6.4 SUM AND DIFFERENCE FORMULAS

(i) $\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$

(ii) $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$

(iii) $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \cdot \tan\beta}$

Make sure you use the signs $\pm$ and $\mp$ in the correct correspondence!

Prove the following identities using the formulas above. In #1 and 2, graph to verify before you show algebraically the identity is true.

1) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$ $\cos\frac{\Pi}{2}\cos\theta + \sin\frac{\Pi}{2}\sin\theta = 0 * \cos\theta + 1 * \sin\theta = \sin\theta$

2) $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\sin\frac{\Pi}{2}\cos\theta - \sin\theta\cos\frac{\Pi}{2} = 1 * \cos\theta - 0 * \sin\theta = \cos\theta$

NO CALCULATORS FROM THIS POINT ON!!!!!

3) $\cos 15° = \cos\frac{\Pi}{4}\cos\frac{\Pi}{6} + \sin\frac{\Pi}{4}\sin\frac{\Pi}{6} = \frac{\sqrt{2}}{2} * \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} * \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

4) $\sin\frac{\pi}{12} = \sin\frac{\Pi}{4}\cos\frac{\Pi}{6} - \sin\frac{\Pi}{6}\cos\frac{\Pi}{4} = \frac{\sqrt{2}}{2} * \frac{\sqrt{3}}{2} - \frac{1}{2} * \frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

5) $\cos\frac{7\pi}{12} = \cos\frac{\Pi}{3}\cos\frac{\Pi}{4} - \sin\frac{\Pi}{3}\sin\frac{\Pi}{4} = \frac{1}{2} * \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} * \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

$$\frac{7\Pi}{12} = \frac{4\Pi}{12} + \frac{3\Pi}{12} = \frac{\Pi}{3} + \frac{\Pi}{4}$$

6) $\cos 80° \cos 20° + \sin 80° \sin 20°$ =cos60°=1/2

7) Given $\sin\beta = -\frac{4}{5}, \quad -\frac{\pi}{2} < \beta < 0$, and $\cos\alpha = \frac{\sqrt{5}}{5}, 0 < \alpha < \frac{\pi}{2}$

Find $\sin(\alpha + \beta)$. $\cos\beta = \frac{3}{5}$, $\sin\alpha = \frac{2\sqrt{5}}{5}$

$$sin\alpha cos\beta + cos\alpha sin\beta = \frac{2\sqrt{5}}{5} * \frac{3}{5} + \frac{\sqrt{5}}{5} * \frac{4}{5} = \frac{6\sqrt{5} - 4\sqrt{5}}{25} = \frac{2\sqrt{5}}{25}$$

8) Prove algebraically that $\tan(\theta + \pi) = \tan\theta$.

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \cdot \tan\beta}$$

$$\frac{tan\theta + tan\pi}{1 - tan\theta tan\pi} = \frac{tan\theta + 0}{1 + tan\theta * 0} = \frac{tan\theta}{1} = tan\theta$$

9) $\tan\left(\frac{7\pi}{12}\right) =$

$\tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right) = \frac{tan\frac{\pi}{4} + tan\frac{\pi}{3}}{1 - tan\frac{\pi}{4}tan\frac{\pi}{3}} = \frac{1+\sqrt{3}}{1-1*\sqrt{3}} = \frac{1+\sqrt{3}}{1-\sqrt{3}} * \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{1+2\sqrt{3}+3}{1-3} = \frac{4+2\sqrt{3}}{-2} = \frac{-2-\sqrt{3}}{2}$

$$\frac{7\Pi}{12} = \frac{4\Pi}{12} + \frac{3\Pi}{12} = \frac{\Pi}{3} + \frac{\Pi}{4}$$

10) $\cos 40° \cos 10° + \sin 40° \sin 10°$ =cos(30)=$\frac{\sqrt{3}}{2}$

11) $\cos\frac{5\pi}{12}\cos\frac{7\pi}{12} - \sin\frac{5\pi}{12}\sin\frac{7\pi}{12}$ =-1

12) $\sin\frac{\pi}{18}\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18}$ =$\frac{\sqrt{3}}{2}$