Now can look at points using Pythagorean theorem in two ways:
- In rectangular world
  Good for lots of things you have done since sixth grade
- In polar world
  Handy for circles and spirograph pictures
  and....

Next: Looking at points as in the complex plane.

<u>Step 1:</u> Plotting complex numbers in rectangular plane grid called the complex plane

In the complex plane the point (2,3) refers to a complex number.

Its real part =2 and its imaginary part =3

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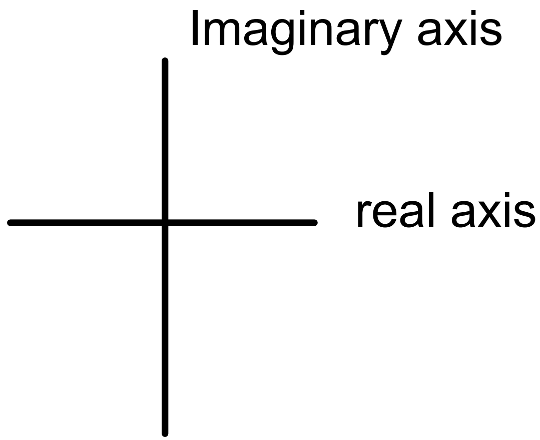
z=2+3i Plots as (2,3)



But if we can look at complex numbers as points in a rectangular grid

then we can look at complex numbers as points in a polar grid

Same idea as before, the point just refers to a complex number, just plotting it on a different grid

z=2+3i Plots on a rectangular, complex plane at (2,3)

Polar directions to the same point: $r=\sqrt{2^2+3^2}=\sqrt{13}$

$$\theta=\tan^{-1}\left(\frac{3}{2}\right)=.9828\ radians$$

$$2=\sqrt{13}\cos(.9828)$$

$$3=\sqrt{13}\sin(.9828)$$

so,

$$z=\sqrt{13}\left(\cos(.9828)+\sin(.9828)i\right)$$

is a possible reformatting of 2+3i. Why do this? We will see.

Rectangular form z=2+3i Polar form: $z=\sqrt{13}\left(\cos(.9828)+\sin(.9828)i\right)$

Z=-4+9i

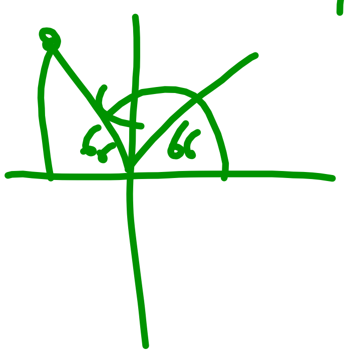
- Write polar coordinates
- Write z in polar form

rect ⇒ $(-4, 9)$

$(\sqrt{97}, 113.9625)$

$\tan^{-1}\left(\frac{9}{4}\right)$

$r=\sqrt{x^2+y^2}$



$\sqrt{97}\left(\cos(113.9625)+i\sin(113.9625)\right)$

$r\left(\cos(\theta)+i\sin(\theta)\right)$

Write $z = 4\left(\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)i\right)$

in rectangular form.

$\frac{1}{2}$ $\frac{\sqrt{3}}{2}$

$\left(4, \frac{\pi}{3}\right)$ Polar coordinates

$r\cos\theta$ $r\sin\theta$

$(2, 2\sqrt{3})$ rect coordinates

$2 + 2\sqrt{3}i$ rect number

So any complex number can be written in polar form
and any complex number in polar form can be written in rectangular form.
(Thinking of it as a point not required, but maybe helpful)

$z=-7 + 23i \Rightarrow$

$$z=24.04(\cos(106.928^{\circ})+i\sin(106.928^{\circ}))$$

So any complex number can be written in polar form.
(Thinking of it as a point not required)

The product of two complex triangles creates a triangle with a hypotenuse equal the product of their hypotenuses and a central angle equal to the sum of their angles.

What good is that?

There is certain math with complex numbers that works really well if you use the polar form of the number.

Yuck

$$z_1 z_2 = (x_1 + y_1 i)(x_2 + y_2 i) = x_1 x_2 + x_1 y_2 i + x_2 y_1 i - y_1 y_2$$

$Z_1 = 2(\cos(40) + i\sin(40))$ $Z_2 = 6(\cos(10) + i\sin(10))$

$$z_1 z_2 = r_1(\cos\theta_1 + \sin\theta_1 i) * r_2(\cos\theta_2 + \sin\theta_2 i)$$

$$= r_1 r_2 \cos\theta_1 \cos\theta_2 + r_1 r_2 \cos\theta_1 i \sin\theta_2 + r_1 r_2 \cos\theta_2 i \sin\theta_1 - r_1 r_2 \sin\theta_1 \sin\theta_2$$

$$= r_1 r_2(\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2) + r_1 r_2 i(\cos\theta_1 i \sin\theta_2 + \cos\theta_2 \sin\theta_1)$$

$$= r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$$

Neat

$12(\cos(50) + i\sin(50))$

$25(\cos(200) + i\sin(200))$

$$z_1 z_2 = r_1 r_2 \left(\cos\left(\theta_1 + \theta_2\right) + i\sin\left(\theta_1 + \theta_2\right)\right)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left(\cos\left(\theta_1 - \theta_2\right) + i\sin\left(\theta_1 - \theta_2\right)\right)$$

De Moivre's Theorem

$$z^n = r^n\left(\cos\left(n\theta\right) + i\sin\left(n\theta\right)\right)$$

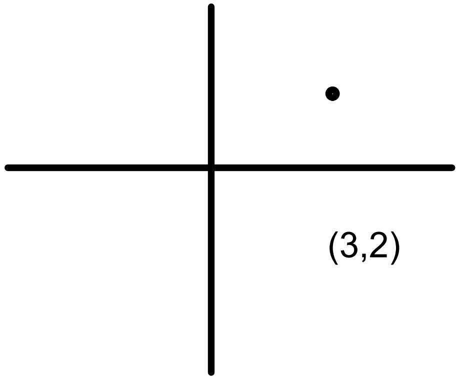
And this is neat too.

$$\left(1+\sqrt{3}i\right)^4$$

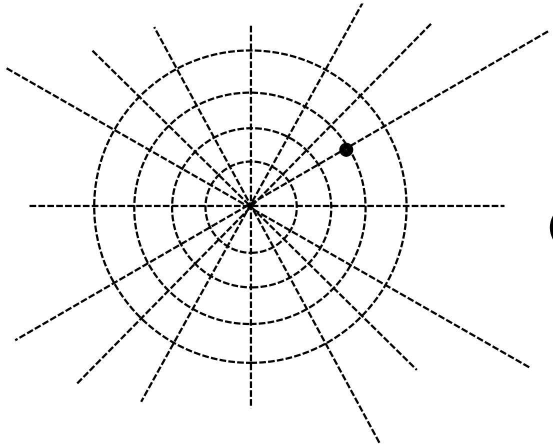
$$z = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$w = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$$

Convert answers to
rectangular form.

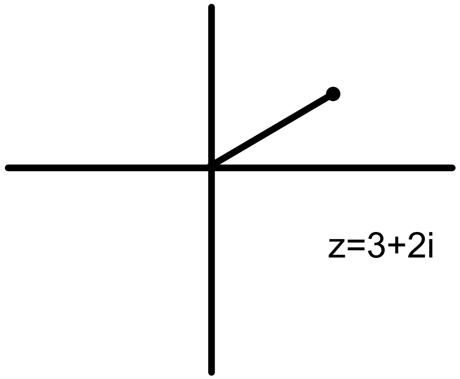


(3,2)



(3,34°)

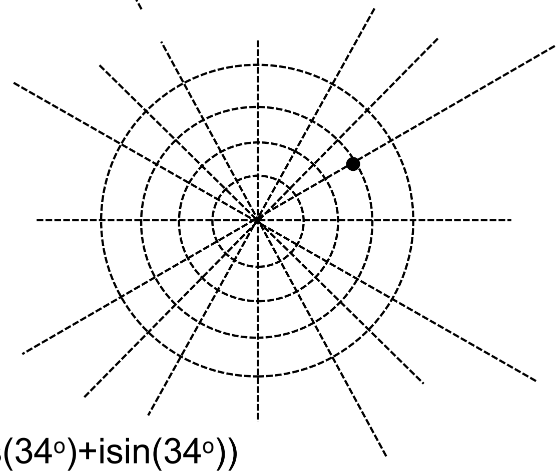
This point can be plotted on a complex plane to represent a complex number



z=3+2i

We can use the geometry of the plane to express math of complex numbers.

Some of this works even better in polar form.



3(cos(34°)+isin(34°))

What the heck would the square root of z=2+3i be?
The third root?

The complex form lets us get an answer.

De Moivre's Theorem

$$z^n = r^n\left(\cos(n\theta) + i\sin(n\theta)\right)$$

$$if \;\; n = 1/m$$

$$z^{1/m} = r^{1/m}\left(\cos\left(\frac{\theta}{m}\right) + i\sin\left(\frac{\theta}{m}\right)\right)$$

BUT

remember taking roots has not worked out to be 1 to 1.
(How many numbers square to make 4?)

Complex numbers have multiple roots. How many?
- Square root produces 2 roots
- Third root produces 3 roots
- Ninth root produces 9 roots

What the heck would the square root of z=2+3i be?
The third root?

To find the roots of a complex number z=x+yi,
convert it to z=r(cosθ+isinθ)
and the find MULTIPLE roots are found as follows:

For the n[th] use this formula, n times:

$$z^{1/n} = \sqrt[n]{r}\left(\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right)$$

First time:

k=0 $$z^{1/n} = \sqrt[n]{r}\left(\cos\left(\frac{\theta_0}{n}\right) + i\sin\left(\frac{\theta_0}{n}\right)\right)$$

Next:

k=1 $$z^{1/n} = \sqrt[n]{r}\left(\cos\left(\frac{\theta_0}{n} + \frac{2*1*\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2*1*\pi}{n}\right)\right)$$

Keep going to n-1

The roots are all the numbers that get back to the original number when you raise them to n.

What the heck would the square root of z=2+3i be?
The third root?

$r = \sqrt{13} \qquad \theta = .9828$

$$z^{1/n} = \sqrt[n]{r}\left(\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right)$$

$$z^{1/3} = \sqrt[3]{13}\left(\cos\left(\frac{.9828}{3} + \frac{2*0*\pi}{3}\right) + i\sin\left(\frac{.9828}{3} + \frac{2*0*\pi}{3}\right)\right)$$

$$z^{1/3} = \sqrt[3]{13}\left(\cos\left(\frac{.9828}{3} + \frac{2*1*\pi}{3}\right) + i\sin\left(\frac{.9828}{3} + \frac{2*1*\pi}{3}\right)\right)$$

$$z^{1/3} = \sqrt[3]{13}\left(\cos\left(\frac{.9828}{3} + \frac{2*2*\pi}{3}\right) + i\sin\left(\frac{.9828}{3} + \frac{2*2*\pi}{3}\right)\right)$$

$$\left(6\left(\cos 120^o + i\sin 120^o\right)\right)^{1/4}$$

$$\frac{120}{4} = 30 \qquad \frac{360}{4} = 90$$

$$6^{1/4}\left(\cos\left(30^o + 90^o k\right) + i\sin\left(30^o + 90^o k\right)\right)$$

$$6^{1/4}\left(\cos\left(30^o\right) + i\sin\left(30^o\right)\right)$$

$$6^{1/4}\left(\cos\left(120^o\right) + i\sin\left(120^o\right)\right)$$

$$6^{1/4}\left(\cos\left(210^o\right) + i\sin\left(210^o\right)\right)$$

$$6^{1/4}\left(\cos\left(300^o\right) + i\sin\left(300^o\right)\right)$$

Raise each to the power to prove it is a root

$$z^n = r^n\left(\cos\left(n\theta\right) + i\sin\left(n\theta\right)\right)$$

$$6\left(\cos\left(120^o\right) + i\sin\left(120^o\right)\right) =$$

$$6\left(\cos\left(480^o\right) + i\sin\left(480^o\right)\right) =$$

$$6\left(\cos\left(840^o\right) + i\sin\left(840^o\right)\right) =$$

$$6\left(\cos\left(1200^o\right) + i\sin\left(1200^o\right)\right)$$

Find the complex cube roots of -8-8i

and back