4.2 EXPONENTIAL FUNCTIONS

Definition: An exponential function is a function of the form

$$f(x) = a^x, \text{ where } a > 0 \text{ and } a \neq 1$$

Where the **domain** of f is the set of all real numbers.

1) Why must **a** be positive? ____when a<0, a^x is not defined for all x._____
2) Why must $a \neq 1$? ___This is simply y=1________________

3) Graph the functions: $y = 2^x$, $y = 3^x$, $y = \left(\frac{3}{2}\right)^x$. These graphs share some common characteristics. Let's see…

a) What are the X- and Y- intercepts?**No x-intercept, y-intercept=(0,1)**
b) What is the range of f? __**{y|y>0}**_________
c) Describe f as an increasing or decreasing function. _**Increasing**
d) Is f one-to-one or many-to-one? _**one-to-one**_________
e) Describe the behavior of f as $x \to \infty$ and as $x \to -\infty$:
As x increases infinitely the functions become infinitely large.
As x decreases infinitely the functions become approach 0 from the positive side.
f) Identify any horizontal asymptotes: _**y=0**______________________
g) Are there any vertical asymptotes? _**No**________________________
h) Discuss "smoothness" and "continuity" of the graph of f. ____________
___**It is smooth and continuous.**______________________________

4) Graph the functions $y = \left(\frac{1}{2}\right)^x$, $y = \left(\frac{1}{3}\right)^x$, and $y = \left(\frac{2}{3}\right)^x$.

Observe that the **bases** of each of these functions are the **reciprocals** of the functions in #3.

a) Describe the behavior of f as $x \to \infty$ and as $x \to -\infty$: **As x <u>de</u>creases infinitely the functions become infinitely large.As x <u>in</u>creases infinitely the functions become approach 0 from the positive side.**
b) Identify a point common to the graphs of all three: _**(0,1)**_________
c) What is the horizontal asymptote? ___**y=0**_____________________

d) Identify the graphs as increasing/decreasing and exponential growth/decay. _**The first set are growth. The second set is decay.**______

e) What can you say when you compare the graphs of

$y = 3^x$ *and* $y = \left(\frac{1}{3}\right)^x$? ___**They are the same graph flipped over the y axis.**___

Now let's see if you are ready to generalize some key concepts…

5) The graph of every exponential function $f(x) = a^x, a > 0$, passes through three points: ___**(-1,1/a)**__, __**(0,1)**__ and ___**(1,a)**___.

6) The graphs of $f(x) = a^{-x}$ *and* $f(x) = \left(\frac{1}{a}\right)^x$ are identical. Why?

_**Yes, because $a^{-x}=(a^{-1})^x=(1/a)^x$**____________________.

7) If the graph of an exponential function $f(x) = a^x$ is decreasing, then **a** must be _**0<a<1**_____________.

8) If the graph of $y = k \cdot a^x$ has y-intercept 5, what is the value of k?

K=5

9) If $f(x) = 2^x$, show that $f(x+1) = 2 \cdot f(x)$ $2^{x+1}=2^x 2^1=2*2^x$

TRUE/FALSE:

10) The graph of an exponential function $f(x) = a^x,\ a < 0$ is increasing.

False, f(x) for negative numbers is often undefined and not increasing.

11) The range of every exponential function $f(x) = a^x,\ a > 0$, is all real numbers.

False, only the positive numbers.

12) Every exponential function of the form $y = a^x$ is one-to-one and, therefore, has an inverse.

False, not true for a = 1

EXPONENTIAL FUNCTIONS AND TRANSFORMATIONS

5) Complete the table below using the graph of the function $f(x) = 2^x$ as your reference. Make a sketch and describe the transformation.

	Sketch	*Describe transformation*
$f(x) = 2^x$		*none*
$f(-x) =$ ________ or 2^{-x} or $(1/2)^x$ ________		Flipped over the y-axis
$f(x) + 1 =$ _______		Up 1
$f(x - 1) =$ ________		Shifted right one
$3 \cdot f(x) =$ ________		Vertically stretched

6) Describe the transformations that have taken place on the graph of $y = 2 \cdot 3^{x-1} + 4$ as compared to the graph of $y = 3^x$.

Right 1, Up 4, Steeper

<u>OPTIONAL:</u> THE NUMBER *e* ≈ 2.71828 18284 59045 23536….

The number *e* is sometimes called Euler's number after the Swiss mathematician Leonhard Euler. The number *e* is one of the most important numbers in mathematics. Let's see why…

11) Interest payments are calculated based on nominal annual rate and the number of payments a year. Interest payments for a nominal annual rate of 8% paid annually are 8%. Interest payments for a nominal annual rate of 8% paid semi-annually are 4% each. When you are paid more than once a year interest is compounded. Suppose you deposit $1000 in an investment account that pays you an 8% nominal annual rate. Write a function to give you the amount of money in the account according to the following scenarios:

<u>Amount after 1 year</u>

a) Compounding once a year.

_$f(x)=\$1000*(1+.08)^{x}$

b) Compounding monthly.

$f(x)=\$1000*(1+.08/12)^{12x}$

c) Compounding daily.

$f(x)=\$1000*(1+.08/365)^{365x}$

d) Compounding every second…

$f(x)=\$1000*(1+.08/3.15569e7)^{3.15569e7x}$

When the **number of times** the amount is compounded approaches infinity ($n \to \infty$), this is referred to as **continuous compounding.**

When the compounding is done continuously, the growth factor is closely related to the number e as follows:

12) Evaluate the expression $\left(1+\frac{1}{n}\right)^n$ for many different "very large" values of n. What do you get? Increasingly closer to e

13) In advanced mathematics, the above could be simply expressed by using the shorthand notation

$$\lim_{x\to\infty}\left(1+\frac{1}{n}\right)^n = \_\_\_\_\_\_\_\_\_\_$$

14) Now try figuring out what this expression will give you:

$$\lim_{x\to\infty}\left(1+\frac{.05}{n}\right)^n = e^{.05}$$

15) Does it make sense to replace the growth factor above by a number with base e????????? (*rhetorical question!)*

Now let's try some problems dealing with Euler's number…

16) You invest $10,000 in an account that pays you an annual rate of 7.5% compounded continuously.

a) Set up a model to represent the growth of your investment over time (measured in years).

$I(x)=\$10{,}000e^{.075t}$

b) How much money will you have in the account after 3 years?

$12,523.23

c) When will your account be worth $15,000?

5.4 years

17) Spreading of Rumors…

A model for the number of people N in a college community who have heard a certain rumor is

$$N = P\left(1 - e^{-.15d}\right)$$

Where P is the total population, d is the number of days since the rumor began. In a community of 1000 students, how many will have heard the rumor after 3 days?

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