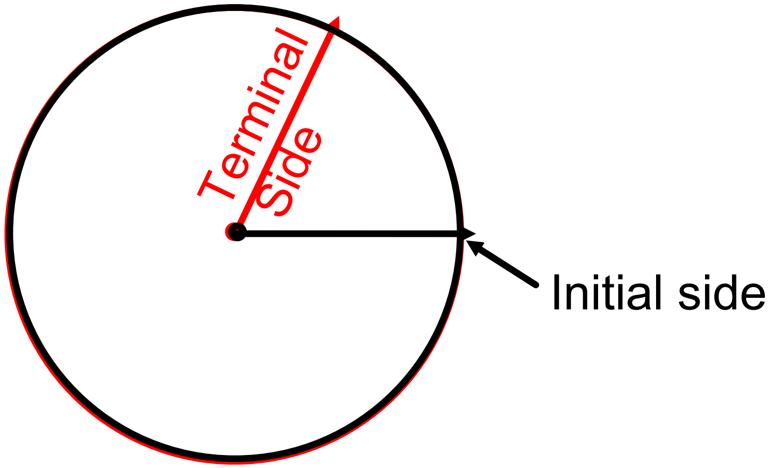
Be able to properly use the following terminology:

- Initial Side
- Terminal Side
- Positive Angle
- Negative Angle
- Standard Position
- Quadrantal Angles
- Coterminal Angles
- Expressions generating coterminal angles

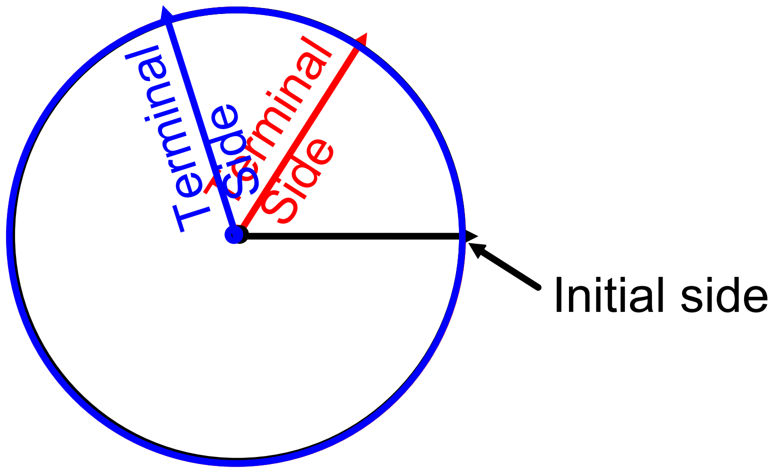
initial angle work.ggb

How does an angle terminate?

An angle formed by one ray rotating away from another.
Where it stops rotating an angle is formed.
The rotated ray is called the terminal side.



Two angles are coterminal if their rotating side ends up in the same place.



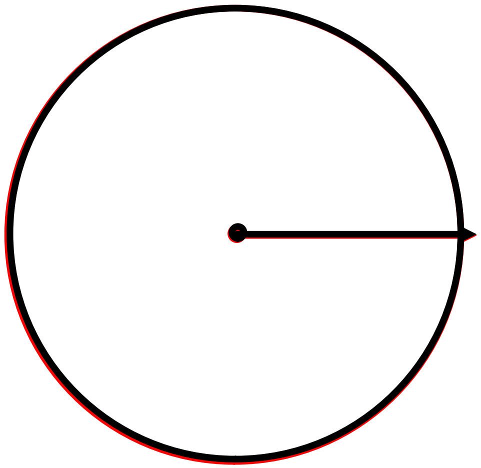
755°

Expression: $35^{o} + 360*x$, where x is an integer

Steps for finding the smallest coterminal angle for any angle:

Approach 1: Keeping adding 360s until the total is just than the number.
The coterminal number is the original angle less a multiple of 360.

Approach 2: Divide the angle by 360^{o}
The remainder of the division represents the smallest coterminal angle.



If a wheel rotates 450 rotations a minute, how many degrees per second does it turn?

A spool of thread has a radius of 1/2 inch. It rolls across the floor dropping 1.5 feet of thread per second. How many rotations per second is that?

What are radians?

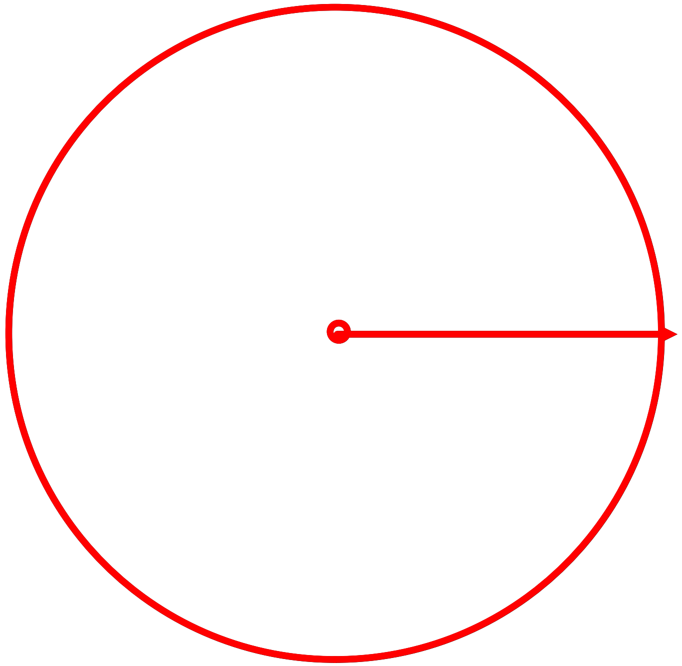
A way to measure rotation, by measuring how many

radii worth of distance along the circumference of the circle are cut off by the rotation

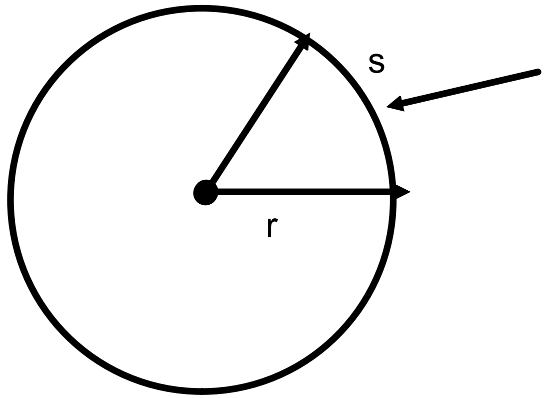
A rotation measures one radian when that rotation cuts off a part of the

circumference that measures one radius long.

- Wheel
- Geogebra



Measuring rotation



Radians: Measure the length of this
and divide it by one radius

This is some amount out of 2π

$$\frac{s}{r} = \theta$$

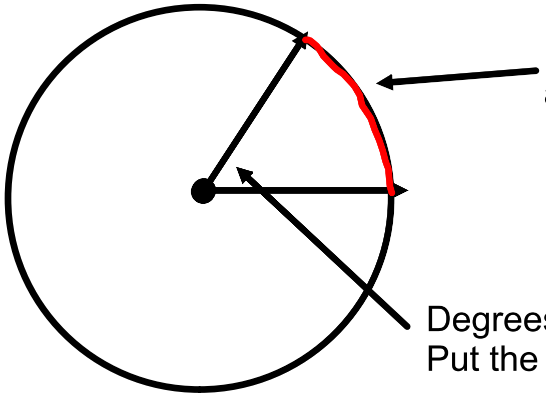
Where theta is in radians

$$s = r\theta$$

r is our radius
s is the length of the arc

Check 5.1 packet top of 7

Measuring rotation



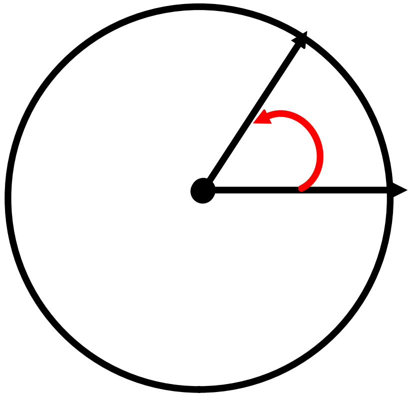
Radians: Measure the length of this
and divide it by one radius

Once around is 2π,

Degrees:
Put the protractor here and measure degrees on protactor.

Once around 360^{o}

Quoting radians:



Radians measure rotation.

Just like 360^{o} is rotating all the way around.
$2\pi^{r}$ is rotating all the way around.

$2\pi^{r} = 6.28^{r}$, but one is in exact terms and one is round.
Because 2π is the total
and we often calculating proportions of the total
and exact terms are better

radian measure often are fractions or multiples of π

- An alternative measure to angles.
- Each degree is $1/360^o$ of the circle
- Each radian is $1/(2\pi^r)$ of a circle.

- How many degrees are there around circle? 360^o
- How many radians are there around a circle? 2π

- $360^o = 2\pi^r$ or $180^o = \pi^r$

- When we think of degrees we think of measuring rotation at the central angle and then describing the arc size by that number also.
- When we think of radians we think of measuring rotation by measuring the arc size and then describing the central angle by that number also.

- Degrees were decided by someone who arbitrarily decide to cut a circle up in 360 slices.
- A radian is decided based on cutting a circle up into units as long as the circle's radius.

Radians often have "π" in them because their lengths are in relation to the circumference of a circle, 2π

Skills:

Convert radians to degrees and degrees to radians

Answer questions about rotation and speed.

Skills: Convert radians to degrees and degrees to radians

$180^{o} = \pi^{r}$

Fast way:

$$\frac{\pi}{12}^{r} = ?^{o}$$

$$\frac{180}{12} = 15^{o}$$

Slow way:

$$\boxed{\frac{rad}{\pi} = \frac{Deg}{180^{o}}}$$

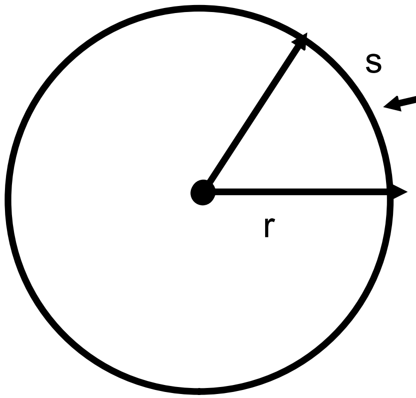
$$\frac{\pi/12^{\,r}}{\pi^{r}} = \frac{x}{180^{o}}$$

$$180^{o} * \frac{\pi}{12} = \pi x$$

$$15^{o} = x$$

Answer questions about rotation and speed.

Measuring rotation



Radians: Measure the length of this and divide it by one radius

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$$\frac{s}{r} = \theta$$

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Convert radians to degrees and degrees to radians

Answer questions about rotation and speed.

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- When we think of degrees we think of measuring rotation at the central angle and then describing the arc size by that number also.
- When we think of radians we think of measuring rotation by measuring the arc size and then describing the central angle by that number also.

1 rotation = 1 revolution => $360^o = 2\pi^r$ => $2\pi r''$

Angular speed/velocity:

$$100\ revolution\ per\ \min. = \frac{100\ rev}{1\min} => \frac{100*360^o}{1\min} = \frac{100*2\pi^r}{1\min}$$

$$\frac{\theta}{t} = \frac{angle\ size}{time}$$

A merry-go-round turns 6 revolutions a minute, what is its angular speed?

If we state our angular velocity in radians, then we are saying how many radians per minute the rotation is happening.

Multiply that speed by the size of radius and what do we get?

$$\frac{radians}{\min} * \frac{inches}{raduis} = \frac{?}{?}$$

What would this mean?

$$Linear\ speed = r\frac{\theta}{t} = \frac{r\theta}{t}$$

1 rotation = 1 revolution => 360^{o} $= 2\pi^{r}$ => $2\pi r''$

Linear speed/velocity:

$$100 \ revolution \ per \ \min. = \frac{100 \ rev}{1 \min} => \frac{100 * 2\pi r''}{1 \min}$$

$$\frac{r\theta}{t} = \frac{radius \ size * number \ or \ radians}{time}$$

If wheel has a radius of 15" and its angular velocity is 32π radians per second what is its linear speed?

If a wheel is moving forward at 20π feet radians per second what is its radius?

In the paper towel factor a machine rolls the paper on to the cardboard tubes. The machine turns the tubes at 300 revolutions per minute. The tubes have a radius of .4 inches. What is the linear speed of the paper moving on to the tube? (Ignore the increasing diameter of the roll as the paper accumulates).

Reconciliation to Packet/Book

Angular velocity = ω
Rotation in radians = θ
Time = t
Radius = r

Angular velocity

$$\varpi = \frac{\theta}{t}$$

Linear speed

$$v = r\frac{\theta}{t}$$

$$v = r\frac{\theta}{t} = \frac{r\theta}{t} = \frac{s}{t}$$

$$\frac{55\,miles}{1\,hour}$$

$v = r\varpi$ Same formula

Rotation: Round things have linear speed and angular speed.

Linear speed/velocity: Think of a spool of thread.
If the spool rolling along the floor dropping thread, how much thread does it drop per minute.

$$\frac{Distance}{Time} = \frac{s}{t} \Rightarrow \frac{Inches}{Minute} \qquad \frac{Distance}{Time} = \frac{radius\ length * \#\ radians}{t} = \frac{r\theta}{t}$$

Angular speed/velocity: Think of a spoke on a wheel.
If you mark a spoke on a bicycle wheel, how many degrees or radians does it rotate through per minute?

$$\frac{Angle\ measure}{Time} = \frac{\theta}{t} \Rightarrow \frac{Radians}{Minute}$$

Revolutions per minute:

$$\frac{\#\ of\ revs}{time} = \frac{\left(\frac{Linear\ Distance}{Circumference}\right)}{Time} = \frac{\left(\frac{Linear\ Distance}{Time}\right)}{Circumference} = \frac{Linear\ Speed}{Circumference} = \frac{\left(\frac{\#\ of\ revs * Circumference.}{Time}\right)}{Circumference} = \frac{\#\ of\ revs}{Time}$$

Sectors: Arc length and areas of a sector can be found by the proportion of radians in the sector.

Think percentage of a pizza

Arc lengths

$$\frac{\text{Your crust length}}{\text{Whole crust length}} = \frac{\text{radians}}{\text{Total radians}}$$

$$\frac{\text{Arc length}}{\text{Circumference}} = \frac{\text{radians}}{2\Pi}$$

$$\frac{s}{2\Pi r} = \frac{\theta}{2\Pi}$$

$$\frac{s}{r} = \theta$$

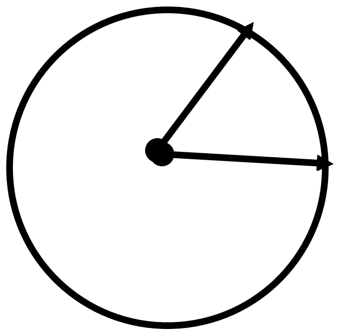
Areas

$$\frac{\text{Area of your slice}}{\text{Area of pizza}} = \frac{\text{Sector radians}}{\text{Total radians}}$$

$$\frac{\text{Sector area}}{\text{Area}} = \frac{\text{radians}}{2\Pi}$$

$$\frac{A}{\Pi r^2} = \frac{\theta}{2\Pi}$$

$$\frac{2A}{r^2} = \theta$$



Attachments

- the sum of sin(n*pi/180) for n = 1 to 359
- sin(arcsin(12/13) + n*pi/4) for n =1 to 8
- initial angle work.ggb