Chapter 8

1) Four ways to manipulate points on the coordinate plane
2) Four results of math geeks thinking too hard about the pythagorean theorem for a few centuries
3) Four useful restatements of two dimensional thinking.

Rectangular coordinates
Complex coordinates
Polar coordinates
Vectors

If it all feels like subtlety going over the same ground again and again, it is.

Some of it is pretty neat. Some of it is very powerful.

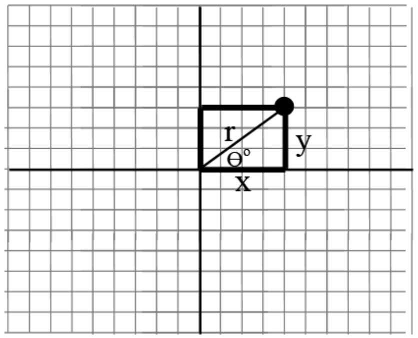
Edit page 6

I) The relationship of rectangular and polar coordinates.

⇨ How many ways could I give a person directions from (0,0) to the point (4,3)?

What we are used to) Rectangular instructions: travel right 4 units and then up 3 units.
A new way) Use a rotation:
- Face right and rotate counter clockwise _____ degrees.

- Travel _____ units along that path.



The new way, the rotational form, corresponds to "polar coordinates."
Polar coordinates are simply a different way of designating a location.
They are useful because some functions and relations that are hard to express in rectangular coordinates are easily expressed in polar coordinates.

Stating what we are used to: A rectangular coordinate, (x,y), shows you a location as a vertex of a rectangle that has one vertex at (0,0) and has sides of length x and y.

The new version: Polar coordinates, (r, θ), identify a location as the endpoint of the hypotenuse of a right triangle that has one angle at the origin that measures θ° and that has a hypotenuse that is r long.

Both versions can be seen in the picture above.

⇨ *Formulas*: The polar coordinates for the rectangular location (8,4) are:

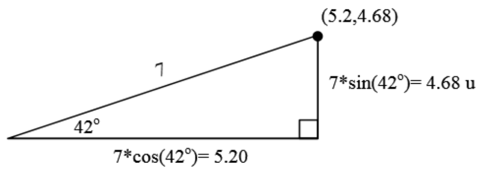
- Rotation = $\theta = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}\left(\frac{4}{8}\right) = 26.57^o$
- Distance = $r = \sqrt{x^2+y^2} = \sqrt{4^2+8^2} = 4\sqrt{5}$ units.

The point with rectangular coordinates (8,4) has polar coordinates of ($4\sqrt{5}$,26.57°).

* What are the polar coordinates of the following rectangular points?
(Sketch the rectangle and the triangle.)
1) (3,6) 2) (-2,7) 3) (3,-7)

<u>??</u> What rectangular coordinates correspond to the polar coordinate (5,30°)?
(Hint: Draw the triangle. Cheat: Follow the model of the triangle below.)

What rectangular coordinate does the polar coordinate (7,42°) indicate?



Polar coordinate (7,42°) indicate the same location as rectangular coordinates (5.2,4.68).

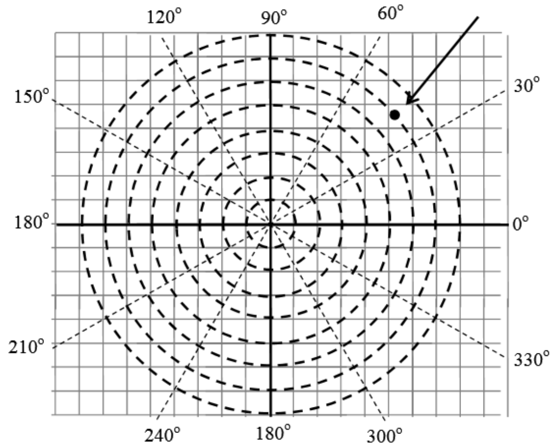
⇨ *Formulas*: The rectangular coordinates of the point (2,50°) are:
- x = $rcos(\theta) = 2 * \cos(50^o) = 1.2856$
- y = $rsin(\theta) = 2 * \sin(50^o) = 1.5321$

The point with polar coordinates (2,50°) has rectangular coordinates of (1.29,1.53).

* What are the rectangular coordinates of the following polar points?
1) (4,85°) 2) (3,400°) 3) (5,-40°)

⇨ Polar coordinates are plotted on polar graph paper. The picture below shows the polar graph paper (dashed) overlapping on rectangular graph paper (solid). The point shown is one of the points worked with in the notes so far. Which one?
- How can you recognize the point on rectangular graph paper?

- How can you recognize the point on the polar graph paper?



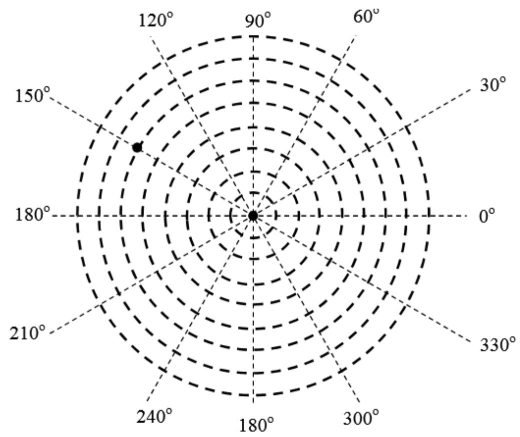
II) Special features of polar coordinates.

Polar coordinates allow multiple ways to refer to a point.
How could that be?

What would an $r = -2$ mean?

What would a $\theta = -30^{\circ}$?

How many angles are co-terminal with a rotation of 30°?



Plot the following points:
1) (4,60°) 2) (-4,240°) 3) (4,420°) 4) (-4, -120°)

5) Write four coordinates of the point shown in the graph above using radians. Include versions using positive and negative values for both r and θ.

III) Restating equations rectangular to polar and vice versa.

You will be asked to restate equations in terms of x and y to equations in terms of r and θ. There are two ways to do this:
- o Think through what the graph looks like and how to express it in the other form.
- o Substitute according to the formula above.

⇨Think through these:
- What does the graph of x = 4 look like? (All the points whose x coordinate is 4.)

- What does the graph of y = 2 look like? (All the points whose y coordinate is 2.)

- What does the graph of r = 3 look like? (All the points whose r coordinate is 3.)

- What does the graph of θ = 30° look like? (All the points whose θ is 30°)

- What does the graph of $x^2 + y^2 = 9$ look like? Can you restate that as a polar equation?

- Can you restate θ = 30° as an equation in x and y? (Hint: What is the slope?)

⇨ Converting by substitution:

$x = r\cos(\theta)$ $\qquad$ $y = r\sin(\theta)$

$$x^2 + y^2 = r^2\cos^2(\theta) + r^2\sin^2(\theta) = r^2\left(\cos^2(\theta) + \sin^2(\theta)\right) = r^2$$
$$\rightarrow x^2 + y^2 = r^2$$
$$\rightarrow \sqrt{x^2 + y^2} = r$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \theta \quad \text{or} \quad \frac{y}{x} = \tan(\theta)$$

Transform each rectangular equation to polar form and each polar equation to rectangular form:

1) x = 4 $\qquad$ 2) y = 2 $\qquad$ 3) r=5 $\qquad$ 4) θ = 30°

Challenge: 5) r = 3cos(θ)*

*Avoid squaring both sides. How can you get an “r” on the right hand side?

p.590(1,5,9,17,21,25,29,33,37,45,49)

- **Match polar points to graph**
- **Plot polar points**
- **Find equivalent polar coordinates**
- **Convert rectangular to polar**
- **Convert polar to rectangular**

⇨ *Formulas*: The polar coordinates of the point (8,4) are:

- Rotation = θ = $tan^{-1}\left(\frac{y}{x}\right)$
- Distance = r = $\sqrt{x^2 + y^2}$

⇨ *Formulas*: The rectangular coordinates of the point (2,50°) are:

- x = $rcos(\theta)$
- y = $rsin(\theta)$

- Translate rectangular equation to polar
- Translate polar equation to rectangular
- Graph linear polar equations
    - Linear
    - Circular
    - cardioid
  - Graph polar equations
    - Make table
    - Use calculator

$$r = \cos\theta$$

$$r^2 = r\cos\theta$$

$$x^2 + y^2 = x$$

$$\left(x^2 - x + .25\right) + y^2 = .25$$

$$\left(x - .5\right)^2 + y^2 = .25$$

$$(x-.5)^2+y^2=.25$$

$$x^2-x+.25+y^2=.25$$

$$x^2-x+y^2=0$$

$$r^2\cos^2\theta-r\cos\theta+r^2\sin^2\theta=0$$

$$r^2\left(\cos^2\theta+\sin^2\theta\right)-r\cos\theta=0$$

$$r^2\left(1\right)=r\cos\theta$$

$$r=\cos\theta$$

$$x^2 + y^2 = \frac{x}{\sqrt{x^2 + y^2}}$$

$$r^2 = \frac{r\cos\theta}{r}$$

$$r^2 = \cos\theta$$

$$r^2 - \cos\theta = 0$$

When do you stop simplifying:

1) Solve for r in polar (or y in rectangular)
2) If not solve for 0 or a constant to consolidate terms on one side.
3) Look for an opportunity to find a known shape. {often redundant with 1)}
4) Get to as few terms as possible.

$$r\cos\theta - 4\sin\theta = 0$$

$$r\cos\theta - \frac{4r\sin\theta}{r} = 0$$

$$x - \frac{4y}{\sqrt{x^2 + y^2}} = 0$$

$$x = \frac{4y}{\sqrt{x^2 + y^2}}$$

$$x\sqrt{x^2 + y^2} = 4y$$

$$x^2\left(x^2 + y^2\right) = 16y^2$$

$$x^4 + x^2y^2 = 16y^2$$

$$x^4 = y^2\left(16 - x^2\right)$$

$$\frac{x^4}{16 - x^2} = y^2$$

$$\frac{x^2}{\sqrt{16 - x^2}} = y$$

$$r\cos\theta = 4\sin\theta$$

$$r = 4\frac{\sin\theta}{\cos\theta}$$

$$r = 4\tan\theta$$

$$\sqrt{x^2 + y^2} = 4\frac{y}{x}$$

$$x\sqrt{x^2 + y^2} - 4y = 0$$

Tools for converting:

Straight substitute:

y=rsinΘ
x=rcosΘ

$$\frac{y}{x} = \tan\theta$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

Two tricks:

Multiply on side by $\frac{r}{r}$

$$3 = \cos\theta$$

$$3 = \frac{r\cos\theta}{r}$$

Multiply both sides by r

$$3r = \cos\theta$$

$$3r^2 = r\cos\theta$$

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Calc skills: Mode "Polar", X=" becoms "r=", X becomes Θ

Exercise

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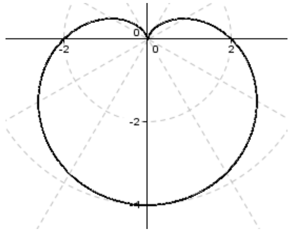
"cos" turns it 90°

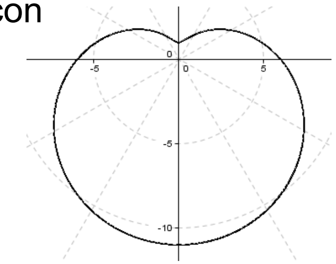
Cardiod



$$r = a \pm a\sin\theta$$

$$a > 0$$

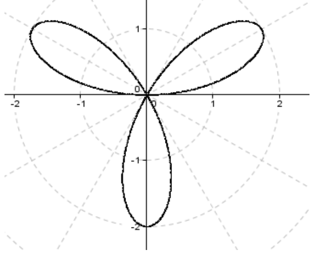
Limacon



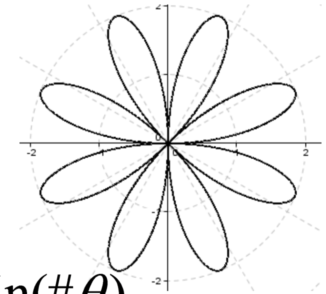
$$r = a \pm b\sin\theta$$

$$0 < a < b$$

Rose: 3 petals



Rose: 8 petals

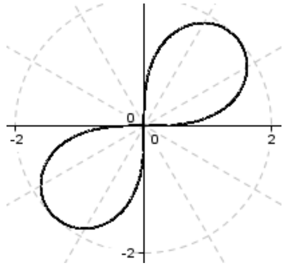


$$r = a\sin(\#\theta)$$

$$a > 0$$

When b is odd, b=# of petals
When b is even,2b=# of petals

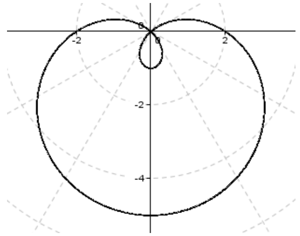
Lemniscate



$$r^2 = a^2\sin(2\theta)$$

$$a > 0$$

Limacon with a loop



$$r = a \pm b\sin\theta$$

$$0 < b < a$$

Dealing with polar equations:

1) Learn to recognize some without calculator
    Circle around the pole
    Lines
2) Translating simple equations to rectangular to recognize

3) More complex forms:
    - Memorize or
    - Use your calculator.
        Cardiod
        Limacon
        Lemniscate
        Rose

Diagonal line

Horizontal line

Vertical line

Circle

Cardiod

Limacon

Lemniscate

Rose

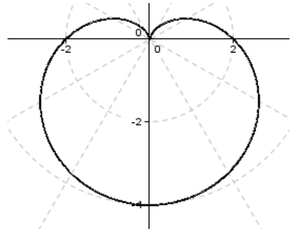
$$r = 2 - 2\sin\theta$$

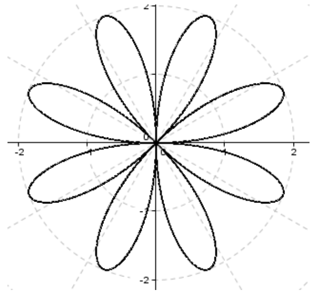
$$r = 2\sin(4\theta)$$

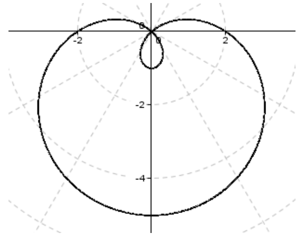
$$r = 2 - 3\sin\theta$$

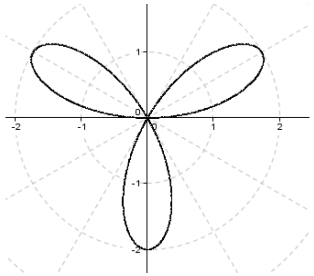
$$r = 6 - 5\sin\theta$$

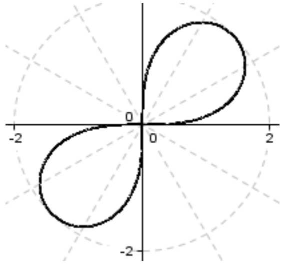
$$r^2 = 2^2 \sin(2\theta)$$

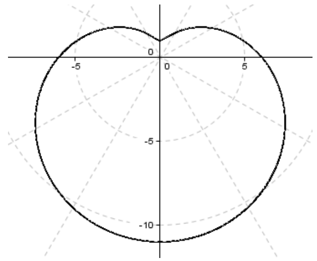












Demo functions
Use calculator skills

Complex same algorithm as rect to polar

r = d(x,y)

Angle = tan-1(y/x)

Product of a complex number: what does it mean to take a product of coordinates

(4,5)

(-2,3)