Sum and difference formulas -

Three uses of the formulas:
- Basic application:Figuring out twelves from memory
- Minor Variation: Backwards problems
- More advanced application: Figuring out two angle problems with triangles
- Conceptual Application: Doing harder identities

Key skills for memory problems.

Recognize how to construct the angle as a difference
(Some fraction work)
Plug-in and evaluate the formula

What if I asked you to evaluate cos(15°) without a calculator?

How would this help?

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(15^o)=\cos(45^o-30^o)=\cos 45^o\cos 30^o-\sin 45^o\sin 30^o$$

1) Some mental gymnastics: 15 can be made from 45 and 30
2) Read/use the formula correctly
{you'll see what I mean when you make a few mistakes.}

Don't write yet!

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\cos(90^o)=\cos(30+60)=\cos(30)\cos(60)-\sin(30)\sin(60)$$

$$\cos(90)=\frac{\sqrt{3}}{2}*\frac{1}{2}-\frac{1}{2}*\frac{\sqrt{3}}{2}$$

$$\sin(90^o)\sin(30+60)=\sin(30)\cos(60)+\cos(30)\sin(60)$$

$$\sin(90)=\frac{1}{2}*\frac{1}{2}+\frac{\sqrt{3}}{2}*\frac{\sqrt{3}}{2}$$

With these formulas we can use the memorized angles:

$$\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\frac{\pi}{2},\frac{2\pi}{3},\frac{3\pi}{4},\frac{5\pi}{6},\pi,\frac{7\pi}{6},\frac{5\pi}{4},\frac{2\pi}{3},\frac{3\pi}{2},\frac{5\pi}{3},\frac{7\pi}{4},\frac{11\pi}{6}$$

To figure out more angles (the twelves):

$$\boxed{\frac{\pi}{12}},\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3},\boxed{\frac{5\pi}{12}},\frac{\pi}{2},\boxed{\frac{7\pi}{12}},\frac{2\pi}{3},\frac{3\pi}{4},\frac{5\pi}{6},\boxed{\frac{11\pi}{12}},\pi$$

$$\boxed{\frac{13\pi}{12}},\frac{7\pi}{6},\frac{5\pi}{4},\frac{2\pi}{3},\boxed{\frac{17\pi}{12}},\frac{3\pi}{2},\boxed{\frac{19\pi}{12}},\frac{5\pi}{3},\frac{7\pi}{4},\frac{11\pi}{6},\boxed{\frac{23\pi}{12}}$$

$$\frac{\pi}{12},\boxed{\frac{2\pi}{12},\frac{3\pi}{12},\frac{4\pi}{12}},\frac{5\pi}{12},\boxed{\frac{6\pi}{12}},\frac{7\pi}{12},\boxed{\frac{8\pi}{12},\frac{9\pi}{12},\frac{10\pi}{12}},\frac{11\pi}{12},\boxed{\frac{12\pi}{12}}$$

$$\frac{13\pi}{12},\boxed{\frac{14\pi}{12},\frac{15\pi}{12},\frac{16\pi}{12}},\frac{17\pi}{12},\boxed{\frac{18\pi}{12}},\frac{19\pi}{12},\boxed{\frac{20\pi}{12},\frac{21\pi}{12},\frac{21\pi}{12}},\frac{24\pi}{12}$$

$$\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}=\frac{4\pi}{12}-\frac{3\pi}{12}=60^o-45^o$$

$$\frac{5\pi}{12}=\frac{\pi}{4}+\frac{\pi}{6}=\frac{3\pi}{12}+\frac{2\pi}{12}=45^o+30^o$$

$$\frac{7\pi}{12}=\frac{\pi}{4}+\frac{\pi}{3}=\frac{3\pi}{12}+\frac{4\pi}{12}=45^o+60^o$$

$$\frac{11\pi}{12}=\frac{3\pi}{4}+\frac{\pi}{6}=\frac{9\pi}{12}+\frac{2\pi}{12}=135^o+30^o$$

$$\frac{13\pi}{12}=\frac{3\pi}{4}+\frac{\pi}{3}=\frac{9\pi}{12}+\frac{4\pi}{12}=135^o+60^o$$

$$\frac{17\pi}{12}=\frac{5\pi}{4}+\frac{\pi}{6}=\frac{15\pi}{12}+\frac{2\pi}{12}=225^o+30^o$$

$$\frac{19\pi}{12}=\frac{5\pi}{4}+\frac{\pi}{3}=\frac{15\pi}{12}+\frac{4\pi}{12}=225^o+60^o$$

$$\frac{23\pi}{12}=\frac{7\pi}{4}+\frac{\pi}{6}=\frac{21\pi}{12}+\frac{2\pi}{12}=315^o+30^o$$

$$\frac{25\pi}{12}=\frac{7\pi}{4}+\frac{\pi}{3}=\frac{21\pi}{12}+\frac{4\pi}{12}=315^o+60^o$$

Where speed comes from in these problems

$$\frac{2\pi}{12}=\frac{\pi}{6}$$

$$\frac{3\pi}{12}=\frac{\pi}{4}$$

$$\frac{4\pi}{12}=\frac{\pi}{3}$$

$$\frac{6\pi}{12}=\frac{\pi}{2}$$

$$\frac{8\pi}{12}=\frac{2\pi}{3}$$

$$\frac{9\pi}{12}=\frac{3\pi}{4}$$

$$\frac{10\pi}{12}=\frac{5\pi}{6}$$

In the packet

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

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$$\cos(90^o)=\cos(30+60)=\cos(30)\cos(60)-\sin(30)\sin(60)$$

$$\cos(90)=\frac{\sqrt{3}}{2}*\frac{1}{2}-\frac{1}{2}*\frac{\sqrt{3}}{2}$$

$$\sin(90^o)\sin(30+60)=\sin(30)\cos(60)+\cos(30)\sin(60)$$

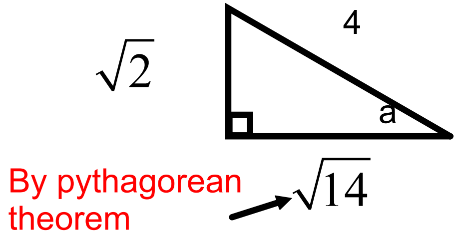
$$\sin(90)=\frac{1}{2}*\frac{1}{2}+\frac{\sqrt{3}}{2}*\frac{\sqrt{3}}{2}$$

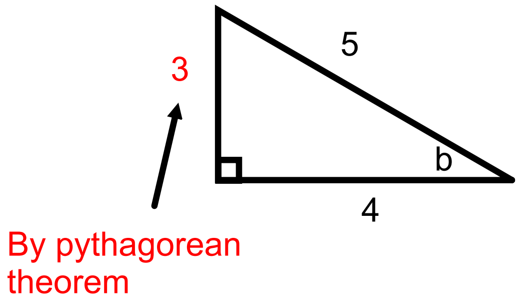
Do packet 1-3,
4 ?

Two triangle problems, just a lot to keep track of USE YOUR PENCIL:

Given: $\sin\alpha = \frac{\sqrt{2}}{4} \quad 0 < \alpha < \frac{\pi}{2}; \cos\beta = \frac{4}{5} \quad -\frac{\pi}{2} < \beta < 0$

Find: $\sin(\alpha+\beta)$ ***<u>KEY: You need BOTH sin and cos for both angles</u>***





Elements:
- Make the triangle so you know all sin/cos
- Get signs correct
- Plug in and solve

$\sin\alpha = \frac{\sqrt{2}}{4} \quad \cos\alpha = \frac{\sqrt{14}}{4}$

And its....Quadrant I, cos is +

$\cos\beta = \frac{4}{5} \quad \sin\beta = \frac{-3}{5}$

And its....Quadrant IV, sin is -

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{\sqrt{2}}{4}\frac{4}{5} + \frac{\sqrt{14}}{4}\frac{-3}{5} = \frac{4\sqrt{2}-3\sqrt{14}}{20}$$

26. $\tan \alpha = \frac{5}{12}, \quad \pi < \alpha < \frac{3\pi}{2}; \quad \sin \beta = -\frac{1}{2}, \quad \pi < \beta < \frac{3\pi}{2}$

24. $\cos\alpha = \dfrac{\sqrt{5}}{5}, \quad 0 < \alpha < \dfrac{\pi}{2}; \quad \sin\beta = -\dfrac{4}{5}, \quad -\dfrac{\pi}{2} < \beta < 0$

30. If $\cos\theta = \frac{1}{4}$, θ in quadrant IV, find the exact value of:

(a) $\sin\theta$

(b) $\sin\left(\theta - \frac{\pi}{6}\right)$

(c) $\cos\left(\theta + \frac{\pi}{3}\right)$

(d) $\tan\left(\theta - \frac{\pi}{4}\right)$

Identities: 32, 42, 48

$$\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}=\frac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}=\frac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta}*\frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}=\frac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\frac{1-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}{1+\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=\frac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\frac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}=\frac{1-\tan\alpha\tan\beta}{1+\tan\alpha\tan\beta}$$

Challenges:

- Mental math tricks.
- Forwards and backwards

How many different angles can you construct by adding or subtracting 30,60, and 45?

How many different angles can you construct by adding or subtracting $\pi/6$,$\pi/3$, and $\pi/4$?

We can get the increments of 15° or $\pi/12$?

30,45,60

cos(60+30)

sin(60+30)

tan(60+30)

cos(60-30)

sin(60-30)

tan(60-30)

sin(45-30
60-45
45+30
45+60