Pre-Calculus 41/0 – 4.2
Modeling Exponential Growth and Decay - Preliminary Concepts
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Name: ____

NATURAL GROWTH/DECAY MODELS - Many quantities such as savings accounts or populations grow (or decay) at a **constant percentage rate** per unit of time. Such growth is called **natural growth.**

In this assignment, use one of the following two models to describe natural growth or decay of a certain quantity, A, over time, t:

Growth: $A(t) = A_o(1+r)^t$ Decay: $A(t) = A_o(1-r)^t$

where A_o = initial amount present

(*) Note: If an initial population of 10 bacteria increase by 3.5% every day, write the model as $A(t) = 40(1.035)^t$ *, and not as* $A(t) = 40(1+.035)^t$

EXTRA 1) Suppose the United States population, starting at 3.9 million in 1790, had continued indefinitely to grow at a constant 3% annual rate.

a) Find a model to describe the growth of the U.S. population where t=0 represents the year 1790.

P(t) = 3.9*1.03^t________________________

b) Use your model to predict the U.S. population in 1890. Compare this with the actual population in 1890 which was 63 million.

3.9*1.03^100=74.9527mm_________

c) Would it be realistic to use this model to predict the US population in 2010? What would it be? How does this figure compare with the actual? _No, there are physical limitations to growth that were not relevant in 1790. The predicted value of 2,601.6810mm is greater than 309 mm.

DO 2) The revenue for the state of Georgia from 1990 to 2001 can be modeled approximately by the natural growth function $R(t) = 6.8(1.068)^t$, where t is years after 1990 and R(t) is measured in billions of dollars.

a) According to this model, what was the State of Georgia's revenue in 1990? __6.8 billion____________

b) What is the growth factor? __1.068___ What is the growth rate? ___6.8%_______

c) Use the model to predict the year in which Georgia's revenue reaches $10 billion. Explain how you use your graphing calculator to solve this problem.

Year is: 1996______________________

d) What is Georgia's revenue in the year 2013?

Revenue in 2013: ___$30.8779 billion ___

THE HALF-LIFE OF CAFFEINE

The **half-life** of any substance is the amount of time it takes the substance to decrease by 50% of its initial amount.

Example: If some radioactive substance decays at the rate of 1.3% every year, then its half-life is solved by picking any initial amount, let's say 10 mg, and solving the equation $5 = 10(0.987)^t$. *Remember that the decay factor is obtained by solving* $1 - r$ *which in this case is* $1 - 0.013$*!*

<u>DO</u> 3) The amount of caffeine in the bloodstream decreases at an hourly rate of approximately 11.5%.

a) A can of Red Bull energy drink contains about the same amount of caffeine as a cup of regular coffee, 100 mg. If you drink a Red Bull or a cup of coffee, how much caffeine will remain in your bloodstream after 2 hours? Show the model you use to answer this question.

Model: ____$f(x)=100(.885)^t$________

Solution: _____78.3225____________

b) What is the half-life of caffeine in a healthy adult? Show the model you use.

Model: ___$.5=.885^t$_________________

Answer: _5.6737_________

<u>EXTRA</u> 4) Many people use the over-the-counter medication ibuprofen for relief from muscle strains and joint pain. The amount of ibuprofen in an adult's bloodstream decreases at an hourly rate of approximately 29%, and a normal adult dose is 400 mg.

a) Find a natural growth model I(t) that gives the amount of ibuprofen in the bloodstream as a function of time t in hours.

___$I(t)=400(.71)^t$__________________

b) What is the half-life of ibuprofen? Show the equation you use.

_$.5=.71^t$_t=2.0238 hrs______

c) How much ibuprofen is in a person's bloodstream after 3 hours?

________143.1644 mg____________

PART II: Algebra Skills

Solve each exponential equation using the properties of exponents. Follow the example and strategy given below.

Strategy for solving exponential equations:

- ***Step 1**: Express each side of the equation as a power of the same base*
- ***Step 2**: Set the exponents equal and then solve*
- ***Step 3**: Check the answer*

Example: Solve a) $16^x = \frac{1}{4}$ b) $5^{4-x} = 25^{x-1}$

Step 1: a) $\left(4^2\right)^x = 4^{-1}$ b) $5^{4-x} = \left(5^2\right)^{x-1}$

$4^{2x} = 4^{-1}$ $5^{4-x} = 5^{2x-2}$

Step 2: $2x = -1$ $4 - x = 2x - 2$

$x = -\frac{1}{2}$ $x = 2$

Step 3: Check by substitution of the value found for x into the original equation…

<u>MANY OF THE SOLUTIONS SHOWN ARE LONG SOLUTIONS. WITH PRACTICE YOU SHOULD SEE THESE ANSWERS WITH FEWER STEPS</u>

<u>DO</u> 1) $3^x = \frac{1}{27}$

$3^x = \frac{1}{3^3} = 3^{=3}$

X=-3

<u>EXTRA</u> 2) $5^x = \sqrt{125}$

$5^x = (5^3)^{1/2} = 5^{3/2}$

x=3/2

<u>EXTRA</u> 3) $8^{2+x} = 2$

$(2^3)^{2+x} = 2^{6+3x} = 2^1$

1=6+3x

-5/3=x

4) $4^{1-x} = 8$

$(2^2)^{1-x} = 2^{2-2x} = 2^3$

2-2x=3

x=-1/2

EXTRA 5) $27^{2x-3} = 3$

$(3^3)^{2x-3} = 3^{6x-9} = 3^1$

6x-9=1

X=5/3

DO 6) $49^{x-2} = 7\sqrt{7}$

$(7^2)^{x-2} = 7^1 7^{1/2}$

$7^{2x-4} = 7^{3/2}$

2x-4=3/2

X=11/4

DO 7) $4^{2x+5} = 16^{x+1}$

$4^{2x+5} = (4^2)^{x+1}$

2x+5=2x+2

No solution

$\left(16^{1/2}\right)^{2x+5} = 16^{x+1}$

X+2.5=X+1

No solution

EXTRA 8) $25^{2x} = 5^{x+6}$

$5^{4x} = 5^{x+6}$

4x=x+6

x=2

EXTRA 9) $6^{x+1} = 36^{x-1}$

x+1=2x-2

x=3

DO 10) $3^{2x} - 6 \cdot 3^x + 9 = 0$

$3^{2x} - 2 * 3^1 * 3^x + 3^2 = 3^{2x} - 2 * 3^{1+x} + 3^2$

$= (3^x - 3^1)^2$

$3^x = 3^1$

X=1