

6.7 TRIGONOMETRIC EQUATIONS

- Trigonometric **Identities** versus Trigonometric **Equations**
What's the difference?

Examples:

1) Find all solutions to the equation $\cos \theta = \frac{1}{2}$.

$$\left\{ \theta \mid \theta = \frac{\pi}{3} + 2\pi k, \theta = \frac{5\pi}{3} + 2\pi k \right\}$$

2) Solve $2 \cos \theta + \sqrt{3} = 0$, $0 \leq \theta < 2\pi$

$$2 \cos \theta + \sqrt{3} = 0$$

$$\cos \theta = \frac{-\sqrt{3}}{2}$$

$$\left\{ \frac{5\pi}{6}, \frac{7\pi}{6} \right\}$$

3) Solve: $\cos(2\theta) = \frac{1}{2}$, $0 \leq \theta < 2\pi$

$$a = 2\theta, \cos(a) = \frac{1}{2}, \left\{ a \mid a = \frac{\pi}{3} + 2\pi k, a = \frac{5\pi}{3} + 2\pi k \right\}$$

$$2\theta = \frac{\pi}{3} + 2\pi k \Rightarrow \theta = \frac{\pi}{6} + \pi k$$

$$2\theta = \frac{5\pi}{3} + 2\pi k \Rightarrow \theta = \frac{5\pi}{6} + \pi k$$

$$\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

4) Use a calculator to solve the equation:

$$\cos \theta = 0.4, \text{ where } 0 \leq \theta < 2\pi$$

$$\{1.1593, 5.1239\}$$

5) Solve: $\sin \theta = -.75$

$$\{3.9890, 5.4351\}$$

Solving a variety of Trigonometric Equations: Quadratic in Form, using identities, and using a graphing calculator.

Solve each equation on the interval $0 \leq \theta < 2\pi$.

1) $2 \sin^2 \theta + \sin \theta = 0$

$$\sin(\theta)(2\sin\theta + 1) = 0$$

$$\sin(\theta) = 0 \Rightarrow \theta = \pi k$$

$$2\sin\theta + 1 = 0 \Rightarrow \sin\theta = \frac{-1}{2} \Rightarrow \theta = \frac{7\pi}{6} + 2\pi k, \theta = \frac{11\pi}{6} + 2\pi k$$

$$\left\{0, \frac{7\pi}{6}, \pi, \frac{11\pi}{6}\right\}$$

2) $(2 \cos \theta - 1)(\sin \theta + 1) = 0$

$$2 \cos(\theta) - 1 = 0 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} + 2\pi k, \theta = \frac{5\pi}{3} + 2\pi k$$

$$\sin\theta + 1 = 0 \Rightarrow \sin\theta = -1 \Rightarrow \theta = \frac{3\pi}{2} + 2\pi k$$

$$\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \frac{3\pi}{2}\right\}$$

3) $2 \cos^2 \theta - \cos \theta - 1 = 0$

$$(2\cos\theta + 1)(\cos\theta - 1) = 0$$

$$2\cos\theta + 1 = 0 \Rightarrow \cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3} + 2\pi k, \theta = \frac{4\pi}{3} + 2\pi k$$

$$\cos\theta - 1 = 0 \Rightarrow \cos\theta = 1 \Rightarrow \theta = \pi + 2\pi k$$

$$\left\{\pi, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$$

4) $(\cot \theta + 1)\left(\csc \theta - \frac{1}{2}\right) = 0$

$$(\cot\theta + 1)\left(\csc\theta - \frac{1}{2}\right) = 0$$

$$\cot\theta + 1 = 0 \Rightarrow \cot\theta = -1 \Rightarrow \theta = \frac{3\pi}{4} + \pi k$$

$$\csc\theta - \frac{1}{2} = 0 \Rightarrow \sin\theta = 2 \Rightarrow \text{no solution}$$

$$\left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$$

5) $\sin(2\theta) = \cos \theta$

$$2\sin(\theta)\cos(\theta) = \cos(\theta)$$

$$2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$$

$$\cos(\theta) * (2\sin(\theta) - 1) = 0$$

$$2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} + 2\pi k, \theta = \frac{5\pi}{6} + 2\pi k$$

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} + \pi k$$

$$\left\{\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}\right\}$$

6) $\sin(2\theta) \sin \theta = \cos \theta$

$$2\sin(\theta)\cos(\theta)\sin(\theta) = 0$$

$$2\sin^2(\theta)\cos(\theta) - \cos(\theta) = 0$$

$$\cos(\theta) * (2\sin^2(\theta) - 1) = 0$$

$$2\sin^2\theta - 1 = 0 \Rightarrow \sin^2\theta = \frac{1}{2} \Rightarrow \sin^2\theta = \pm \frac{\sqrt{2}}{2}, \theta = \frac{\pi}{4} + \frac{\pi}{2}k$$

$$\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2} + \pi k$$

$$\left\{\frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$

In items 7-8: solve for x in the interval $-\pi \leq x \leq \pi$.

7) Solve the equation $2 \sin x = 0.7x$ by graphing an "appropriate" function and looking for the x -intercepts.

$$\{-2.2403, 0, 2.2403\}$$

8) Solve the equation $\sin x - \cos x = x$ (graphically).

$$x = -1.259$$