6.3 TRIGONOMETRIC IDENTITIES

What is an Identity?

DEFINITION of IDENTITY:

> Two functions f and g are said to be **identically equal** if
>
> $$f(x) = g(x)$$
>
> For every value of x in the domains of both f and g.

Examples:

1) $(x + 3)^2 = x^2 + 6x + 9$ is an identity, by the definition above. Also, the sides can be restated to match based on a definition.

2) $2x + 3 = 11$ is not an identity, because the two sides are only equal if x has a certain value. It is a **conditional equation.**

Which are identities? If not an identity, explain why.

3) $\sin^2 x + \cos^2 x = 1$ Identity because the left side = 1 for all x's

4) $\sec x = \dfrac{1}{\cos x}$ Identity because no matter what the x the two sides are equal.

5) $x^2 + 4x + 3 = 0$ Not an identity. For many x's the two sides are not equal

SUMMARY OF TRIGONOMETRIC IDENTITIES

QUOTIENT IDENTITIES:

$\tan\theta = \underline{\quad\quad}$ $\cot\theta = \underline{\quad\quad}$

Sinθ/Cosθ Cosθ/sinθ

RECIPROCAL IDENTITIES:

$\cos\theta = \frac{1}{\quad}$ $\sec\theta = \underline{\quad\quad}$ $\cot\theta = \underline{\quad\quad}$

1/Secθ 1/cosθ 1/tanθ

EVEN-ODD IDENTITIES:

$\sin(-\theta) =$ -sin(θ) $\cos(-\theta) =$ cos(θ) $\tan(-\theta) =$ -tan(θ)

$\csc(-\theta) =$ -csc(θ) $\sec(-\theta) =$ sec(θ) $\cot(-\theta) =$ -cot(θ)

PYTHAGOREAN IDENTITIES:

$\sin^2\theta + \cos^2\theta = \underline{\quad\quad}$ 1

$\tan^2\theta + 1 = \underline{\quad\quad\quad\quad}$ $Sec^2\theta$

$1 + \cot^2\theta = \underline{\quad\quad\quad\quad}$ $Csc^2\theta$

PROBLEMS

Some of the identities above can be rewritten with minor variations.
Complete as appropriate:

1) $\sin^2 x = \underline{\quad\quad\quad}$ $1-\cos^2x$ 2) $\sec^2 x - 1 = \underline{\quad\quad\quad}$ $\tan^2x$

3) $1 - \csc^2 x = \underline{\quad\quad\quad}$ $-cotan^2x$

Establish the identities: (**meaning: prove that the statement is true by applying definitions/identities)**

4) $\mathbf{\sec\theta \cdot \cot\theta = \csc\theta}$ $\frac{1}{cos\theta} * \frac{cos\theta}{sin\theta} = \frac{1}{sin\theta} = csc\theta$

5) $\mathbf{\sec\theta \cdot \sin\theta = \tan\theta}$ $\frac{1}{cos\theta} * sin\theta = \frac{sin\theta}{cos\theta} = tan\theta$

6) $\mathbf{\sin\theta(\cot\theta + \tan\theta) = \sec\theta}$ $sin\theta\left(\frac{cos\theta}{sin\theta} + \frac{sin\theta}{cos\theta}\right) = cos\theta + \frac{sin^2\theta}{cos\theta} = \frac{cos^2\theta + sin^2\theta}{cos^2\theta} = \frac{1}{cos\theta} = sec\theta$

7) $\mathbf{\sin(-\theta) + \sin(\theta) = 0}$ -sin(θ)+sin(θ)=0

8) $\mathbf{1 + \cot^2(-\theta) = \csc^2\theta}$

Lots of ways to do this one.

Way 1: $1 + \left(cot(-\theta)\right)^2 = 1 + \left(-cot(\theta)\right)^2 = 1 + cot^2(\theta) = csc^2(\theta)$

Way 2:

$1 + \frac{cos^2(-\theta)}{sin^2(-\theta)} = 1 + \frac{cos^2(\theta)}{(-\sin(\theta))^2} = 1 + \frac{cos^2(\theta)}{sin^2(\theta)} = \frac{sin^2(\theta) + cos^2(\theta)}{sin^2(\theta)} = \frac{(\sin(\theta))^2 + (cos(\theta))^2}{(\sin(\theta))^2} = \frac{1}{sin^2(\theta)} = csc^2(\theta)$

9) $\mathbf{\sin\theta\csc\theta - \cos^2\theta = \sin^2\theta}$ $sin\theta * \frac{1}{sin\theta} - cos^2\theta = 1 - cos^2\theta = sin^2\theta + cos^2\theta - cos^2\theta = sin^2\theta$

10) $\mathbf{(\csc\theta - 1)(\csc\theta + 1) = \cot^2\theta}$ $\left(\frac{1}{sin\theta} - 1\right)\left(\frac{1}{sin\theta} + 1\right) = \frac{1}{sin^2\theta} - 1 = \frac{1 - sin^2\theta}{sin^2\theta} = \frac{sin^2\theta + cos^2\theta - sin^2\theta}{sin^2\theta} = \frac{cos^2\theta}{sin^2\theta} = cot^2\theta$

11) $\mathbf{(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = 1}$ $\left(\frac{1}{sin\theta} - \frac{cos\theta}{sin\theta}\right)\left(\frac{1}{sin\theta} + \frac{cos\theta}{sin\theta}\right) = \frac{1}{sin^2\theta} - \frac{cos^2\theta}{sin^2\theta} = \frac{1 - cos^2\theta}{sin^2\theta} = \frac{sin^2(\theta) + cos^2(\theta) - cos^2(\theta)}{sin^2(\theta)} = \frac{sin^2\theta}{sin^2\theta} = 1$

12) $\mathbf{\frac{1 + \tan\theta}{1 + \cot\theta} = \tan\theta}$

$\frac{1 + \frac{sin\theta}{cos\theta}}{1 + \frac{cos\theta}{sin\theta}} = \frac{\frac{cos\theta + sin\theta}{cos\theta}}{\frac{sin\theta + cos\theta}{sin\theta}} = \frac{cos\theta + sin\theta}{cos\theta} * \frac{sin\theta}{sin\theta + cos\theta} = \frac{sin\theta(cos\theta + sin\theta)}{cos\theta(sin\theta + cos\theta)} = \frac{sin\theta(cos\theta + sin\theta)}{cos\theta(cos\theta + sin\theta)} = \frac{sin\theta}{cos\theta} = tan\theta$

13) $\csc\theta - \cot\theta = \dfrac{\sin\theta}{1+\cos\theta}$ $\frac{1}{sin\theta} - \frac{cos\theta}{sin\theta} = \frac{1-cos\theta}{sin\theta} = \frac{1-cos\theta}{sin\theta} * \frac{1+cos\theta}{1+cos\theta} =$
$\frac{1-cos^2\theta}{sin\theta(1+cos\theta)} = \frac{cos^2\theta+sin^2\theta-cos^2\theta}{sin\theta(1+cos\theta)} = \frac{sin^2\theta}{sin\theta(1+cos\theta)} = \frac{sin\theta}{1+cos\theta}$

14) $1 - \dfrac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$ $\frac{1-cos\theta-sin^2\theta}{1-cos\theta} = \frac{sin^2\theta+cos^2\theta-cos\theta-sin^2\theta}{1-cos\theta} =$
$\frac{-cos\theta+cos^2\theta}{1-cos\theta} = \frac{-cos\theta(1-cos\theta)}{1-cos\theta} = -cos\theta$