1) Given that $\sin\theta = \frac{4}{5}$ *and* $\frac{\pi}{2} < \theta < \pi$:

a) Find **sin(2θ)** $\cos(\theta) = \frac{-3}{5}$

$$sin(2\theta) = 2sin\theta cos\theta = 2 * \frac{4}{5} * \frac{-3}{5} = \frac{-24}{25}$$

b) Find **cos(2θ)** $= 1 - 2sin^2\theta = 1 - 2 * \left(\frac{4}{5}\right)^2 = \frac{-7}{25}$

2) Develop the formula for **tan(2θ)** in terms of tan**θ**.

$$tan(2\theta) = \frac{sin(2\theta)}{cos(2\theta)} = \frac{2sin\theta cos\theta}{cos^2\theta - sin^2\theta} * \frac{\frac{1}{cos^2\theta}}{\frac{1}{cos^2\theta}} = \frac{2\frac{sin\theta}{cos\theta}}{1 - \frac{sin^2\theta}{cos^2\theta}}$$

$$= \frac{2tan\theta}{1 - tan^2\theta}$$

3) Use your formula to find **tan(2θ)** if θ in an angle in the 3rd quadrant such that $\sin\theta = -\frac{3}{4}$

$$tan(2\theta) = \frac{2tan\theta}{1 - tan^2\theta} = \frac{2 * \frac{3}{\sqrt{7}}}{1 - \left(\frac{3}{\sqrt{7}}\right)^2} = \frac{\frac{6}{\sqrt{7}}}{\frac{7-9}{7}} = \frac{6}{\sqrt{7}} * \frac{-7}{2} = \frac{-42}{2\sqrt{7}}$$

$$= \frac{-42 * \sqrt{7}}{14} = -3\sqrt{7}$$

(*i*) $\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$

(*ii*) $\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$

(*iii*) $\tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

(see original functions, solve for the right thing. III is I over II

HALF-ANGLE FORMULAS

(i) $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ (depending on quadrant!)

(ii) $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$

(iii) $\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

Use Half-angle formulas to find the exact value:

1) $\sin(-15°)$ $\frac{\alpha}{2} = -15^{o}$ $\alpha = -30^{o}$ $sin(-15^{o}) = -\sqrt{\frac{1-cos(-30^{o})}{2}} =$

$-\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{3}}{4}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

2) $\cos 22.5°$ $\frac{\alpha}{2} = 22.5^{o}$ $\alpha = 45^{o}$ $cos(22.5^{o}) = \sqrt{\frac{1+cos(45^{o})}{2}} =$

$\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{2}$

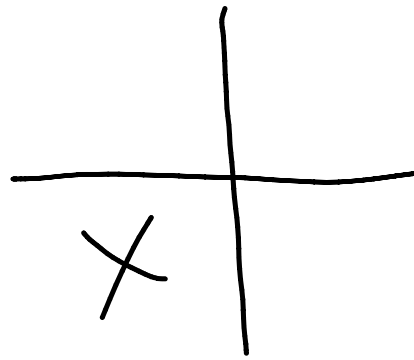
3) $\tan(67.5°)$ $\frac{\alpha}{2} = 67.5^{o}$ $\alpha = 135^{o}$ $tan(67.5^{o}) = \sqrt{\frac{1-cos(135^{o})}{1+cos(135^{o})}} =$

$\sqrt{\frac{1+\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}}} = \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{\frac{2-\sqrt{2}}{2}}} = \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}} = \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}} * \frac{2+\sqrt{2}}{2+\sqrt{2}}} = \sqrt{\frac{4+4\sqrt{2}+2}{4-2}} = \sqrt{3+2\sqrt{2}}$

4) $\sin\left(\frac{7\pi}{8}\right)$ $\frac{\alpha}{2} = \frac{7\pi}{8}$ $\alpha = \frac{7\pi}{4}$ $sin\left(\frac{7\pi}{8}\right) = \sqrt{\frac{1-cos\left(\frac{7\pi}{4}\right)}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} =$

$\sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$

5) If $\tan\theta = \frac{1}{2}$, $\pi < \theta < \frac{3\pi}{2}$, find: $sin\theta = \frac{-\sqrt{5}}{5}$ $cos\theta = \frac{-2\sqrt{5}}{5}$



a) $\sin\frac{\theta}{2} = -\sqrt{\frac{1-cos(\theta)}{2}} = -\sqrt{\frac{1+\frac{2\sqrt{5}}{5}}{2}} = -\sqrt{\frac{5+2\sqrt{5}}{10}}$

b) $\cos\frac{\theta}{2} = -\sqrt{\frac{1+cos(\theta)}{2}} = -\sqrt{\frac{1-\frac{2\sqrt{5}}{5}}{2}} = -\sqrt{\frac{5-2\sqrt{5}}{10}}$