5.5 Graphs of Tangent, Cotangent, Cosecant, and Secant Functions

TANGENT AND COTANGENT FUNCTIONS

What you need to remember:

Reciprocal Identities:	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

1) Complete the table below for the tangent and cotangent values (in decimal form):

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\tan\theta$	0	1	Und	-1	0	1	Und	-1	0
$\cot\theta$	Und	1	0	-1	Und	1	0	-1	Und

Period of Tangent and Cotanget:

2) You will notice that the values of $\tan\theta$ repeat every π units. Therefore, the period of the function is _π___. Since $\cot\theta$ is the reciprocal of $\tan\theta$ both functions will have the same period.

Behavior of Tangent for angle values near 90°

Notice also the "behavior" of $\tan\theta$ when the values of θ get closer and closer to $\frac{\pi}{2}$. As the values of θ "approach" $\frac{\pi}{2}$ coming from the left (that is, for angles that are less than $\frac{\pi}{2}$ but getting closer and closer to it), the values of $\tan\theta$ get very large (use your calculator to verify this…). We summarize using the following notation:

$$\text{as } \theta \to \frac{\pi}{2}^-, \quad \tan\theta \to \infty$$

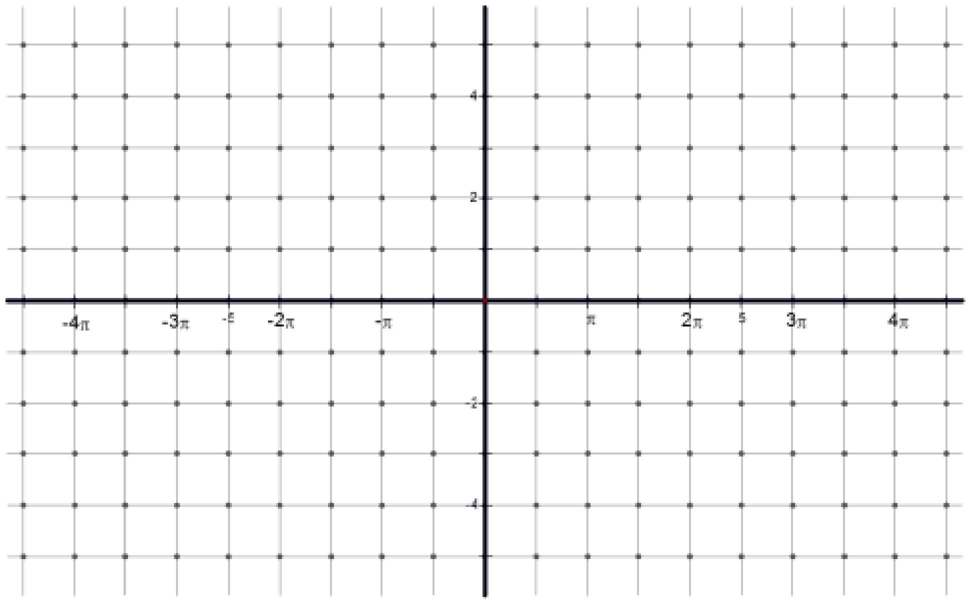
Likewise,

$$\text{as } \theta \to \frac{\pi}{2}^+, \quad \tan\theta \to -\infty$$

We are now ready to graph…

THE GRAPH OF Y=TAN X

3) Use the values of the table above to sketch the graph of y = tan x.



Summary of the graph of y = tan x:

4) Domain: _All reals exc. $\frac{\pi}{2}+\pi K$ where K is an int. 5) Range: _All reals

6) Vertical Asymptotes: _ $\frac{\pi}{2}+\pi K$ where K is an int._____

7) Even/Odd Properties (explain): Odd, -tan(x)=tan(-x)_____

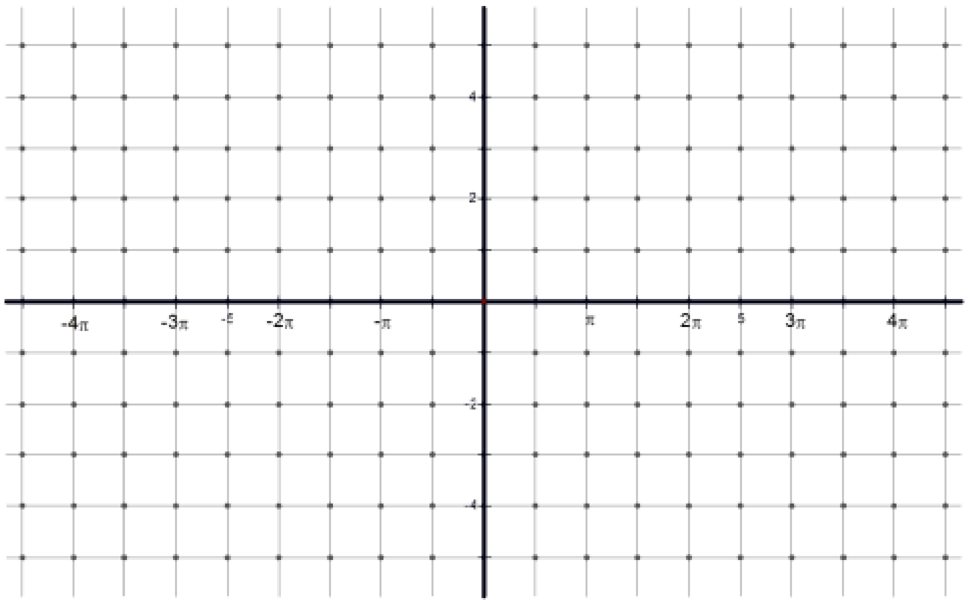
8) x-intercepts: __(0+πK where K is an integer,0)___

9) y-intercepts: __(0,0)________________

10) Period: __π_______

THE GRAPH OF THE COTANGENT FUNCTION Y = COT X

11) Since $\cot \theta = \dfrac{\cos \theta}{\sin \theta} = \dfrac{1}{\tan \theta}$ (the cotangent is the reciprocal of the tangent), the vertical asymptotes for this graph will occur at x-values of the form __0+πK where K is an integer______



Summary of the graph of Y = Cot X:

4) Domain: _All reals exc. π+πK where K is an int. 5) Range: _All reals

6) Vertical Asymptotes: _ π+πK where K is an int.____

7) Even/Odd Properties (explain): Odd, -cotan(x)=cotan(-x)____

8) x-intercepts: __($\frac{\pi}{2}$+πK where K is an integer,0)___

9) y-intercepts: __None_______________

10) Period: __π______

THE GRAPH OF THE COSECANT FUNCTION Y = CSC X

Recall that the cosecant is the **reciprocal** of the sine function:

$$\csc\theta = \frac{1}{\sin\theta}$$

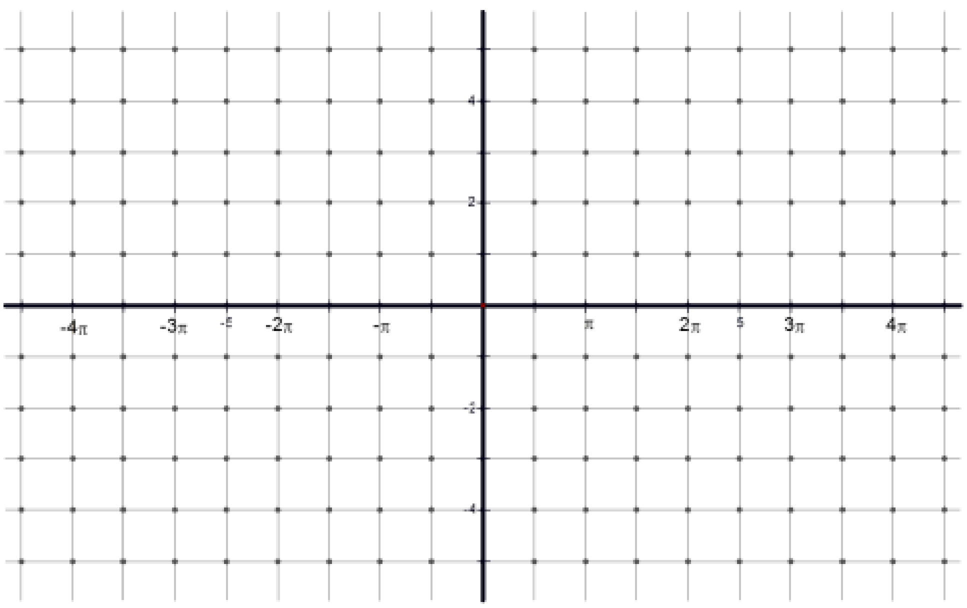
It is natural to expect some "relationship" between the graphs of sine and cosecant. For example, if the **period** of the sine is 2π , we should expect the cosecant to have the same period.

19) Let's complete the table before attempting to graph:

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin\theta$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$\frac{-\sqrt{2}}{2}$	-1	$\frac{-\sqrt{2}}{2}$	0
$\csc\theta$	Und	$\sqrt{2}$	1	$\sqrt{2}$	Und	$-\sqrt{2}$	-1	$-\sqrt{2}$	Und

20) The **vertical asymptotes** of y = csc x will be 0+πK where K is an integer

21) Graph Y = CSC X below (graph first the sine and use it as a guideline…)



22) Domain: All reals exc. πK where K is an int. 23) Range: _(-∞,-1]∪[1,∞)_

24) Vertical Asymptotes: πK where K is an int.__

25) Even/Odd Properties (explain): Odd, -csc(x)=csc(-x)__

26) x-intercepts: __None___________________

27) y-intercepts: ___None_________________

28) Period: __2π______

THE GRAPH OF THE SECANT FUNCTION:

The secant is the **reciprocal** of the cosine function: $\sec\theta = \dfrac{1}{\cos\theta}$

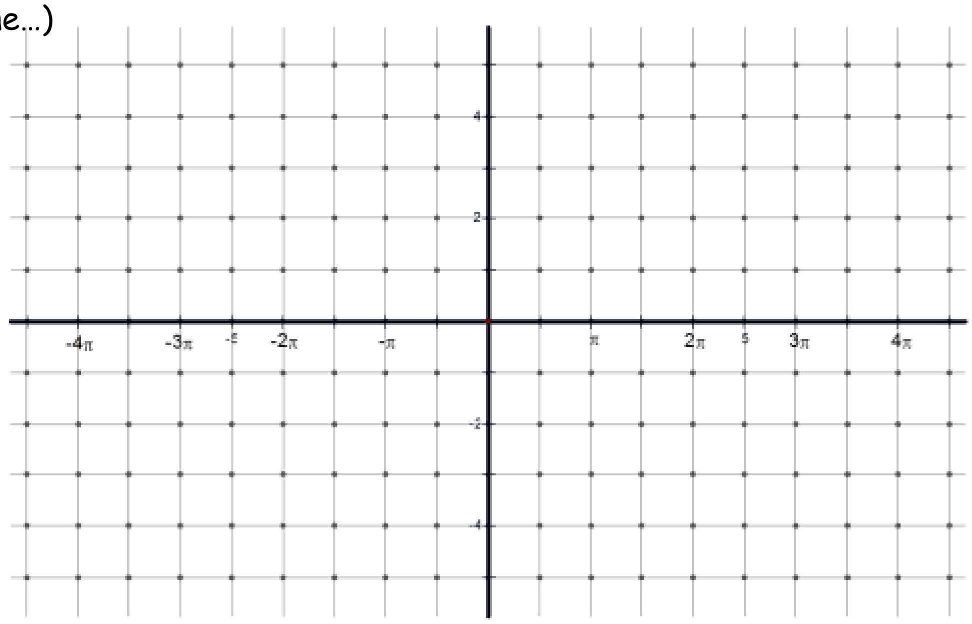
Hence, there will be a "relationship" between the graphs of cosine and secant. For example, the **periods** of both cosine and secant functions is ______.

29) Let's complete the table before attempting to graph:

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\cos\theta$	1	$\frac{\sqrt{2}}{2}$	0	$\frac{-\sqrt{2}}{2}$	-1	$\frac{-\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
$\sec\theta$	1	$\sqrt{2}$	Und	$-\sqrt{2}$	-1	$-\sqrt{2}$	Und	$\sqrt{2}$	1

30) The **vertical asymptotes** of y = sec x will be π/2 +πK where K is an int.

31) Graph Y = SEC X below (graph first the cosine and use it as a guideline…)



22) Domain: All reals exc. $\frac{\pi}{2}$+πK, K is an int. 23) Range: _(-∞,-1]∪[1,∞)____

24) Vertical Asymptotes: $\frac{\pi}{2}$+πK where K is an int.__

25) Even/Odd Properties (explain): Even, sec(x)=sec(-x)__

26) x-intercepts: __None___________________

27) y-intercepts: ___(0,1)________________

28) Period: __2π______