4.4 PROPERTIES OF LOGARITHMS

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The laws of logarithms are closely related to the laws of exponents because the logarithmic function $y = \log_a x$ and the exponential function $y = a^x$ are inverses. Let's see…

Laws of Logarithms

Suppose that $\log_a M = x$ and $\log_a N = y$. Verify each of these laws and give an example to illustrate ($M, N, a > 0,\ a \neq 1$)

1) $\log_a M \cdot N = \log_a M + \log_a N$ (The log of a product = product of logs)

$$log_2 32 = log_2 4 + log_2 8$$

5=2+3

2) $\log_a \frac{M}{N} = \log_a M - \log_a N$ (log of a quotient = difference of logs)

$$log_3 \frac{27}{9} = log_3 27 - log_3 9$$

1=3-2

3) $\log_a M^k = k \cdot \log_a M$, for any real number k.

$$log_4 2^6 = 6 log_4 2$$

$$3 = 6\frac{1}{2}$$

4) $\log_a M = \log_a N \Leftrightarrow M = N$ (Equality of Logs)

$$log_3 27 = log_3 27$$

Furthermore…

5) $\log_a a^k$ = __k______ and 6) $a^{\log_a k}$ = ___k_____

Let's try a few... 

Write as a rational number, if possible, or as a single logarithm.

7) $\log_5 2 + \log_5 3 + \log_5 4$ $= log_5 24$

8) $\log_2 8 + \log_2 6 - \log_2 3$ $= log_2 16 = 4$

9) $\frac{1}{2} log_6 9 + \log_6 12$ $= log_6 36 = 2$

10) $\log_2 48 - \frac{1}{3}\log_2 27$ $= log_2 16 = 4$

11) $2\log_3 6 - 2\log_3 2$ $= log_3 9 = 2$

12) $\log A + 2\log B - 3\log C$ $= log \frac{A * B^2}{C^3}$

13) $\frac{1}{2}(\log M + \log N - \log P) = log \frac{(MNP)^{\frac{1}{2}}}{P}$

14) $\frac{1}{3}(2\log M - \log N - \log P) = log \frac{(MNP)^{\frac{2}{3}}}{NP}$

Find the value of X. (Use a calculator here...)

15) $\log X = 5.7419$ =551,950.33

16) $\log X = 0.7332 - 2$ =.0541

Express y in terms of x:

17) $\log y = 2\log x$ $y=x^2$

18) $\log y = \log 4 - 2\log x$ $y = \left(\frac{2}{x}\right)^2$

19) $\log y = -\log x$

Y=1/X

20) $\log y = 2\log x + \log 2$

$y=2x^2$

If $\log_8 3 = a$ *and* $\log_8 5 = b$, express the given logarithm in terms of a and b.

21) $\log_8 75$ a+2b 22) $\log_8 225$ 2a+2b 23) $\log_8 0.12$
.12=3/25=>a-2b

Solve.

24) $\log_2(x+2) + \log_2 5 = 4$ $\log_2 5(x+2)=4 \Rightarrow 2^4=5x+10 \Rightarrow 16=5x+10 \Rightarrow x=6/5$

25) $\log_3 x + \log_3(x-2) = 1$ $\log_3 x(x-2)=1 \Rightarrow 3^1=x^2-2x \Rightarrow 0= x^2-2x-3 \Rightarrow x=3$

CHANGE OF BASES

Some exponential equations can be easily solved without a calculator by changing both sides of the equation to the same base. For example,

Solve the equation $9^{2x} = 3\sqrt[3]{3}$ $(3^2)^{2x}=3*3^{1/2}=3^{3/2} \Rightarrow 4x=3/2 \Rightarrow x=3/8$

Others cannot be easily expressed as powers of the same base. These equations require that you take logarithms of both sides. You can use any base log, but base 10 and base e are the most useful because they are available in your . Let's try to solve a few… Express answers correct to 3 decimal places. (note: no graphical solution accepted here…)

26) $3^x = 12$ $\frac{ln12}{ln3} = x = 2.262$

27) $4^x = 20$ $\frac{ln20}{ln4} = x = 2.161$

28) $e^{.05t} = 3$ ln3=.05t=>21.972

29) $30(5)^x = 120$ $\frac{ln4}{ln5} = x = .861$

30) $e^{x-1} = 3$ x=2.099

31) $\ln x + \ln(x+3) = \ln 10$
$x^2 + 3x = 10$ x= -5 or 2=> x=2

32) The value of a new $30,000 car depreciates 30% per year. What is its value after 3 years? (use the continuous growth/decay formula).

$30{,}000 * .7^3 = 10{,}290$ or $30{,}000e^{-.3567t} = 10{,}390$

33) A radioactive element has a half-life(*) of 6 years. If you have 10 kg of the element now, how much will be left after 2 years?

$.5 = e^{r6}$ $\ln(.5) = r6$ $r = -.1155$

$10e^{-.1155*2} = 7.937$ kg or $10 * .5^{6/2} = 7.937$

(*) Half-life means that it takes 6 years for the element to decay to half or 50% of the original amount.

34) A $10,000 certificate of deposit at a certain bank will double in value in 9 years (those good old days…). Assuming a continuous exponential model, (i) Find the annual rate of interest; (ii) Give a formula for the accumulated amount t years after the investment is made, and (iii) how long does it take for the money to triple in value?

i) $2 = (1+r)^9 \Rightarrow 2^{1/9} - 1 = .0801\%$ ii) $10{,}000(1.0801)^t = TV$ or $10{,}000 * e^{.077t} = TV$

iii) $3 = e^{.077t} \Rightarrow 14.2647$

Last but not least…

The <u>CHANGE OF BASE</u> **formula** enables you to find the logarithm of a number in one base if logs in a different base are known.

$$\log_a c = \frac{\log_b c}{\log_b a}$$

Solve by changing from the given base to either common logs or natural logs:

35) $\log_7 120$

$$\frac{\ln(120)}{\ln(7)} = 2.4603$$

36) $\log_4 300$

$$\frac{\ln(300)}{\ln(4)} = 4.1144$$

37) Use the change of base formula to graph the function $y = \log_2 x$. Then, compare it with $y = 2^x$, by entering each in Y1 and Y2. ZOOM SQUARE and write your (obvious) comments below.

They are inverses