

Find the fourth roots of  $z=-7-24i$ . Answer in polar form.

Convert to polar form:

$$r = \sqrt{(-7)^2 + (-24)^2} = 25$$

$$\theta = \tan^{-1}\left(\frac{-24}{-7}\right) = 73.7398^\circ$$

Check angle quadrant: WRONG QUADRANT!

Add  $180^\circ = 253.7398^\circ$

$$r(\cos\theta + i\sin\theta) = 25(\cos(253.7398^\circ) + i\sin(253.7398^\circ))$$

Take the root:

Formula:  $z^{1/n} = r^{1/n} \left( \cos\left(\frac{\theta}{n} + \frac{360^\circ k}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{360^\circ k}{n}\right) \right)$  for  $k=0$  to  $n-1$

Plug in:

$$z^{1/4} = r^{1/4} \left( \cos\left(\frac{253.7398^\circ}{4} + \frac{360^\circ k}{4}\right) + i\sin\left(\frac{253.7398^\circ}{4} + \frac{360^\circ k}{4}\right) \right)$$
$$z^{1/4} = 25^{1/4} (\cos(63.4349^\circ + 90^\circ * k) + i\sin(63.4349^\circ + 90^\circ * k))$$

Root 1:  $k=0, z^{1/4} = 5^{1/2}(\cos(63.4349^\circ) + i\sin(63.4349^\circ))$   
add  $90^\circ$

Root 2:  $k=1, z^{1/4} = 5^{1/2}(\cos(153.4349^\circ) + i\sin(153.4349^\circ))$   
add  $90^\circ$

Root 3:  $k=2, z^{1/4} = 5^{1/2}(\cos(243.4349^\circ) + i\sin(243.4349^\circ))$   
add  $90^\circ$

Root 4:  $k=3, z^{1/4} = 5^{1/2}(\cos(333.4349^\circ) + i\sin(333.4349^\circ))$   
add  $90^\circ$

CHECK WITH CALCULATOR:

$$25^{1/4} = 2.2361$$

$$\begin{aligned} \cos(63.4349^\circ) &= .4472, \sin(63.4349^\circ) = .8944, \\ \cos(153.4349^\circ) &= -.8944, \sin(153.4349^\circ) = .4472, \\ \cos(243.4349^\circ) &= -.4472, \sin(243.4349^\circ) = -.8944, \\ \cos(333.4349^\circ) &= .8944, \sin(333.4349^\circ) = -.4472, \end{aligned}$$

Root 1:  $2.23621 * .4472 + (2.23621 * .8944)i = 1+2i$   
 $(1+2i)^4 = -7-24i$

Root 2:  $2.23621 * -.8944 + (2.23621 * .4472)i = -2+1i$   
 $(-2+1i)^4 = -7-24i$

Root 3:  $2.23621 * -.4472 + (2.23621 * -.8944)i = -1-2i$   
 $(-1-2i)^4 = -7-24i$

Root 4:  $2.23621 * .8944 + (2.23621 * -.4472)i = 2-1i$   
 $(2-1i)^4 = -7-24i$