

1) Given that  $\sin \theta = \frac{4}{5}$  and  $\frac{\pi}{2} < \theta < \pi$ :

a) Find  $\sin(2\theta)\cos(\theta) = \frac{-3}{5}$

$$\sin(2\theta) = 2\sin\theta\cos\theta = 2 * \frac{4}{5} * \frac{-3}{5} = \frac{-24}{25}$$

b) Find  $\cos(2\theta) = 1 - 2\sin^2\theta = 1 - 2 * \left(\frac{4}{5}\right)^2 = \frac{-7}{25}$

2) Develop the formula for  $\tan(2\theta)$  in terms of  $\tan\theta$ .

$$\begin{aligned}\tan(2\theta) &= \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} * \frac{\frac{1}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \frac{2\frac{\sin\theta}{\cos\theta}}{1 - \frac{\sin^2\theta}{\cos^2\theta}} \\ &= \frac{2\tan\theta}{1 - \tan^2\theta}\end{aligned}$$

3) Use your formula to find  $\tan(2\theta)$  if  $\theta$  is an angle in the 3<sup>rd</sup> quadrant such that  $\sin \theta = -\frac{3}{4}$

$$\begin{aligned}\tan(2\theta) &= \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2 * \frac{3}{\sqrt{7}}}{1 - \left(\frac{3}{\sqrt{7}}\right)^2} = \frac{\frac{6}{\sqrt{7}}}{\frac{7-9}{7}} = \frac{6}{\sqrt{7}} * \frac{-7}{2} = \frac{-42}{2\sqrt{7}} \\ &= \frac{-42 * \sqrt{7}}{14} = -3\sqrt{7}\end{aligned}$$

#### IMPORTANT VARIATIONS OF THE DOUBLE-ANGLE FORMULAS

$$(i) \quad \sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$(ii) \quad \cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$(iii) \quad \tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

4) Establish identity (i), (ii), and (iii) below.