What???????

If I ask you, "What is f(#)?", then one way to find out is to:
- Divide by (x-#)
- Take the remainder

If f(x)=$4x^2$ - 12x + 4

What is f(5)?

$$
\begin{array}{r}
4x+8 \\
x-5\overline{\smash{)}4x^2-12x+4} \\
\underline{4x^2-20x} \\
8x+4 \\
\underline{8x-40} \\
+44
\end{array}
$$

Divide and discover:

$$f(x)=4x^2-12x+4=\left(4x+8\right)\left(x-5\right)+44$$

So, when we plug 5 in on the right:

$$f(x)=4\left(5\right)^2-12\left(5\right)+4=\left(28\right)\left(0\right)+44$$

We can either plug 5 in or divide to get the answer

If I ask you, "What is f(#)?", then one way to find out is to:
- Divide by (x-#)
- Take the remainder

What is f(2) for $3x^2-8x$?

$$
\begin{array}{r}
3x-2 \\
x-2 \overline{\smash{\big)}\,3x^2-8x} \\
\underline{3x^2-6x} \\
-2x \\
\underline{-2x+4} \\
-4
\end{array}
$$

$3x^2-8x=(3x-2)(x-2)-4$

Why?

There is a function:

$$(3x-2)(x-2)$$

For which 2 is a zero.

By how much does our original function miss this functions by?

The remainder.

Put another way, what is the difference between this function and the original function? => The remainder

Objective
- Learn rules to help find zeros
- Gain skill at using rules and synthetic division to quickly find zeros.

$F(X)=2X^3-2X^2-82X+210$ $= (x-5)(2x^2+8x-42)$

5? => (x-5) => to synthetic division

f(x)=2(x-5)(x-3)(x+7)

Switch signs

| | | | | |
|---|---|---|---|---|
| | 5 \| 2 | -2 | -82 | 210 |
| | | 10 | 40 | -210 |
| 2*5 | 2 | 8 | -42 | 0 |

$2x^2+8x-42$ no remainder

$2x^2+8x-42=2x^2+14x-6x-42$

$=2x(x+7)-6(x+7)=(2x-6)(x+7)=2(x-3)(x+7)$

A) A biconditional:

1) If $f(c)=0$, then $x-c$ is a factor of $f(x)$
2) If $x-c$ is a factor of $f(x)$ then $f(c) =0$

Use: If you find a zero, can be divided out to create a simpler polynomial for further study.

B) The degree of the polynomials tells you the maximum number of real zeros.
Use: You know when to stop.

C) IF A POLYNOMIAL FUNCTION HAS INTEGER COEFFICENTS
then make two lists:
1) factor the constant (call these p's)
2) factor the leading coefficient (call these q's)
3) make all the +/- fractions you can putting the p's/q's
ALL YOU REAL RATIONAL ZEROS ARE IN THIS LIST.
Use: It gives you all the possibilities.
- A useful list
- A guide on window size

D) IF THE LEADING COEFFICIENT OF A POLYNOMIAL IS 1
then all the zeros are between
-M to +M where M is
The smaller of
o The sum of the absolute value of the coefficients
and
o 1 + the coefficient with the biggest absolute value
Use: You know how big a window to set on the calculator.

## E) End behavior and turning points are still useful.

## F) Intermediate value theorem to find irrationals.

$f(x) = 2x^4+2x^3-14x^2-2x+12$

factors of 12 {1,2,3,4,6,12}
factors of 2 {1,2}

p/q's => $\{\pm 12, \pm 6, \pm 4, \pm 3, \pm\frac{3}{2}, \pm 2, \pm 1, \pm\frac{1}{2}\}$

Everything is even, I like 2.

| | | | | | |
|---|---|---|---|---|---|
| 2 \| | 2 | 2 | -14 | -2 | 12 |
| | | 4 | 12 | -4 | 0 |
| | 2 | 6 | -2 | -6 | |

$2x^3+6x^2-2x-6$

$2x^3+6x^2-2x-6$

I like 3 or (x-3)

| | | | | |
|---|---|---|---|---|
| 3 \| | 2 | 6 | -2 | -6 |
| | | 6 | | |
| | 2 | 12... I don't like 3. | | |

Try again.....

$2x^3+6x^2-2x-6$

I like -3 or (x+3)

| | | | | |
|---|---|---|---|---|
| -3 \| | 2 | 6 | -2 | -6 |
| | | -6 | 0 | 6 |
| | 2 | 0 | -2 | 0 |

$2x^2-2=2(x^2-1)=2(x+1)(x-1)$

$f(x)=2(x-2)(x+3)(x+1)(x-1)$

$$F(X)=2X^3-2X^2-82X+210$$

factors of 210=>{1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210}

factors of 2=>{1, 2}

{+/-1, +/-2, +/-3, +/-5, +/-6, +/-7, +/-10, +/-14, +/-15, +/-21, +/-30, +/-35, +/-42, +/-70, +/-105, +/-210, +/-1/2, +/-3/2, +/-5/2, +/-7/2, +/-15/2, +/-21/2, +/-35/2, +/-105/2}}

5? => (x-5) => to synthetic division

| | | | |
|---|---|---|---|
| 5 \| 2 | -2 | -82 | 210 |
| | 10 | 40 | -210 |
| 2 | 8 | -42 | 0 |

$2x^2+8x-42$

$$f(x) = x^5 - x^4 - 9x^3 + 13x^2 + 8x - 12$$

12 6 4 3 2 1

$$\begin{array}{r|rrrrrr} -1 & 1 & -1 & -9 & +13 & +8 & -12 \\ & & -1 & 2 & 7 & -20 & +12 \\ \hline & 1 & -2 & -7 & 20 & -12 & 0 \end{array}$$

$$(x+1)(x^4 - 2x^3 - 7x^2 + 20x - 12)$$

$$\begin{array}{r|rrrrr} 1 & 1 & -2 & -7 & +20 & -12 \\ & & 1 & -1 & -8 & 12 \\ \hline & 1 & -1 & -8 & 12 & 0 \end{array}$$

$$(x+1)(x-1)(x^3 - x^2 - 8x + 12)$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -8 & +12 \\ & & 2 & 2 & -12 \\ \hline & 1 & +1 & -6 & 0 \end{array}$$

$$(x+1)(x-1)(x^2 + x - 6)(x-2)$$

$$(x+1)(x-1)(x-2)^2(x+3) = f(x)$$

Find the k such that $x^4 - kx^3 + kx^2 + 1$ has a factor of $x+2$

| | | | | | |
|---|---|---|---|---|---|
| -2 \| | 1 | -k | +k | +0 | +1 |
| | | -2 | 2k+4 | -6k-8 | 12k+16 |
| | 1 | -k-2 | 3k+4 | -6k-8 | 12k+17 |

12k+17 = 0

k=-17/12

$$x^3 + \frac{-7}{12}x^2 + \frac{-3}{12}x + \frac{6}{12}$$

$$x+2 \,\big)\, x^4 + \frac{17}{12}x^3 - \frac{17}{12}x^2 + 1$$

$$\underline{x^3 + 2x^3}$$

$$\frac{-7}{12}x^3$$

$$\underline{\frac{-7}{12}x^3 + \frac{-14}{12}x^2}$$

$$\frac{-3}{12}x^2$$

$$\underline{\frac{-3}{12}x^2 + \frac{-6}{12}x}$$

$$\frac{6}{12}x$$

$$\underline{\frac{6}{12}x + \frac{12}{12}}$$

$$0$$

$$x^3 + (-k-2)x^2$$

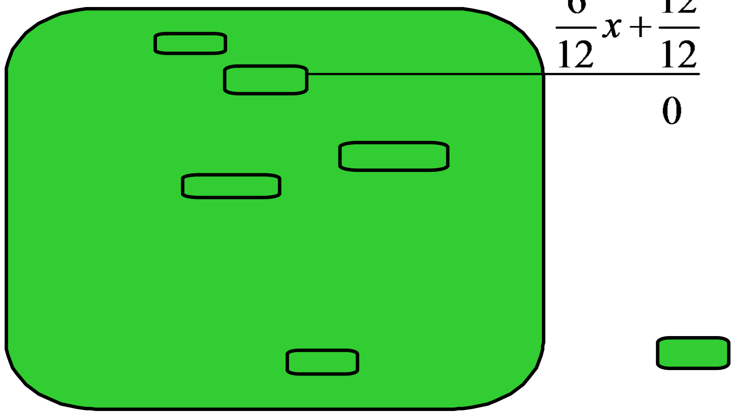
$$x+2 \,\big)\, x^4 - kx^3 + kx^2 + 1$$

$$\underline{x^3 + 2x^3}$$

$$(-k-2)x^3$$

$$\underline{(-k-2)x^3 + (-2k-4)x^2}$$

$$(3k+4)x^2$$



What are the zeros of:

$x^2+1$?

$x^2+2x+20$?

$x^3+3x^2+4x+12$?