

6.1-6.2 - THE INVERSE TRIGONOMETRIC FUNCTIONS

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Recall the following from section 4.1:

- the horizontal test.
- by $f^{-1}(x)$.
- x function, that is:
$$f(f^{-1}(x)) = f^{-1}(f(x)) = x$$
- The domain of f = the range of f^{-1} and the range of f = the domain of f^{-1} .
- $y=x$.
- $y=x$.

Part I: THE INVERSE SINE FUNCTION

1) $\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ, \pi/6$

2) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -60^\circ, -\pi/3$

3) $\sin^{-1}(1) = 90^\circ, \pi/2$

4) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -45^\circ, -\pi/4$

5) Undefined. Sin can not be >1

6) That is outside the range.

7) $\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = .5$

8) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \pi/6$

9) $5\pi/6$ is not in the range of \sin^{-1} . \sin^{-1} maps .5 to $\pi/6$.

Part II: THE INVERSE COSINE FUNCTION

Exercises: Evaluate without a calculator.

$$1) \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ, 2\pi/3 \quad 2) \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ, \pi/6$$

$$3) \cos^{-1}(-1) = \pi \quad 4) \cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) = 3\pi/4$$

$$5) \cos\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = -\sqrt{3}/2 \quad 6) \cos^{-1}\left(\cos\frac{7\pi}{4}\right) = \pi/4$$

7) $7\pi/4$ is outside the range of \cos^{-1} .

$$8) \sin\left(\cos^{-1}\left(-\frac{1}{2}\right)\right) = \sqrt{3}/2$$

$$9) \cos\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = 1/2$$

$$10) \cos\left(\sin^{-1}\frac{4}{5}\right) = a$$

$$11) \cos\left(\sin^{-1}\frac{1}{4}\right) = d$$

$$(a) \frac{\sqrt{15}}{2}$$

$$(b) \frac{4\sqrt{15}}{2}$$

$$(c) \frac{2\sqrt{15}}{15}$$

$$(d) \frac{\sqrt{15}}{4}$$

Problems: Give the value without using a calculator.

1) $\tan^{-1}(1) = 45^\circ$ 2) $\tan^{-1}(0) = 0^\circ$ 3) $\tan^{-1}(-1) = -45^\circ$

4) $\tan^{-1}(\tan 45^\circ) = 45^\circ$ 5) $\tan^{-1}(\tan 135^\circ) = -45^\circ$

6) $\tan(\tan^{-1}(\sqrt{3})) = \sqrt{3}$ 7) $\tan(\tan^{-1}(-1)) = -1$

8) Explain why $\tan(\tan^{-1} x)$ will always equal x , but $\tan^{-1}(\tan x)$ will not necessarily equal x . Because the domain of $\tan x$ is larger than the range of inverse \tan .

9) A

10) B

11) B

12) ___ The domain of \sin^{-1} is -1 to 1, the domain of \tan^{-1} is all reals ___

13) $\cos(\tan^{-1} 0.5) = .89$ 14) $\tan(\sin^{-1} .12251014) = .12$

15) $\tan\left(\cos^{-1} -\frac{3}{5}\right) = -4/3$

16) $\sin\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = 5/13$