4.8 Fitting Exponential, Logarithmic and Logistic Models to Data

Demo 1: Exponential Model

An accident at a nuclear power plant happened on February 1st, 2000, and left the surrounding area polluted with radioactive material that decays exponentially. The amount still present (in SU, safe units) was measured thereafter at bimonthly intervals as follows:

Month	April	June	August	October	December
SU	12.8	10.8	9.2	7.8	6.7

a) Use your calculator to find the exponential function of best fit for the amount A(t) of radioactive material (in SU) remaining after **t** months.

Model: _$A(t)=14.9830*e^{-.1620t}$_____

b) Suppose it will not be safe for people to return to the area until $A = 1\ su$. In what **month** of what **year** will this occur? Explain how you used your calculator to answer this question.

Month and Year: __October, 16 2016____

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DEMO 2: Logarithmic Model

When your blood pressure is measured, you receive a result of the form "122 over 74", which lies in the normal range for healthy adults. The larger of the two numbers is your systolic blood pressure, recorded in millimeters of mercury. The systolic blood pressure ***p*** of a healthy child is lower than that of an adult, and is known to be essentially of the form

$$p = a + b \cdot \ln w$$

Where ***w*** is the child's weight in pounds.

a) Use the experimental data below to determine the required numerical coefficients ***a*** and ***b***. Write the model below.

w	41	67	78	93	125
P	89	100	103	107	110

Model: __p(t)=17.9502+19.3775ln(x)_____

b) Use your model to find the expected systolic blood pressure of a healthy 55-pound child.

__95.6022 __________________________

Demo 3: Fitting Logistic Models to Data

Logistic functions were first used around 1840 to model human population growth by the Belgian mathematician and demographer P.F.Verhulst. He sought to use U.S. census population data for the first half of the nineteenth century to predict the future growth of the United States.

In this problem, we will use census data for the last half of the nineteenth century to attempt to project U.S. population growth during the twentieth century. We will then try different models of population growth to see which is the best fitting one. The following table lists population figures for the 1850-1890 period.

YEAR	U.S. Population (millions)
1850	23.192
1860	31.443
1870	38.558
1880	50.189
1890	62.980

a) Find the best-fitting logistic model for the 1850-1890 U.S. population data.

Model: _

p(t)= 307.67278/(1 + 12.03963$e^{-0.02828x}$)_____________________

where x is years with 1850 = 0

b) According to the logistic model, at what population level does the U.S. population appear to level off? _307.67278 million_____________

c) Using this model, predict the U.S. population in 1990 and 2000?

1990 Population: __250.20236 million_ Actual: 248,710,000

2000 Population: __262.270 million___ Actual: 281,421,906

d) According to the 1990 census, the U.S. population was 248,710,000. The 2000 census reported the U.S. population at 281,421,906. Compare these actual figures with the figures obtained in the model. Your comments will be needed in a little while…

The model predicted low values.

e) Find a continuous growth model to fit the U.S. population data for the years 1850 and 1890.

Model: __$j(x) = 23.74624\,(1.02496^x)$__ $j(x) = 23.74624\,(e^{.02466x})$

f) According to the continuous growth model, what is the rate of growth of the U.S. population? ___2.466% a year________

g) Using the continuous growth model, compute the U.S. population in 1990 and 2000.

1990 population according to continuous model: __749.435 million__
2000 population according to continuous model: __958.992 million__

h) How do the predictions with the Logistic model compare to the predictions with the continuous growth model? Write your comments below.

____The predictions are below and the above. __

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i) According to the logistic model, when will the U.S. population reach its carrying capacity (as described by the model)?

Never, ti approaches it asymptotically.