5.4 THE GRAPH OF THE SINE AND COSINE FUNCTIONS

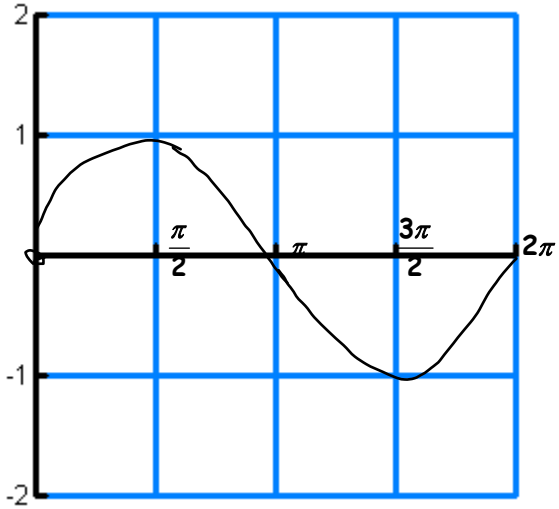
A single wave-length

Angle x	$y = \sin x$	(x, y)
0	0	$(0,0)$
$\pi/2$	1	(π/2,1)
π	0	(π,0)
$3\pi/2$	-1	(3π/2,-1)
2π	0	(2π,0)

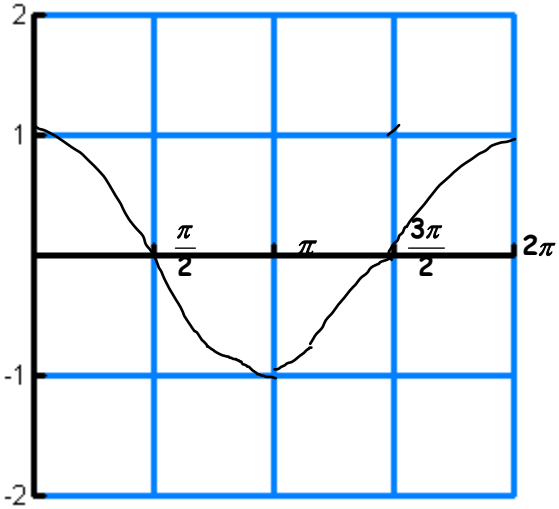
Angle x	$y = \cos x$	(x, y)
0	1	$(0,1)$
$\pi/2$	0	(π/2,0)
π	-1	(π,-1)
$3\pi/2$	0	(3π/2,0)
2π	1	(2π,1)

The graphs of sine and cosine over one period (one wave length)

$y = \sin x$



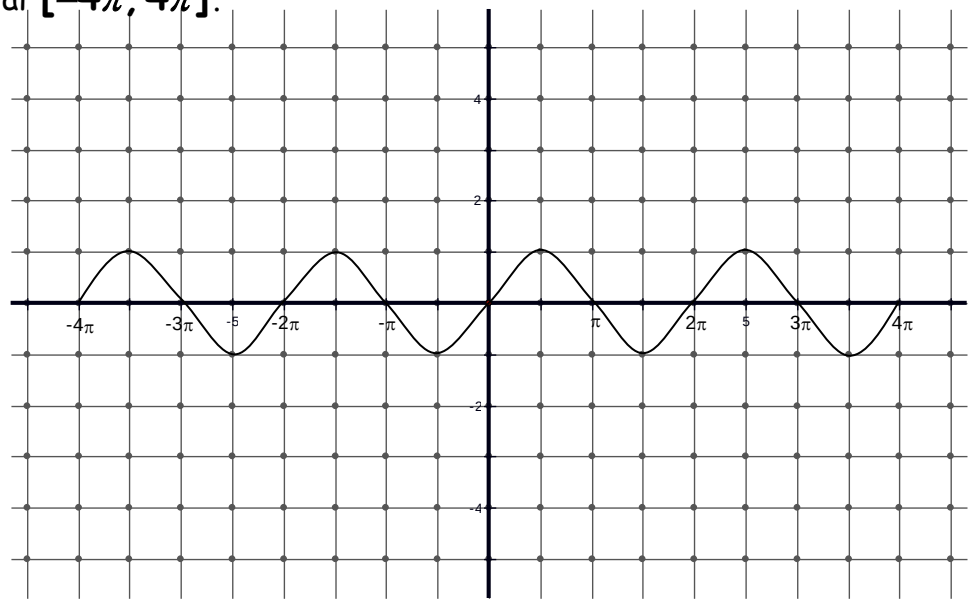
$y = \cos x$



The sine and cosine functions are **periodic** with a fundamental period of 360° or 2π. Their graphs are called **sine waves** or **sinusoids** because of their repeated pattern.

1) <u>The Graph of Y = Sin X</u>

Using the grid below, let's sketch the graph of $y = \sin x$ in the interval $[-4\pi, 4\pi]$.



a) **Domain**: __All reals or <u>[-4π,4π]</u> b) **Range**: _-1,1_____

c) **X**-intercepts: 0 + πK, where k is an int. d) **Y**-intercept: (0,0)_____

e) The **maximum** value of 1 occurs at X = _π/2 + 2πk, where k is an int.___

f) The **minimum** value of -1 occurs at X = _3π/2 + 2πk, where k is an int.___

g) The sine function is _____ (<u>ODD</u>/**EVEN**) because $\sin(-x) = -\sin(x)$_ for every x in its domain; the graph is symmetric about the _origin___

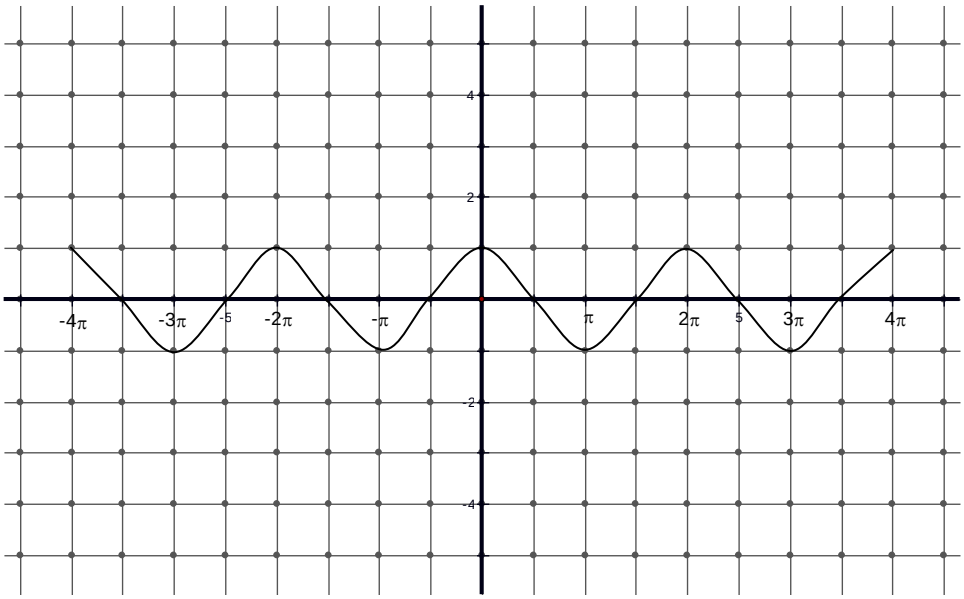
h) The **period** (one complete cycle) of the sine function is _2π_____

i) The equation of the **midline** of the graph is __y=0_____

j) The **amplitude** (distance between the midline and the points of maximum or minimum values) is ___1_____

2) The Graph of Y = cos x

Using the grid below, let's sketch the graph of $y = \cos x$ in the interval $[-4\pi, 4\pi]$.



a) **Domain**: __All reals or [-4π,4π] b) **Range**: _[-1,1]_____

c) **X**-intercepts: π/2 + πK, where k is an int. d) **Y**-intercept: (0,1)_____

e) The **maximum** value of 1 occurs at X = _0 + 2πk, where k is an int.____

f) The **minimum** value of -1 occurs at X = _ π + 2πk, where k is an int.____

g) The cosine function is _____ (**ODD/EVEN**) because cos(-x)=cos(x) _ for every x in its domain; the graph is symmetric about the _y-axis____

h) The **period** (one complete cycle) of the cosine function is _2π_____

i) The equation of the **midline** of the graph is __y=0_____

j) The **amplitude** (distance between the midline and the points of maximum or minimum values) is ___1_____

Amplitude, Vertical Stretches and Shifts

The amplitude of a sinusoidal can be found by using the formula:

$$Amplitude = \frac{(maximum)-(minimum)}{2}$$

3) Using your graphing calculator, graph the functions below. Find the amplitude, and the equation of the midline of the graph.

Equation	Amplitude	Equation of Midline
$y = 3\sin x$	3	Y=0
$y = 5\sin x + 2$	5	Y=2
$y = \frac{1}{2}\sin x - 3$	1/2	Y=-3

4) Write an equation to obtain the midline.

Y = (maximum + minimum)/2

Generalizations on the graph of $y = A\sin x + B$ or $y = A\cos x + B$

5) Discuss the roles played by parameters A and B:

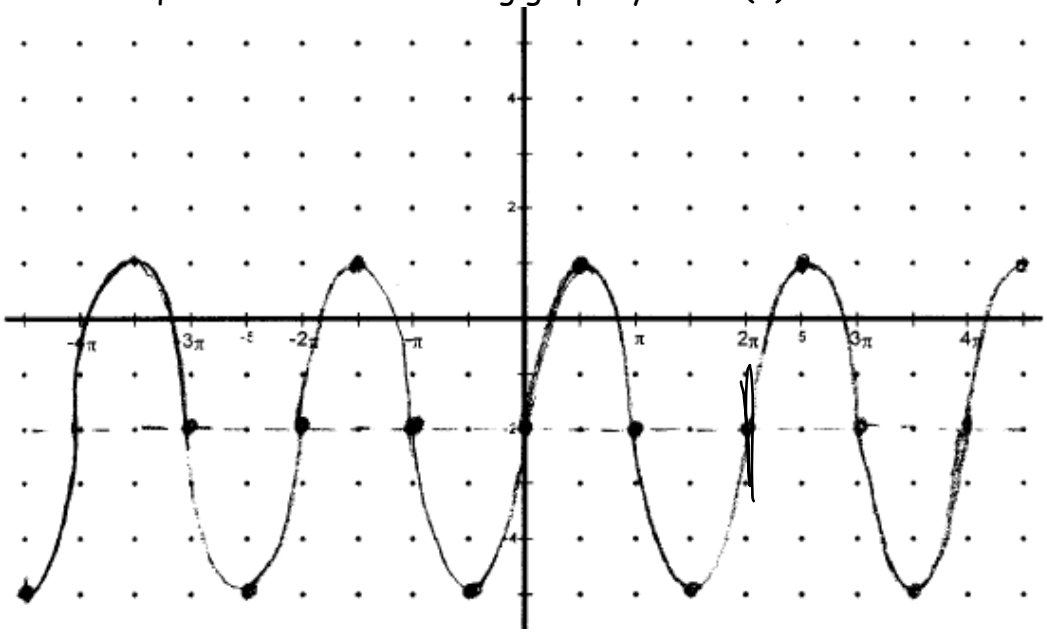
a) The role of A: _____Sets amplitude_____

b) The role of B: _____Vertical shift_____

c) Does B affect the amplitude of the sine wave? Explain.

_____No, it is applied a simple addition after the sin is calculated_

6) Find an equation for the following graph: y = 3sin(x) - 2



Period of the Sine and Cosine Functions

Background: The period (one complete cycle) along the x-axis of the functions $y = \sin x$ *or* $y = \cos x$ is __2π__ radians or _360°__ degrees.

7) Does the amplitude of the sinusoidal function affect the period? Explain.
_____No, period is determined before sine is calculated._____
_____

8) Graph and compare the functions $y = \sin 2x$ *and* $y = \sin x$ on the same graph. Explain what happens to the period when x is multiplied by 2. (use the terms horizontal **stretch** or **compression**)
_____The period is horizontally compressed by a factor of 2._____
_____The period is cut in half._____

9) Graph and compare the graphs of $y = \cos \frac{x}{2}$ *and* $y = \cos x$. Explain
What happens to the period when x is multiplied by $\frac{1}{2}$.
___The period doubles._____

Now we are ready to draw some generalizations…

10) The period of a sinusoid of the type $y = \sin \varpi x$ *or* $y = \cos \varpi x$, for any real number ϖ is given by the formula:

Period T = _2π/ω____

11) Furthermore, the following effects are observed when $\varpi > 1$ and when $0 < \varpi < 1$:

$\varpi > 1$ ___It is horizontally compressed_____

$|\varpi| < 1$ ___It is horizontally streched_____

12) Let $f(x) = 2\sin(3x) - 4$.

a) The amplitude is __2_____.

b) The equation of the midline is ___y=-4_____.

c) The period is ___2π/3_____.

d) The Domain is ___All reals_____ and the range _[-6,-2]____.