5.2 THE OTHER TRIGONOMETRIC FUNCTIONS (Part II)

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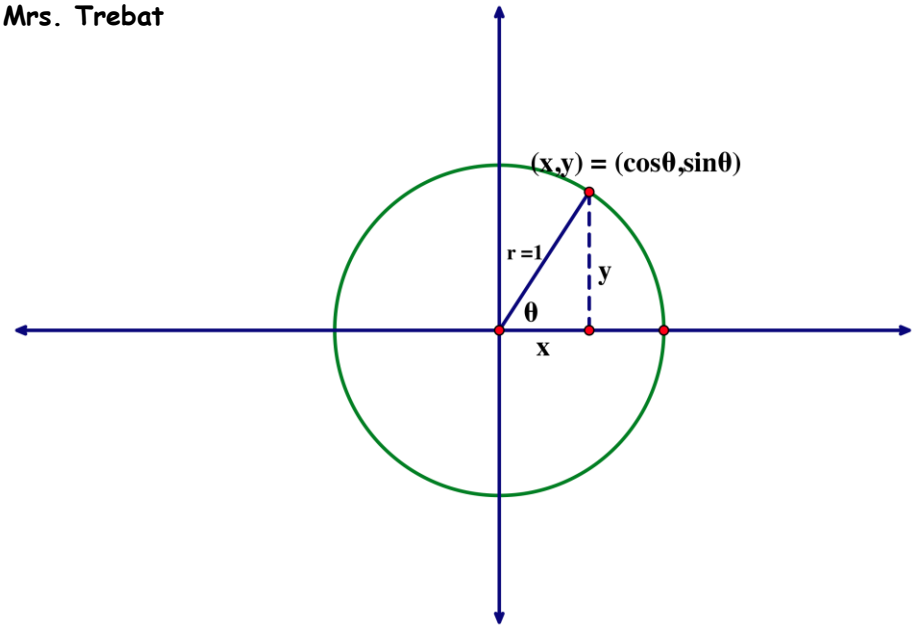


We are now ready to define four other trigonometric functions of an angle θ in terms of the x- and y-coordinates of a point on the terminal ray of θ.

These are **tangent, cotangent, secant,** and **cosecant**. They are defined as follows:

$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y} \quad (y \neq 0)$
$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x} \quad (x \neq 0)$
$\tan\theta = \frac{y}{x} \quad (x \neq 0)$	$\cot\theta = \frac{x}{y} \quad (y \neq 0)$

Notice that

(i) Because of the restrictions on the denominators in the definitions of tangent, cosecant, secant and cotangent, some angles will have undefined function values.

(ii) Sine and Cosecant are **reciprocals** and so are cosine and secant, and tangent and cotangent. Hence, the following **reciprocal identities** are true for any angle θ. This is given in the table below.

RECIPROCAL IDENTITIES:

$\sin\theta = \frac{1}{\csc\theta}$	$\csc\theta = \frac{1}{\sin\theta}$
$\cos\theta = \frac{1}{\sec\theta}$	$\sec\theta = \frac{1}{\cos\theta}$
$\tan\theta = \frac{1}{\cot\theta}$	$\cot\theta = \frac{1}{\tan\theta}$

* ***Identities*** *are equations that are true for all values of the variable.*

Examples:

Find each function value.

1) $\cos\theta$ ***if*** $\sec\theta = \frac{5}{3}$

2) $\sin\theta$ ***if*** $\csc\theta = -\frac{\sqrt{12}}{2}$

3) The terminal side of an angle θ in standard position goes through the point (-8, 15). Find the values of the six trigonometric functions of angle θ.

4) Let P be a point on the **unit circle** which is on the terminal side of angle θ. Assume P has coordinates $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$. Find the six trigonometric function values of θ.

5) Find the exact value without using a calculator:

a) $\tan\frac{3\pi}{4}$

b) $\sec\left(\frac{2\pi}{3}\right)$

c) $\csc\left(\frac{7\pi}{6}\right)$

d) $\cot\left(\frac{5\pi}{4}\right)$

6) Verify the **identity** $\frac{\sin\theta}{\cos\theta}=\tan\theta$. You will use this frequently!

7) At what values of θ is $\tan\theta$ equal to 1? At what values of is $\tan\theta$ equal to -1?

THE SIX TRIG FUNCTION VALUES OF QUADRANTAL ANGLES

8) If the terminal side of a quadrantal angle lies along the **y-axis**, the **tangent** and **secant** functions are undefined. If it lies on the **x-axis**, the **cotangent** and **cosecant** functions are undefined. Explain why.

_____

9) Complete the table for the quadrantal angles of 0°, 90°, 180°, 270°, and 360° and use it for future reference:

QUADRANTAL ANGLES

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
0° or 0						
90° or $\frac{\pi}{2}$						
180° or π						
270° or $\frac{3\pi}{2}$						
360° or 2π						

10) For any non-quadrantal angle θ, $\sin\theta$ and $\csc\theta$ will have the same sign. Explain why. _____

11) $\cos 90° + 3\sin 270° =$ _____

12) $\csc 270° + 2 \cdot \tan(135°) =$ _____

THE SIGNS OF THE TRIG FUNCTIONS

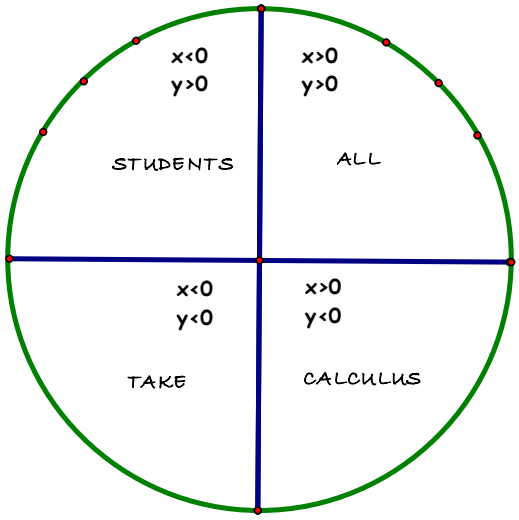
A point (x,y) in quadrant II has $x < 0$ and $y > 0$. This makes the values of sine and cosecant positive for quadrant II angles, while the other

four functions take on negative values. Summarize the results for the other quadrants in the table below.

SIGNS OF FUNCTION VALUES

θ in quadrant	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
I						
II						
III						
IV						

OR



13) Identify the quadrant (or quadrants) for any angle θ that satisfies $\sin\theta > 0$ ***and*** $\tan\theta < 0$. _____

14) If θ is a second-quadrant angle and $\tan\theta = -\frac{3}{4}$, find the values of the other five trigonometric functions. Make a sketch to help.

15) If $\sec 15^\circ \approx 1.035$, give the approximate value of (**NO** calculators!)

a) $\sec(-15^\circ)$ b) $\sec 165^\circ$ c) $\sec 345^\circ$

16)Use your calculator to find the value of each to 4 decimal places.

a) tan 2 b) cot 185° c) csc 3 d) sec (-22°)