Phase shift:
Sign and convention for expressing

In packet item a "trig" sin and cos is 2π cot and tan is π

Write sinusoidal function given features of the data:
Amplitude, Period, Phase shift, Vertical Shift
Construct sinsoidal function given the data
Graph the function vs. the data in the calculator

To graph sinusoidal functions from the equation y =Asin(ωx-ϕ) or Acos(ωx-ϕ)

Step 1: Factor out ω, making $y = A\sin\left(\omega\left(x + \frac{\phi}{\omega}\right)\right)$ or the cos version.

p440.

Step 2: Determine the amplitude |A| and period $T = \frac{2\pi}{\omega}$

Step 3: Mark the shifted start point of the first cycle of the graph at $\frac{\phi}{\varpi}$

Step 4: Mark the ending point of the first cycle one period after the start at $\frac{\phi}{\varpi} + \frac{2\pi}{\varpi}$

Step 5: Divide the interval into quarters, in half and then half again
Step 6: Put x-intercepts and maximum/minimum at the quarter points as appropriate.
Step 7: Sketch the graph.
Step 8: Extend the graph in each direction to make it complete.

Function from features

1) Amplitude: 12 2) Phase shift: 6 3) Midline: 48 4) Period = 4 5) Sin period = 2π

$$y = A\sin(\omega x + \phi) + B$$

Way 1

$$y = A\sin\left(\frac{2\pi}{P}(x-D)\right)+B$$

$$y = 12\sin\left(\frac{2\pi}{4}(x-6)\right)+48$$

$$y = 12\sin\left(\frac{\pi}{2}(x-6)\right)+48$$

$$y = 12\sin\left(\frac{\pi}{2}x-\frac{6\pi}{2}\right)+48$$

$$y = 12\sin\left(\frac{\pi}{2}x-3\pi\right)+48$$

Way 2

1) $P = \frac{2\pi}{\omega}$

$4 = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{4}$

$\omega = \frac{\pi}{2}$

2) $D = \frac{\phi}{\omega}$

$6 = \frac{\phi}{\pi/2}$

$6\pi/2 = \phi$

$3\pi = \phi$

3) $y = A\sin(\omega x - \phi) + B$

$$y = 12\sin\left(\frac{\pi}{2}x-3\pi\right)+48$$

To write the function from a picture

1) Estimate Amplitude: $A = \dfrac{\max - \min}{2}$

2) Estimate vertical shift and midline: $B = \dfrac{\max + \min}{2}, y = B$

4) Estimate ϕ as follows:

- Find the x associated with the start of the period either as:
  where the function crosses the midline for sine
  or find a max or min for cosine
- $\phi = -(starting\ x) * \varpi$

5) Estimate ω as follows:

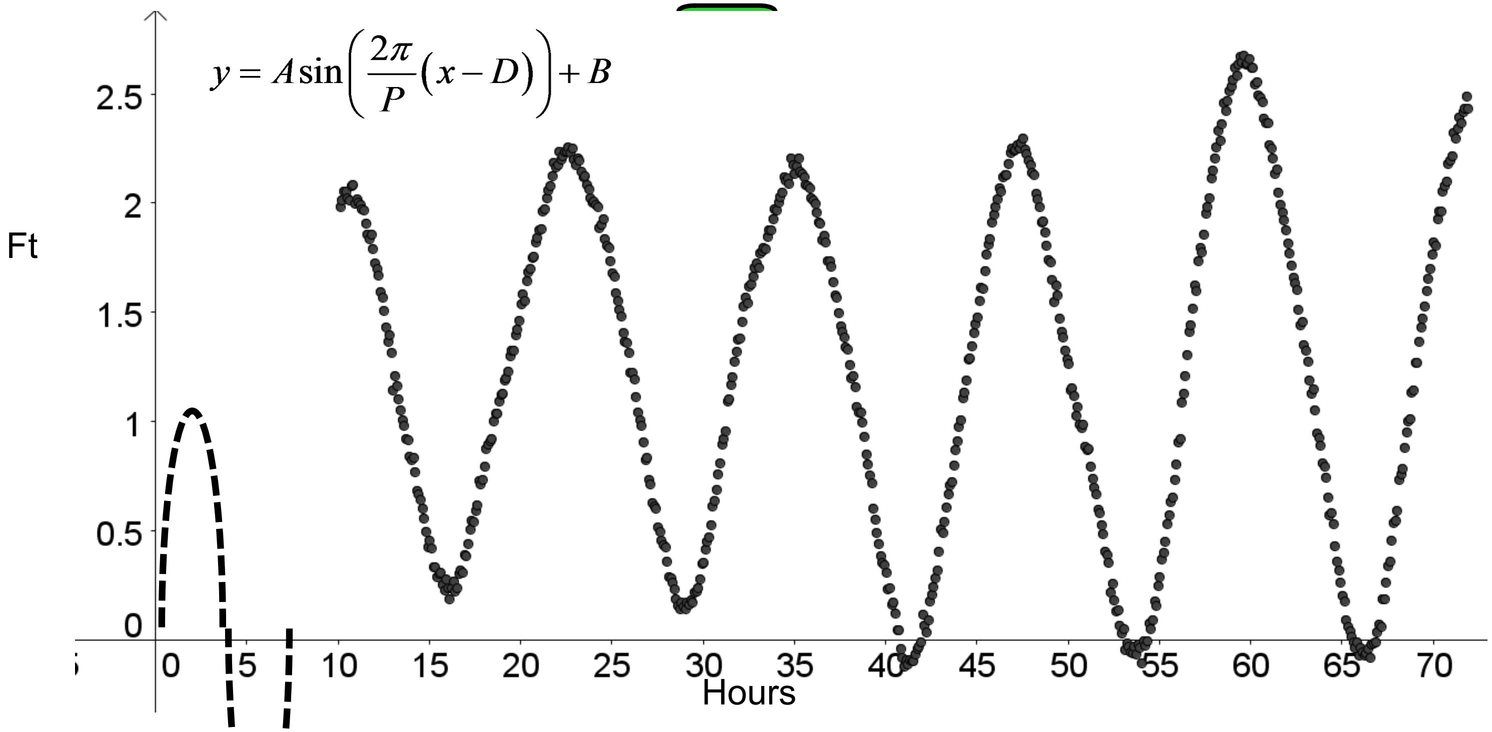
- Find x associated with the next restart of the period.

$$\frac{2\pi}{(ending\ x - starting\ x)} = \varpi$$

Facts:

1) Amplitude: 1.4 2) Phase shift: 19.25 3) Midline: 1.22 4) Period = 11.9 5) Sin period = 2π

$$y = A\sin\left(\frac{2\pi}{P}(x-D)\right)+B$$



Max=2.68 Min=-.12 Average=1.22 First peak:10.8 Second peak:22.7 First start: 19.25

Facts:

1) Amplitude: 1.4 2) Phase shift: 19.25 3) Midline: 1.22 4) Period = 11.9 5) Sin period = 2π

Way 1

$$y = A\sin\left(\frac{2\pi}{P}(x-D)\right)+B$$

$$y = 1.4\sin\left(\frac{2\pi}{11.9}(x-19.25)\right)+1.22$$

$$y = 1.4\sin\left(.5280(x-19.25)\right)+1.22 \leftarrow\!\!\bullet$$

$$y = 1.4\sin\left(.5280x-10.1640\right)+1.22$$

Way 2

1) $P = \dfrac{2\pi}{\omega}$

$$11.9 = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{11.9}$$

$$\omega = .5280$$

2) $D = \dfrac{\phi}{\omega}$

$$19.25 = \frac{\phi}{.5280}$$

$$10.1640 = \phi$$

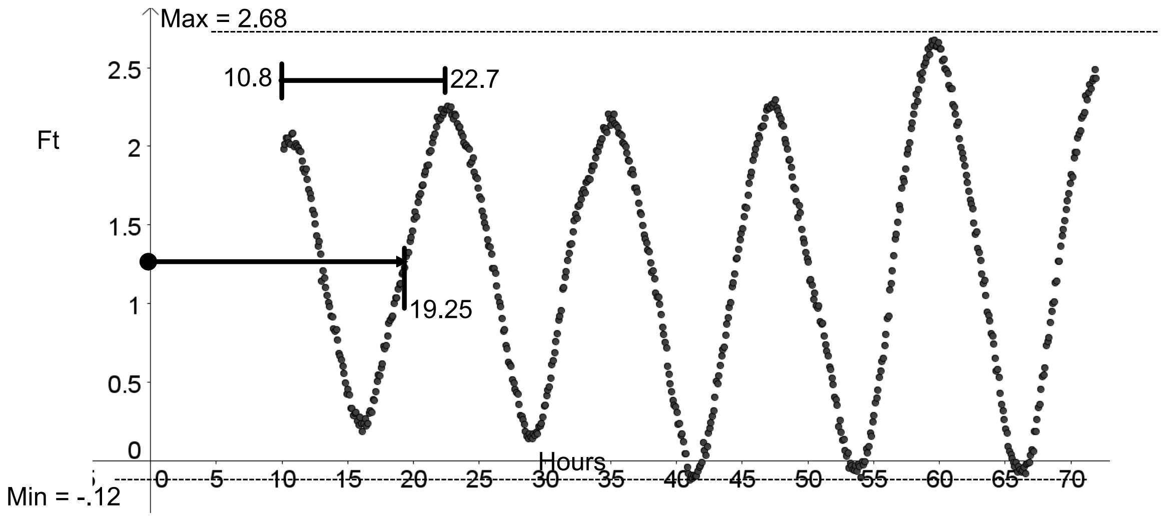
3) $y = A\sin(\omega x - \phi)+B$

$$y = 1.4\sin(.5280x-10.1640)+1.22$$

Target formulation: $y = A\sin\left(\frac{2\pi}{P}(x-D)\right)+B$

Facts:
1) Amplitude: 1.4=A   2) Phase shift: 19.25 = D   3) Midline: 1.28 = B
4) Period:11.9=P   5) 2π = Sine period

$$y=1.4\sin\left(\frac{2\pi}{11.9}(x-19.25)\right)+1.28=1.4\sin\left(\frac{2\pi}{11.9}x-\frac{2*\pi*19.25}{11.9}\right)+1.28=1.4\sin(.5280x-10.1640)+1.28$$



Max=2.68 ft   Min=-.12 ft $\frac{(Max+\min)}{2}=1.28\ ft=B$ $\frac{(Max-\min)}{2}=1.4\ ft=A$

First peak is at 10.8 hours, second peak is at 22.7 hours. Period = 11.9 hours = P*

First start of a sine wave (curve crossing the midline while going up): 19.25. Shift = 19.25 = D**

* You can measure the repeating cylce in lots of different places:
Max to next max, min to next min, midpoint to next midpoint

** The phase shift is most easily observed by looking for the first point where the rising curve crosses the midline, but you can use the next place or the place after that. The infinite model will work out well from any legitimate start of the cycle.

Facts:

1) Amplitude: 31 2) Phase shift: 3 3) Midline: 45 4) Period = 12 5) Sin period = 2π

$$y = A\sin(\omega x + \theta) + B$$

Way 1

$$y = A\sin\left(\frac{2\pi}{P}(x - D)\right) + B$$

$$y = 31\sin\left(\frac{2\pi}{12}(x - 3)\right) + 45$$

$$y = 31\sin\left(\frac{\pi}{6}(x - 3)\right) + 45$$

$$y = 31\sin\left(\frac{\pi}{6}x - \frac{3\pi}{6}\right) + 45$$

$$y = 31\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 45$$

Way 2

1) $P = \dfrac{2\pi}{\omega}$

$$12 = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{12}$$

$$\omega = \frac{\pi}{6}$$

2) $D = \dfrac{\phi}{\omega}$

$$3 = \frac{\phi}{\pi/6}$$

$$3\pi/6 = \phi$$

$$\pi/2 = \phi$$

3)

$$y = A\sin(\omega x - \phi) + B$$

$$y = 31\sin\left(\frac{\pi}{6}x - \frac{\pi}{2}\right) + 45$$

To write the function from the data.

Test includes 5.3 identities

Run regression in the calculator

1) Read amplitude

2) Read vertical shift

3) Factor $y = A\sin\left(\omega\left(x + \frac{\phi}{\omega}\right)\right)$ to identify phase shift

4) Calculate period $Period = \frac{2\pi}{\omega}$

To write the function from the data.

1) Estimate Amplitude: $A = \frac{\max - \min}{2}$

2) Estimate vertical shift and midline: $B = \frac{\max + \min}{2}, y = B$

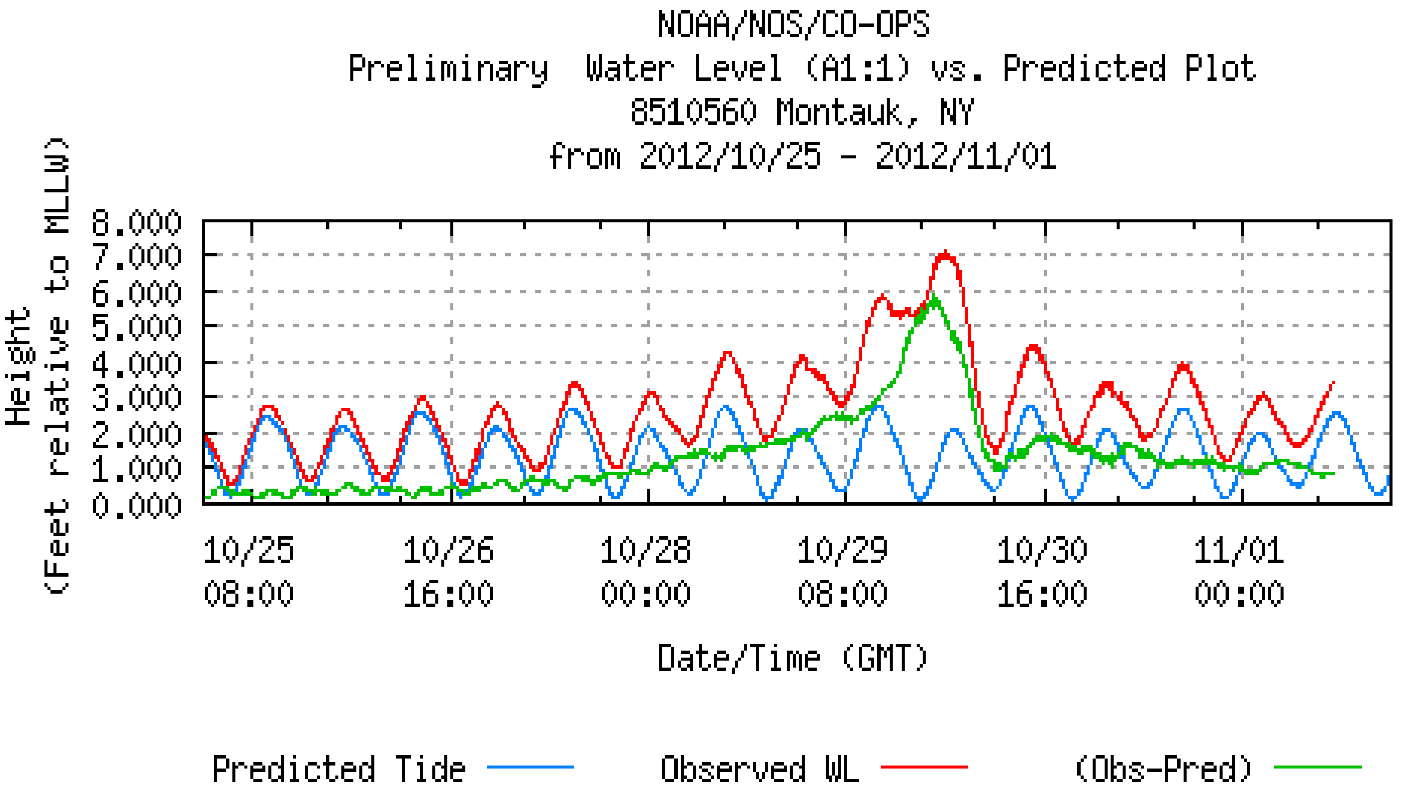
4) Find ω as follows:

- Find the x associated with the start of the period either as:
  where the function crosses the midline for sine
  or find a max or min for cosine
- Find x associated with the next restart of the period.

$$\frac{2\pi}{(ending\ x - starting\ x)} = \varpi$$

5) Find ϕ by solving for ϕ:

- choose one (x,y) point and solve $y = A\sin(\varpi x - \phi)$ for ϕ


NOAA/NOS/CO-OPS
Preliminary Water Level (A1:1) vs. Predicted Plot
8510560 Montauk, NY
from 2012/10/25 - 2012/11/01
Height
(Feet relative to MLLW)
8.000
7.000
6.000
5.000
4.000
3.000
2.000
1.000
0.000
10/25 08:00
10/26 16:00
10/28 00:00
10/29 08:00
10/30 16:00
11/01 00:00
Date/Time (GMT)
Predicted Tide
Observed WL
(Obs-Pred)