5.6 SINUSOIDAL CURVE FITTING

Graded Assignment

Problem 1: Monthly Temperature

The data in the table represent the average monthly temperatures for Baltimore, Maryland. It was collected over many years which tells us the data will not vary much from year to year. Essentially, the data will repeat each year and so it is cyclical or **periodic.**

Periodic data can be modeled by a sine function (a sinusoid). Other examples of periodic data include tides, hours of daylight, and so on.

The goal of this problem is to come up with a good model for the data.

Month, x	Average Monthly Temperature, °F
January, 1	31.8
February, 2	34.8
March, 3	44.1
April, 4	53.4
May, 5	63.4
June, 6	72.5
July, 7	77.0
August, 8	75.6
September, 9	68.5
October, 10	56.6
November, 11	46.8
December, 12	36.7

Source: U.S. National Oceanic and Atmospheric Administration.

1) Using your calculator, draw a scatter diagram of the data for one period (12 months, of course…).

   steps:

- Using STAT-EDIT, input data into lists L1 and L2; make sure you clear the lists before you start; Enter the months (from 0-12) in L1 and the average monthly temperatures in L2;
- Set up Stat Plot: 2nd STAT PLOT ON, type is scattergram (1st option), XLIST is L1, YLIST is L2;
- Set your window: XMIN=0, XMAX=15, XSCL=1; YMIN=0, YMAX=80 (the max average temperature is 77 degrees in July), YSCL=5;
- Graph…

Observe that you have **one period** of the graph, from January to December of a given year; what would the next sine wave (period) represent?

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2) Find a sinusoidal function (analytically) of the form

$y = A\cos(\varpi x - \phi) + B$ that fits the data.

Step 1: Determine the amplitude A of the function.

$$A = \frac{\text{maximum value} - \text{minimum value}}{2} = \_\_\_\_\_\_\_\_$$

Step 2: Determine the average value B (the vertical shift...)

$$B = \frac{\text{maximum value} + \text{minimum value}}{2} = \_\_\_\_\_\_\_\_$$

Step 3: Determine ϖ. In this case the period is 12 months (the time it takes for the data to repeat). Using the formula $P = \frac{2\pi}{\varpi}$, we cross-multiply to get $\varpi = \frac{2\pi}{P}$. For our data, ϖ = ________.

Step 4: Determine the **phase shift** by observing that in the parent cosine function ($y = \cos\theta$), the high point happens when t=0. Therefore, when modeling with the cosine, the phase shift (or delay) is the length of time between t=0 and the first time the function reaches its maximum value. This happens in July or when t=6. Set $\frac{\phi}{\varpi} = 6$ and solve it to get ϕ. Write the model below.

The cosine function is: ___________________________.

3) Draw the sinusoidal function found in part 2 on the scatter diagram. Is it a pretty good fit? Why or why not?

4) Find a **sine** function that could be used to model the problem. All you need to do is adjust the phase shift. Recall that in the parent sine function,

the average value is attained when t=0. Hence, observe the length of time between t=0 (January) and the first time the function reaches its average value…

The sine function is ______________________________

5) Now use your calculator to generate a sine regression for this problem.

Steps:

- STAT → CALC, down to C (sine regression), enter
- Y=, down to Y2, VARS, down to #5 (statistics), → EQ, enter (the sine regression is placed in Y2)
- GRAPH

How do you compare the graphs of the calculator generated function and the ones you obtained analytically? ____________________________

Use the example above to find a sinusoid that can be used as a model for tides in the problem that follows.

PROBLEM 2 -- TIDES

Suppose that the length of time between consecutive high tides is approximately 12.5 hours. According to the *National Oceanic and Atmospheric Administration*, on Saturday, June 28, 1997, in Juneau, Alaska, high tide occurred at 8:11 a.m. (8.1833 hours) and low tide occurred at 2:14 PM (14.2333 hours). Water heights are measured as the amounts above or below the mean lower low water. The height of the water at high tide was 13.2 feet and the height of the water at low tide was 2.2 feet.

1) Explain what the variables **x** and **y** represent in the sinusoidal model we are trying to find for the problem, which is $y = A \sin(\varpi x - \phi) + B$.

X represents ____

Y represents ____

2) Approximately when will the next high tide occur? _______________

3) Find the sinusoidal model that fits the data. Show work below.

The function is: ___________________________________

Problem 3: Extraterrestrial Being Problem

Researchers find a creature from an alien planet. Its body temperature is varying sinusoidally with time. 35 minutes after they start timing, it reaches a high of 120°F. 20 minutes after that it reaches its next low, 104°F.

a) Sketch a graph of this sinusoid. Include all critical information (high, low, sinusoidal axis, etc…).

b) Write a sinusoidal function that expresses temperature in terms of time (in minutes) since they started timing.

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c) Find the first three times after they started timing at which the temperature was 114°F.

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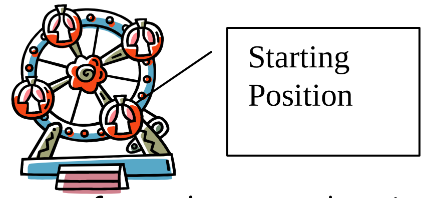
Problem 4: Ferris Wheel Problem



As you ride the Ferris wheel, your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, your seat is at the position shown in the figure above. Let **t** be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 ft above ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 ft.

a) Sketch a graph of this sinusoid.

b) What is the lowest you go as the Ferris wheel turns, and why is this number greater than zero? ________________________________

c) Write an equation of this sinusoid. ___________________________

d) Find the first 2 times at which you are 23 ft above ground.