5.3 PROPERTIES OF TRIGONOMETRIC FUNCTIONS

Reciprocal Identities

1) $\csc\theta = \frac{1}{\sin\theta}$ 2) $\sec\theta = \_\_\_\_\_ \frac{1}{\cos\theta}$ 3) $\cot\theta = \_\_\_\_\_ \frac{\cos\theta}{\sin\theta}$

Quotient Identities

(1) $\tan\theta = \frac{\sin\theta}{\cos\theta}$ (2) $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Identities

(1) $\sin^2\theta + \cos^2\theta = \_\_\_\_ 1$

(2) $1 + \tan^2\theta = \_\_\_\_\_ \sec^2\theta$

(3) $1 + \cot^2\theta = \_\_\_\_\_ \operatorname{cosec}^2\theta$

Applying…

1) If $\sin\theta = \frac{2\sqrt{2}}{3}$ and $\cos\theta = -\frac{1}{3}$, what is

a) $\tan\theta = -2\sqrt{2}$ b) $\cot\theta = \frac{-\sqrt{2}}{4}$ c) $\sec\theta = -3$

d) In which quadrant is θ? II

2) What is $\sin^2(35°) + \cos^2(35°)$ = ? 1

3) If $\tan\theta = \frac{5}{3}$, what is $\sec^2\theta$? $\sec\theta$? $\pm\frac{\sqrt{34}}{3}$

PERIODIC FUNCTIONS – a function whose values repeat…

Definition: A function f is called periodic if there is a positive number p such that

$$f(\theta + p) = f(\theta)$$

Whenever θ *and* $\theta + p$ are in the domain of f.

Periodic Properties:

$$\sin(\theta + 2\pi k) = \sin\theta$$
$$\cos(\theta + 2\pi k) = \cos\theta$$
$$\tan(\theta + \pi k) = \tan(\theta)$$

(**Reciprocal Functions** have the same period…)

Examples:

1) $\sin(750°) = 1/2$ 2) $\tan(135° + 1800°) = -1$

3) $\sin\frac{13\pi}{6} = 1/2$ 4) $\cos\frac{13\pi}{4} = -\frac{\sqrt{2}}{2}$

3) If $\tan\theta = 2$, *what is* ...

$$\tan(\theta) + \tan(\theta + \pi) + \tan(\theta + 2\pi) = 6$$

4) If $\cot(\theta) = -2$, find

$$\cot(\theta) + \cot(\theta - \pi) + \cot(\theta - 2\pi) = -6$$

EVEN-ODD PROPERTIES

A function is **even** $\Leftrightarrow f(-x) = f(x)$

A function is **odd** $\Leftrightarrow f(-x) = -f(x)$

a) Is the sine function odd or even? Make a sketch to verify.

$\therefore \sin(-\theta) =$ _____ -sin(θ)

b) What about the reciprocal of the sine function?

$\therefore \csc(-\theta) =$ _____ -csc(θ)

Complete with even/odd properties:

- $\sin(-\theta) = -\sin(\theta) \Rightarrow \csc(-\theta) = -\csc(\theta)$
- $\cos(-\theta) =$ _____ $\Rightarrow \sec(-\theta) =$ _____
  cos(θ) sec(θ)
- $\tan(-\theta) =$ _____ $\Rightarrow \cot(-\theta) =$ _____
- -tan(θ) -cot(θ)

Problems:

1) If $f(\theta)=\sin\theta$ *and* $f(a)=\frac{2}{5}$, find the exact value

a) $f(-a)=\frac{-2}{5}$

b) $f(a)+f(a+6\pi)=\frac{4}{5}$

2) If $f(\theta)=\cos\theta$ *and* $f(b)=\frac{1}{3}$, find the exact value

a) $f(-b)=\frac{1}{3}$

b) $f(b)+f(b+2\pi)+f(b-2\pi)=1$

3) What is the exact value of…

$\sin 1^\circ+\sin 2^\circ+\sin 3^\circ+\ldots\sin 358^\circ+\sin 359^\circ$?

0

4) If *If* $f(\theta)=\cot\theta$ *and* $f(a)=-3$, *then*

a) $f(-a)=3$

b) $f(a)+f(a+\pi)+f(a+6\pi)=-9$

5) Use even-odd properties to find:

a) $\tan(-30°)$=-tan(30°) $= -\left(\frac{1/2}{\sqrt{3}/2}\right) = \frac{-\sqrt{3}}{3}$

b) $\sec(-45°)$=sec(45°) $= -\left(\frac{1}{\sqrt{2}/2}\right) = -\sqrt{2}$

c) $\csc(-120°)$= -csc(120°) $= -\left(\frac{1}{\sqrt{3}/2}\right) = \frac{-2\sqrt{3}}{3}$

d) $\cot(-210°)$=-cot(210°) $= -\left(\frac{-\sqrt{3}/2}{-1/2}\right) = -\sqrt{3}$

e) If $\sin\theta = 0.15$, what is $\sin(-\theta)$? -.15

f) If $\sec\theta = -\frac{5}{3}$, what is $\cos(-\theta)$?= cos(θ)=$\frac{1}{sec\theta} = \frac{-3}{5}$

cos(-θ)=cos(θ)

NOW LET'S LOOK AT THE DOMAIN AND RANGE OF THE TRIGONOMETRIC FUNCTIONS…

Let P(x,y) $\Leftrightarrow (\cos\theta, \sin\theta)$ be a point on the unit circle that corresponds to θ.

	DOMAIN (values of θ)	RANGE (values of function)
$f(\theta) = \sin\theta$	All real numbers	{y|-1≤0≤1}
$f(\theta) = \cos\theta$	All real numbers	{y|-1≤0≤1}
$f(\theta) = \tan\theta$	All reals except $\frac{\pi}{2}+k\pi$, where k is an integer	All reals
$f(\theta) = \sec\theta$	All reals except $\frac{\pi}{2}+k\pi$, where k is an integer	{y|-∞≤-1∪1≤∞}
$f(\theta) = \csc\theta$	All reals except $0+k\pi$, where k is an integer	{y|-∞≤-1∪1≤∞}
$f(\theta) = \cot\theta$	All reals except $0+k\pi$, where k is an integer	All reals