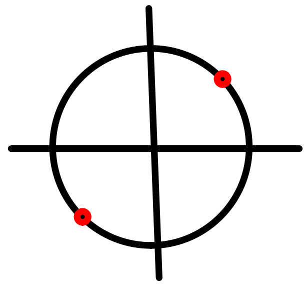
Solving non-one-to-one functions: You have done it before.

Instructions- "Solve:" $\tan\theta = 1$

This asks "What angles have a tangent of 1?"

By analysis of the circle there are infinite answers:

$$\ldots,\frac{\pi}{4},\frac{3\pi}{4},\frac{9\pi}{4},\frac{11\pi}{4},\ldots$$



$$\theta = \frac{\pi}{4} + k\pi$$

How would we write the steps?

$$\tan\theta = 1$$

$$\tan^{-1}(\tan\theta) = \tan^{-1}(1)$$

$$\theta = \tan^{-1}(1)$$

Taking the inverse, not evaluating an inverse function

$$\theta = \ldots,\frac{\pi}{4},\frac{3\pi}{4},\frac{9\pi}{4},\frac{11\pi}{4},\ldots = \frac{\pi}{4} + \pi k$$

By cirlce analysis

By coterminal angles

Solve:

$$\tan\theta = 1$$

$$\tan^{-1}(\tan\theta) = \tan^{-1}(1)$$

$$\theta = \tan^{-1}(1)$$

$$\theta = \ldots, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \ldots = \frac{\pi}{4} + \pi k$$

Solve:

$$x^2 = 4$$

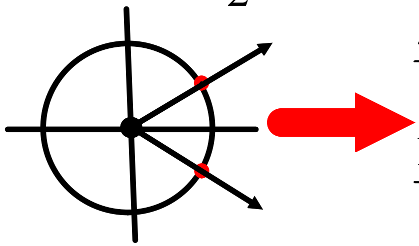
$$\sqrt{x^2} = \pm\sqrt{4}$$

$$x = \pm 2$$

Taking the inverse, not evaluating an inverse function

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ gives one answer.

BUT $\cos\theta = \frac{\sqrt{3}}{2}$ has multiple solutions



$\frac{\pi}{6} + 2\pi k$ *where...*

or

$\frac{11\pi}{6} + 2\pi k$ *where...*

PART 2: Solving non-one-to-one functions: You have done it before.

Solving complex problems.

a) Solve for the embedded not one-to-one function.
b) Identify the non-function's multiple answers
c) Set up equations to solve for those answers

$$a)\ 2\,|3x-6|+5=17$$

$$|3x-6|=6$$

$$b-c)\ 3x-6=6 \qquad 3x-6=-6$$

x=4 x=0

$$a)\ 3(2x-3)^2-4=23$$

$$(2x-3)^2=9$$

$$b-c)\ 2x-3=+\sqrt{9} \qquad 2x-3=-\sqrt{9}$$

x=2 x=0

PART 2: Solving non-one-to-one functions: You have done it before.

Solving complex problems.

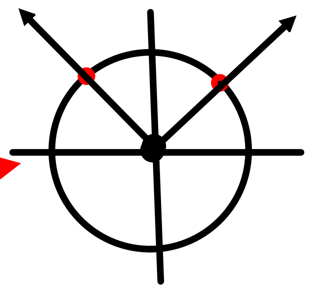
a) Solve for the embedded not one-to-one function.
b) Identify the non-function's multiple answers
c) Set up equations to solve for those answers

$$a)\ 10\sin(\theta) - 4 = 1$$

$$10\sin(\theta) = 5$$

$$\sin(\theta) = \frac{1}{2}$$

$$b-c)\ \theta = \sin^{-1}\left(\frac{1}{2}\right)$$



$$\theta = \frac{\pi}{6} + 2k\pi \ where...$$

$$or$$

$$\theta = \frac{5\pi}{6} + 2k\pi \ where...$$

YOU MUST BE GOOD AT UNIT CIRCLE ANALYSIS

"General form" versus "solving on the interval"

General form:

$$\sin(2\theta) = \frac{\sqrt{2}}{2}$$

$$\sin(A) = \frac{\sqrt{2}}{2}$$

$$A = \frac{\pi}{4} + 2\pi k$$

$$A = \frac{3\pi}{4} + 2\pi k$$

$$\theta = \frac{\pi}{8} + \pi k \quad or$$

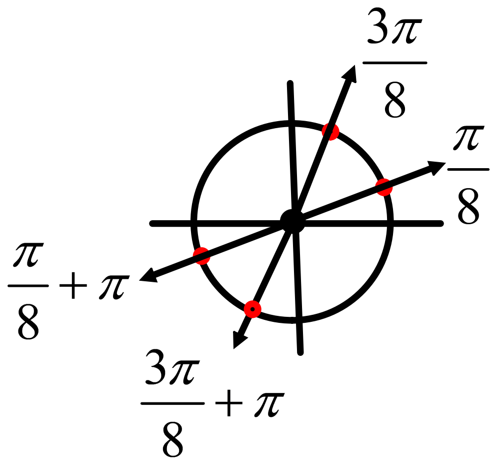
$$\theta = \frac{3\pi}{8} + \pi k$$

On the interval:

$$0 \le \theta < 2\pi$$

Same analyis, but instead of describing the infinite list of answers, just name the ones in this interval.

BUT YOU HAVE TO FIGURE THESE OUT.



General form is a lot less work.

Solve the equation on the interval

$$0 \le \theta < 2\pi$$

$$1 - \cos\theta = \frac{1}{2}$$

$$4\cos^2\theta - 3 = 0$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \pm\sqrt{\frac{3}{4}}$$

$$\cos\theta = \pm\frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{6} + 2\pi k$$

$$\theta = \frac{11\pi}{6} + 2\pi k$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

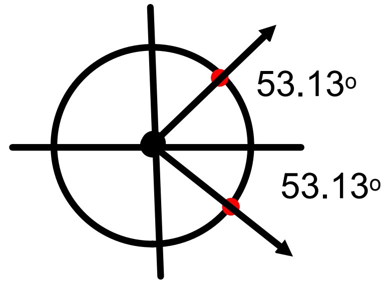
$$\theta = \frac{5\pi}{6} + 2\pi k$$

$$\theta = \frac{7\pi}{6} + 2\pi k$$

$$\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$$

$$\cos\theta = .6$$

By calculator:



$$\cos^{-1}(.6) = 53.13^o$$

$$\theta = 53.13^o + 360k$$

or

$$\theta = 306.87^o + 360k$$

0) Original equation

$$a * \sin(bx + d) - e = g$$

milestone 1) Use algebra to get:

$$\sin\left(A\right) = \#$$

milestone 2) Use circle analyis to get:

$$A = \frac{\#\pi}{\#} + 2\pi k \ OR \ A = \frac{\#\pi}{\#} + 2\pi k \ OR...$$

milestone 3) Undo substitute and solve to get.

$$\theta = \frac{\#\pi}{\#} + \#\pi k \ OR \ \theta = \frac{\#\pi}{\#} + \#\pi k \ OR...$$

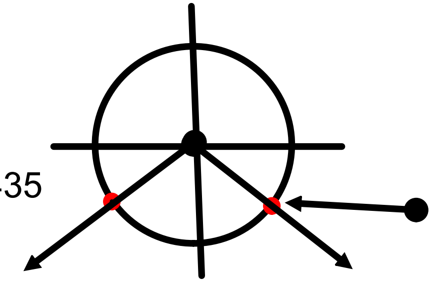
milestone 4) If needed find values in range from each list: {#,#,#}

Use your calculator to solve:

sinθ=.6

sinθ=−.6

Hard, requires clever use of circle



Factoring and left hand side:

$$2\cos\theta\sin\theta = \sin\theta$$
$$2\cos\theta\sin\theta - \sin\theta = 0$$
$$\sin\theta(2\cos\theta - 1) = 0$$

Use identities

$$\cos(2\theta) = 1$$
$$2\cos^2\theta - 1 = 1$$
$$\cos^2\theta = 1$$

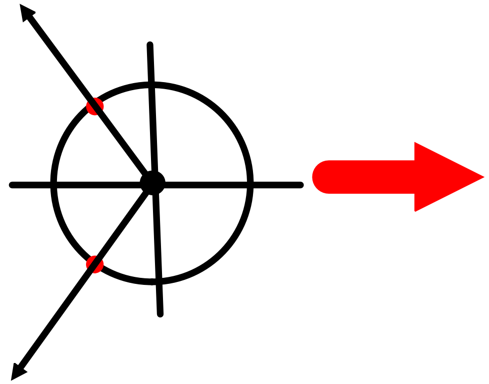
Another variation:

Solve:

$$\cos(4\theta) = \frac{-1}{2}$$

$$say\ A = 4\theta$$

$$\cos(A) = \frac{-1}{2}$$



$$i) A = \frac{2\pi}{3} + 2k\pi \ where...$$

$$ii) A = \frac{4\pi}{3} + 2k\pi \ where...$$

$$i) 4\theta = \frac{2\pi}{3} + 2\pi k \qquad ii) 4\theta = \frac{4\pi}{3} + 2\pi k$$

$$\theta = \frac{\pi}{6} + \frac{1}{2}\pi k \qquad \theta = \frac{\pi}{3} + \frac{1}{2}\pi k$$

These angles, when multiplied by 4, have a cos of -1/2.

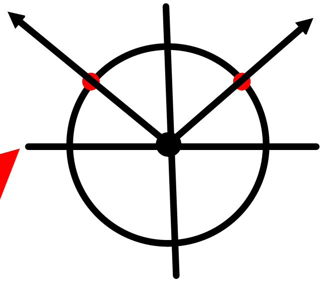
$$2\sin\left(\frac{1}{3}\theta-\frac{\pi}{2}\right)+4=5$$

$$A=\frac{1}{3}\theta-\frac{\pi}{2}$$

$$2\sin(A)=1$$

$$\sin(A)=\frac{1}{2}$$

$$A=\sin^{-1}\left(\frac{1}{2}\right)$$



$$i)A=\frac{\pi}{6}+2\pi k \text{ where...}$$

$$ii)A=\frac{5\pi}{6}+2\pi k \text{ where...}$$

$$i)\frac{1}{3}\theta-\frac{\pi}{2}=\frac{\pi}{6}+2\pi k$$

$$\frac{1}{3}\theta=\frac{4\pi}{6}+2\pi k$$

$$\theta=2\pi+6\pi k$$

$$ii)\frac{1}{3}\theta-\frac{\pi}{2}=\frac{5\pi}{6}+2\pi k$$

$$\frac{1}{3}\theta=\frac{8\pi}{6}+2k\pi$$

$$\theta=4\pi+6k\pi$$

$$2\cos^2\theta+\cos\theta-1=0$$
$$2\cos^2\theta+2\cos\theta-1\cos\theta-1=0$$
$$2\cos\theta\left(\cos\theta+1\right)-\left(\cos\theta+1\right)=0$$
$$\left(2\cos\theta-1\right)\left(\cos\theta+1\right)=0$$

$$\cos^2\theta-\sin^2\theta+\sin\theta=0$$
$$1-\sin^2\theta-\sin^2\theta+\sin\theta=0$$
$$1-2\sin^2\theta+\sin\theta=0$$
$$-2\sin^2\theta+\sin\theta+1=0$$
$$-2\sin^2\theta+2\sin\theta-\sin\theta+1=0$$
$$2\sin\theta(1-\sin\theta)+\left(1-\sin\theta\right)=0$$
$$\left(2\sin\theta+1\right)\left(1-\sin\theta\right)=0$$

$$\sin^2\theta = 2\cos\theta + 2$$

$$1 - \cos^2\theta = 2\cos\theta + 2$$

$$0 = \cos^2\theta + 2\cos\theta + 1$$

$$0 = \left(\cos\theta + 1\right)^2$$

$\sin\theta = \csc\theta$

When does sin=1 or -1

$$\cos(6\theta)+4\sin^2(3\theta)-\sin(3\theta)-1=0$$

$$A=3\theta$$

$$\cos(2A)+4\sin^2(A)-\sin(A)-1=0$$

$$1-2\sin^2(A)+4\sin^2(A)-\sin(A)-1=0$$

$$2\sin^2(A)-\sin(A)=0$$

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$$\cos(2\theta)+\cos(4\theta)=0$$