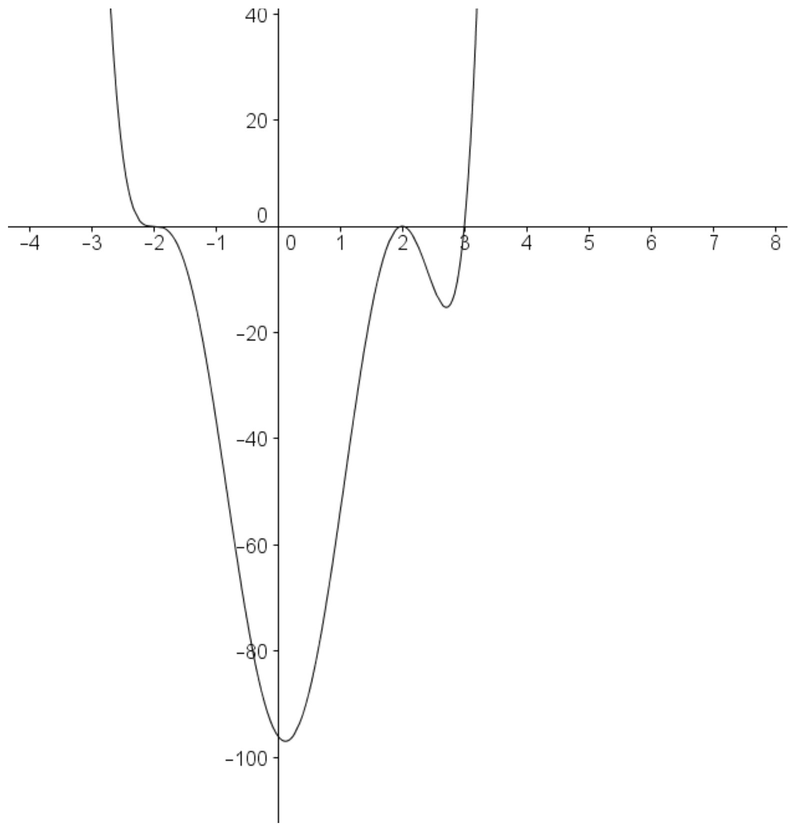


-3 * (-2)^2 * 2^3 = -3*2^5 = -3*32=-96

$(x-3)(x-2)^2(x+2)^3=f(x)$

Problem 70

Average rate of change between points a and b:

$$ARC = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

L3	L4	L5 4
0	100	------
5	128.1	
10	144	
13	153.5	
17	161.2	
18	162.6	
20	166.3	

L4(1)=100

A **rational function** is a function of the form

$$R(x) = \frac{p(x)}{q(x)}$$

where p and q are polynomial functions and q is not the zero polynomial. The domain consists of all real numbers except those for which the denominator q is 0.

Our goals:

To know how the features of the graph resulting from features of the function.

SO:

- We must know what inverse function look like
- We must be able recognize the features in a graph
- We must be able to identify the important features of the function

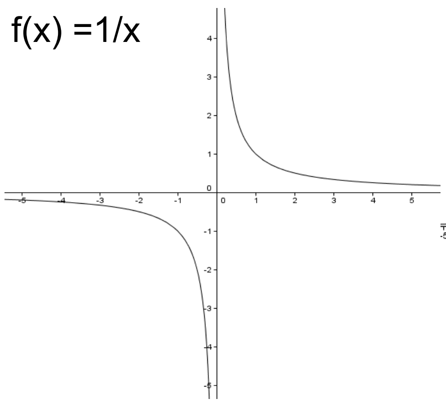
<u>A transformation</u>
A ratio of two products (factorable polynomials)
A set of features

Before: End shape of odd and even power function

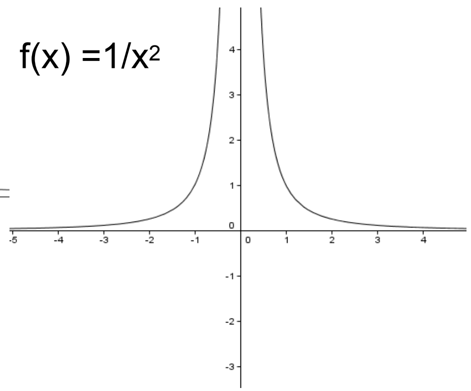
Add: Shape of inverse functions - odd and even

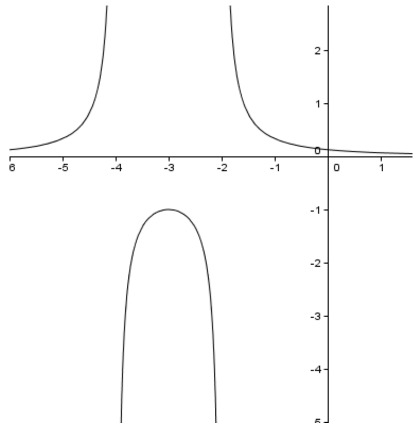
$f(x)=1/(x^2+6x+8)=1/((x+2)(x+4))$

$f(x) = 1/x$

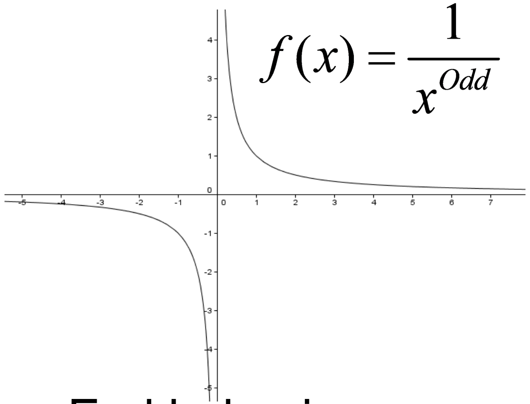


$f(x) = 1/x^2$



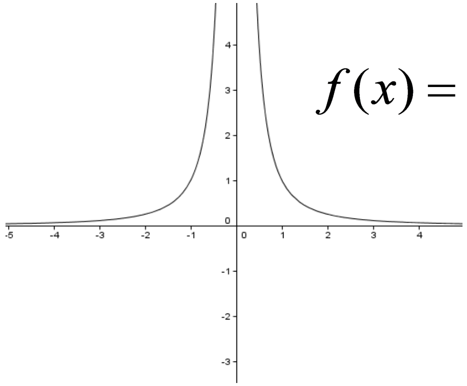


Establishing general patterns of inverse functions



$$f(x) = \frac{1}{x^{Odd}}$$

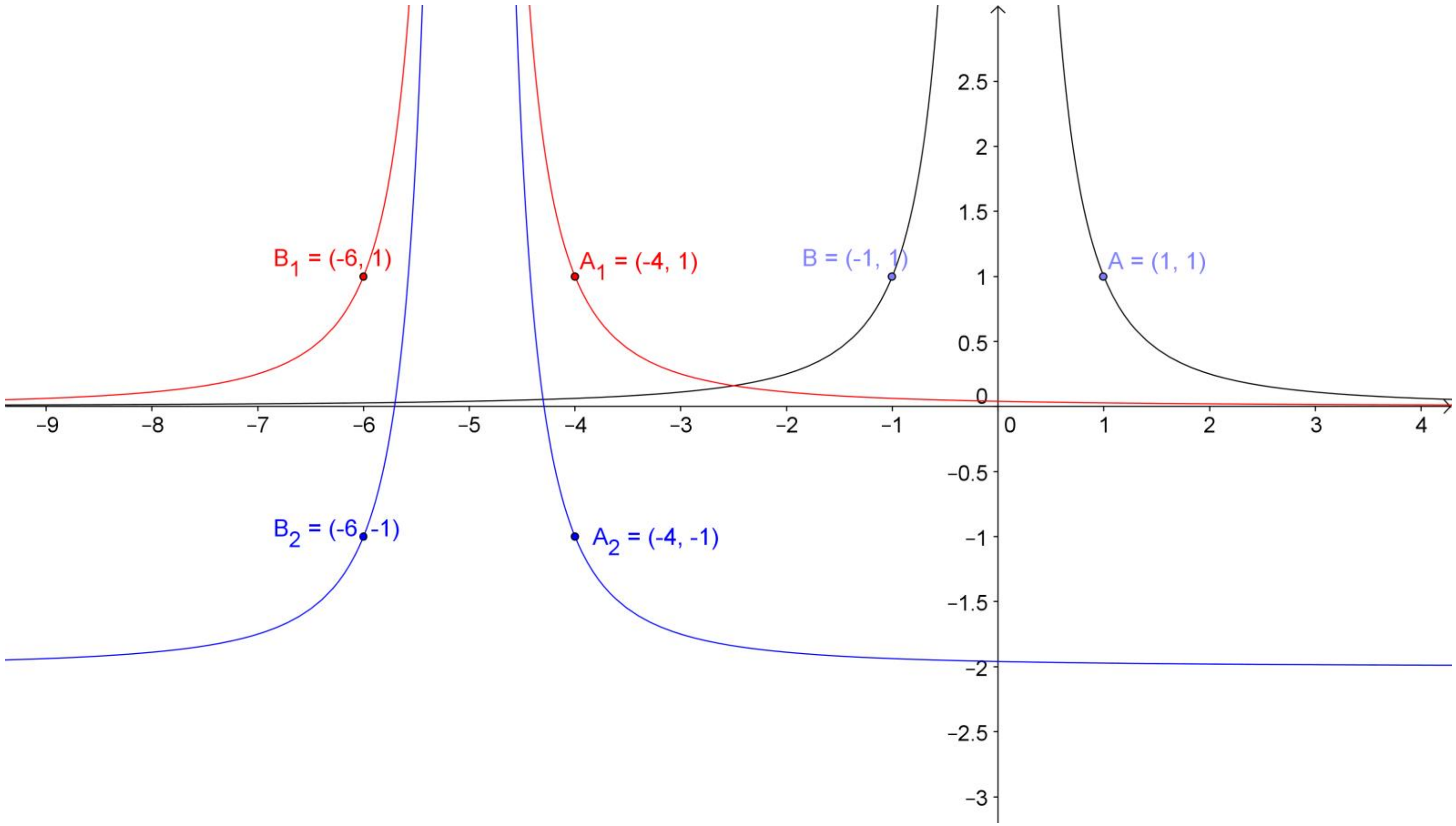
- o End behavior: Approaches x-axis from top and bottom
- o Contains (1,1) and (-1,-1)



$$f(x) = \frac{1}{x^{Even}}$$

- o End behavior: Approaches x-axis from top.
- o Contains (1,1) and (-1,1)

Plot f(x)= 1/(x+5)2-2 by tranformation



Objectives:
- Establishing general patterns of inverse functions
- Graphing by transformation
- Identifying features of a rational function

Graphing by transformation

- Use same old rules

af(bx-c)+d

- Transform known features by rules
  - o Points: (1,1) => $\left(\frac{1+c}{b}, a*1+d\right)$
  - o Horizontal asymptotes move up or down with d
  - o Vertical asymptotes move left or right with c/b

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Identifying features of a rational function

Feature:	Source:
General shape	Reduced form
Domain	Denominator
Range	Inspection/ See asymptotes
All Intercepts	Numerator for x, all for y
Vertical Asymptotes	Denominator
Horizontal/Oblique Asymptotes	Relation of numerator and denominator

First step: Factor numerator and denominator

<u>You do not reduce to exclude</u>

$$R(x) = \frac{(x-2)(x+3)}{(x-2)(x-5)}$$

Is not the same function as

$$R(x) = \frac{(x+3)}{(x-5)}$$

Because they do not have the same domain

$$f(x)=\frac{1}{(x+4)^2} \qquad g(x)=\frac{2x-4}{x^2-2x-24} \qquad h(x)=\frac{x^2-6x+5}{3x^2+21x+18} \qquad i(x)=\frac{x^3-3x^2+12}{x^2+3x-4}$$

$$f(x)=\frac{1}{(x+4)^2} \qquad g(x)=\frac{2(x-2)}{(x-6)(x+4)} \qquad h(x)=\frac{(x-5)(x-1)}{3(x+6)(x+1)} \qquad i(x)=\frac{x^3-3x^2+12}{(x+4)(x-1)}$$

Domain	Denominator
Range	Inspection/ See asymptotes
All Intercepts	Numerator for x, all for y
Vertical Asymptotes	Denominator
Horizontal/Oblique Asymptotes	Relation of numerator and denominator

Analyzing the function

Key term - Asymptote:

An asymptote is a line which a curve approaches, getting closer and closer but never touching.

Asymptotes:

- Closely entwined with understanding domain and range
  - Vertical asymptotes relate to domain
  - Horizontal relate to range
- Key guidance to sketching a graph

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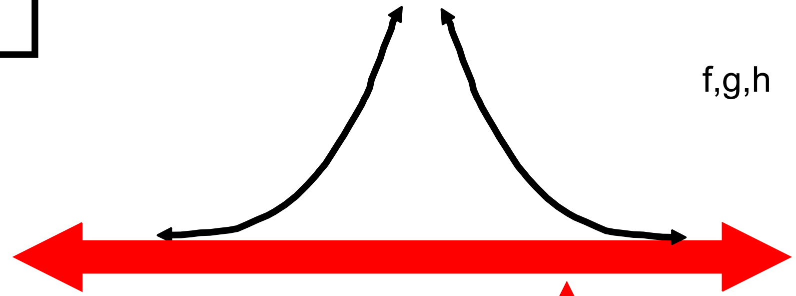
f,g,h



Horizontal asymptote: a ceiling or a floor

If

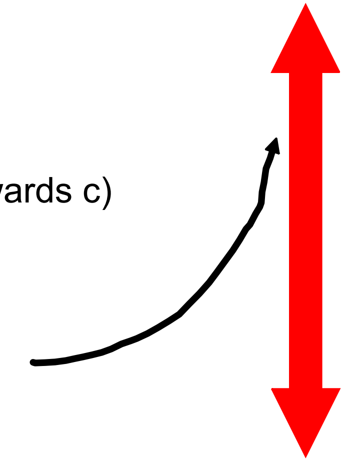
as x ->-ω (as you move left along the x axis) OR

as x ->ω (as you move right along the x axis)

f(x) approaches a horizontal line Y=L

then Y=L is a horizontal asymptote.

Vertical asymptote: a wall



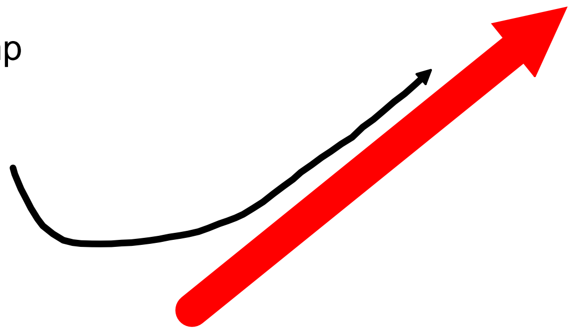
If

as x -> some number c (as you move left or right along the x axis towards c)

f(x) ->-ω OR f(x) ->ω

then the vertical line x=c is a vertical asymptote.

Oblique asymptote: a ramp



ONE SIDE OF THE COIN:

Domain:

Based on the denominator:
2 cases:
- If the zero does reduce out, exclusions are a hole
- If the zero does not reduce out exclusions create an asymptote

f,g,h,i

Range:

- Understanding of base graphs helps.
- Horizontal asymptotes give exclusions and MAY indicate gaps.
- Check by inspection.

h,i

THE OTHER:

Asymptotes:

Vertical
- Irreducible zeros of the denominator:

Horizontal and Oblique: Complex
Based on the degree of the numerator
vs
the degree of the denominator

A proper rational function:

Proper rational functions

2 Cases:

1) Degree of numerator < Degree of denominator

f,g

=> y=0 is a horizontal asymptote

2) Degree of numerator >= Degree of denominator

Improper rational functions

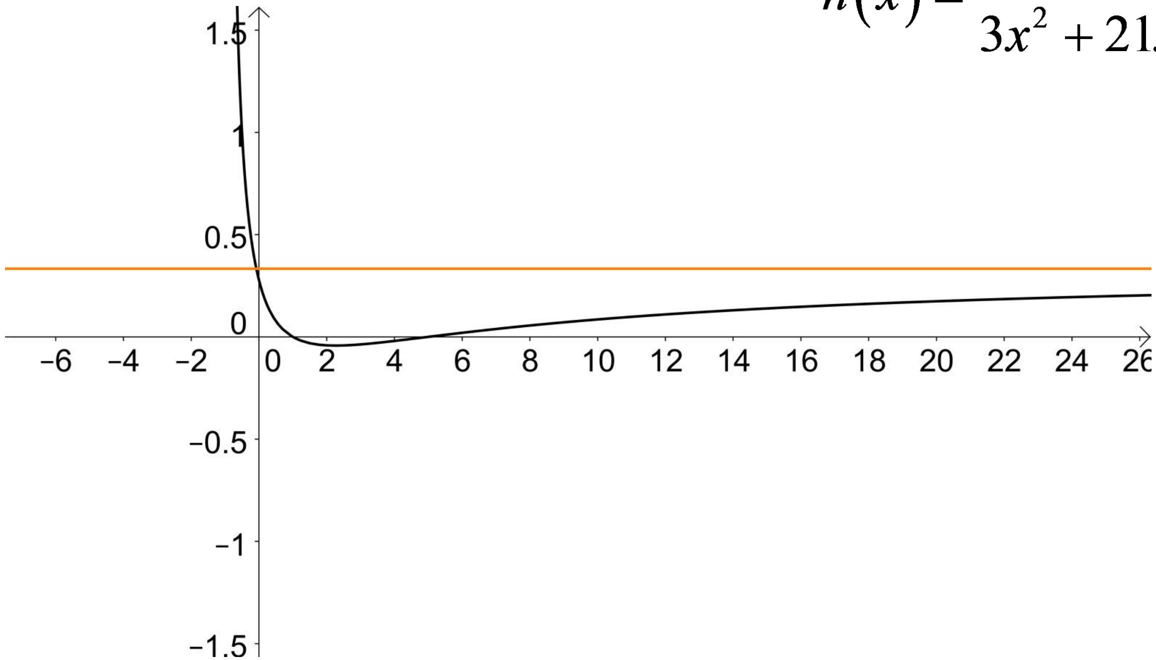
o Divide the function and investigate quotient
Three possibilities:

A) Constant + Remainder
(Deg. N =Deg D)
===> Horizontal asymptote at y=constant

B) mx+b + Remainder
(Deg. N. = Deg. D + 1)
===> Oblique asymptote of y=mx+b

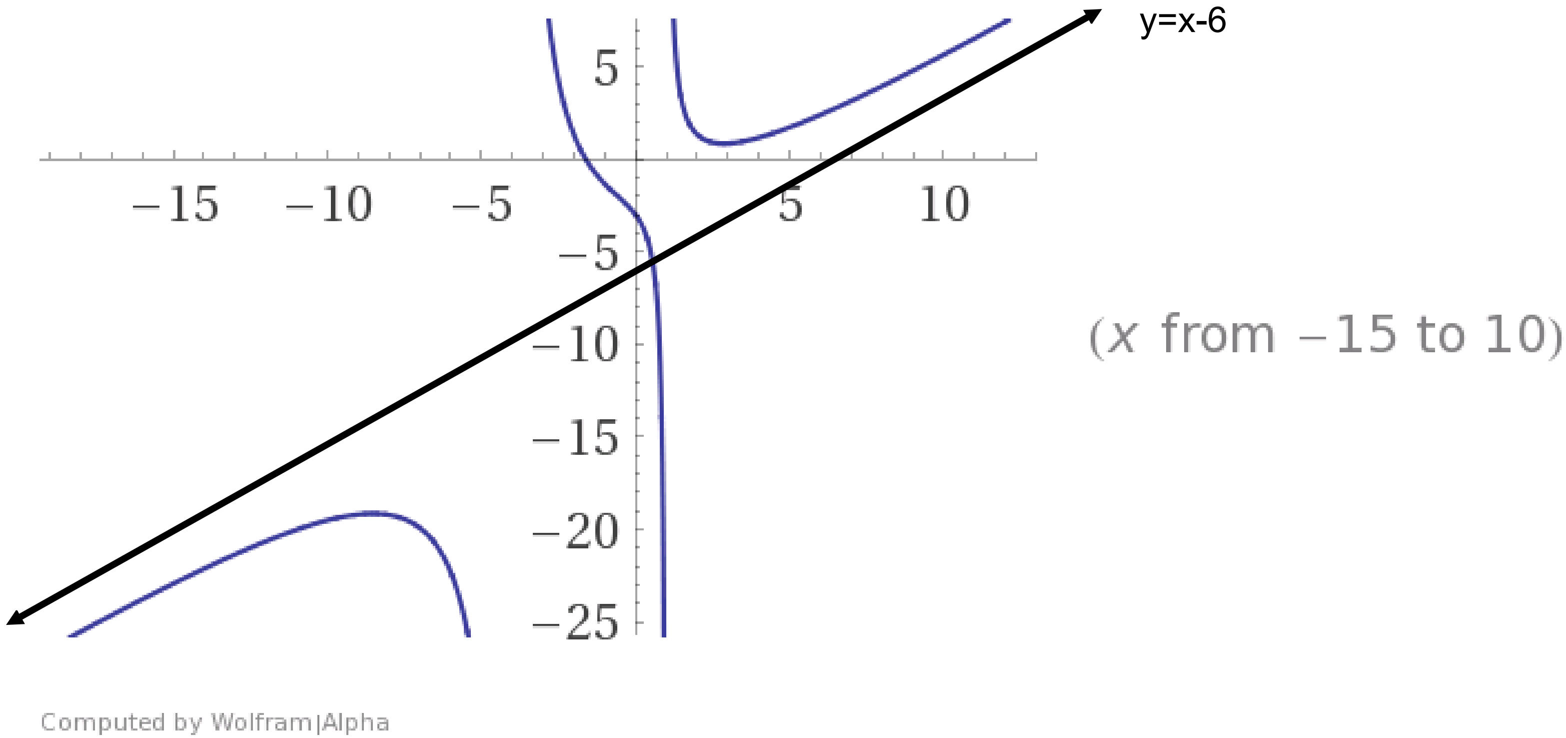
C) Higher order polynomial + Remainder
(Deg. N. > Deg. D + 2)
===> No asymptote. (Graph approaches polynomial

$$h(x)=\frac{x^2-6x+5}{3x^2+21x+18}$$



$$
\begin{array}{r}
x-6 \\
x^2+3x-4\overline{\big) x^3-3x^2+0x+12} \\
\underline{x^3+3x^2-4x} \\
-6x^2+4x+12 \\
\underline{-6x^2-18x+24} \\
22x-12
\end{array}
$$

$$x-6+\frac{22x-12}{x^2+3x-4}$$


y=x-6
5
−15
−10
−5
5
10
−5
−10
−15
−20
−25
(x from −15 to 10)
Computed by Wolfram|Alpha

$$x^3 + x^2 - 8x - 12 \overline{\big)\, x^4 + 2x^3 - 21x^2 - 62x - 40}$$

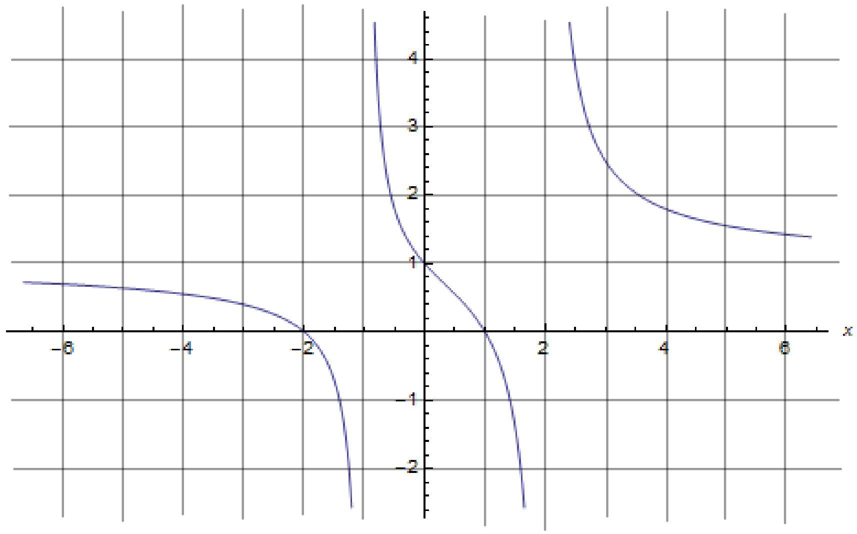
Create a rational function with:

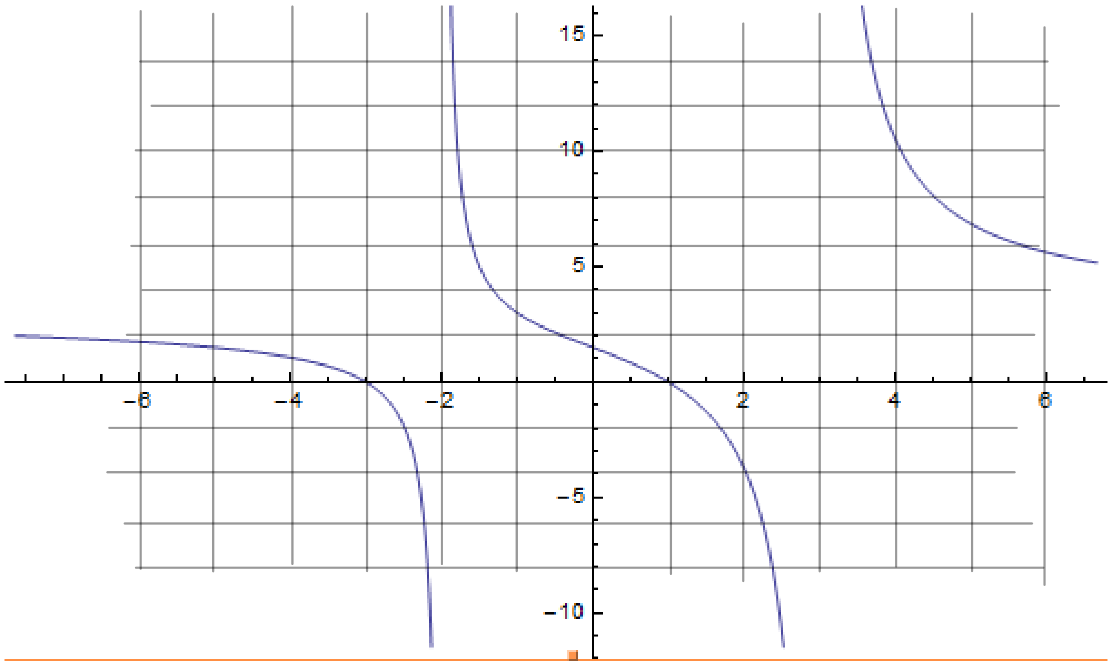
Vertical asymptotes of x=4 and x=-2

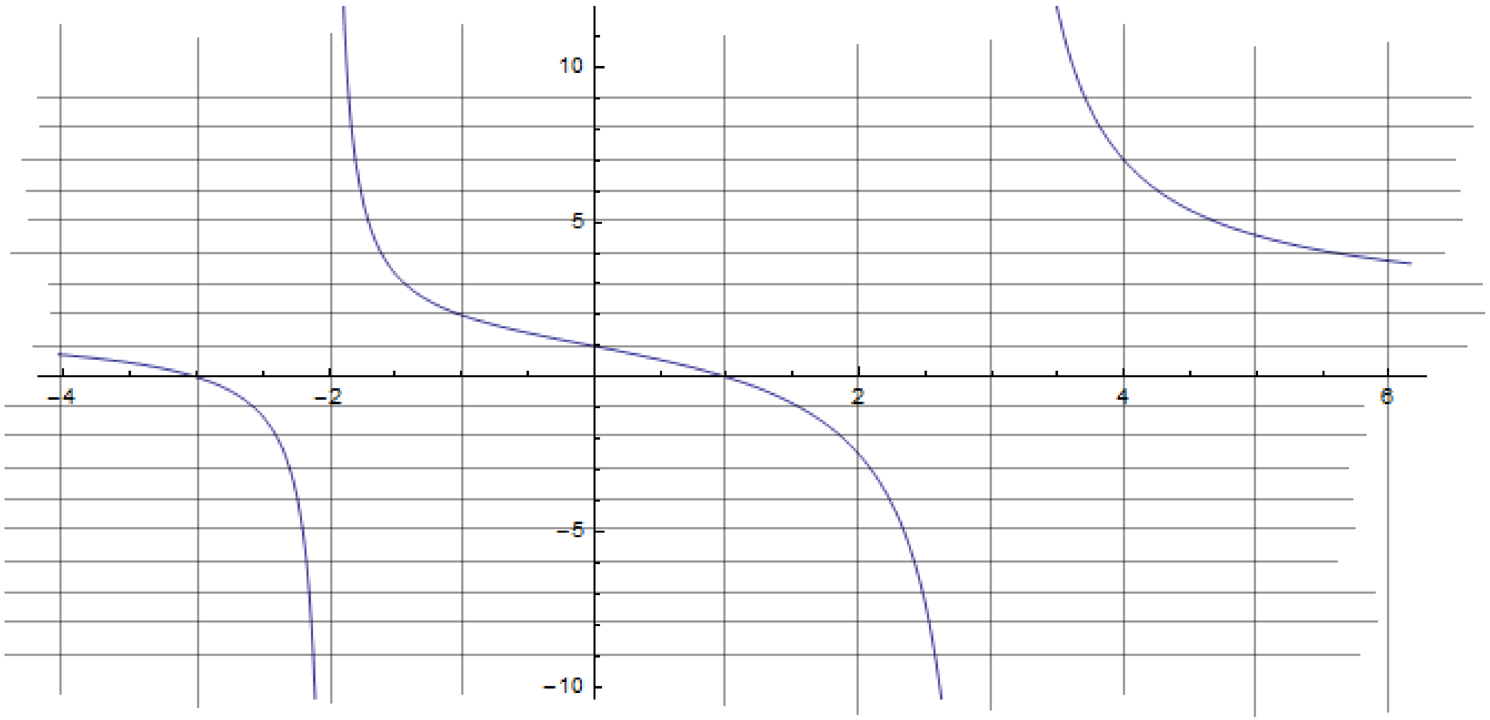
A horizontal asymptote at y=3

and

Zeros at -2 and 6




15
10
5
−5
−10
−6
−4
−2
2
4
6


10
5
−5
−10
−4
−2
2
4
6

$$d(x)=\frac{x^2-2x}{x-3}$$

$$e(x)=\frac{2x^2-32}{x^2-4x-21}$$

$$f(x) = \frac{x^4 - 6x^2 + 8}{x^2 - x - 6}$$

$$(x^2 - 4)(x^2 - 2)$$

$$\left(x+2\right)\left(x-3\right)$$

$$g(x) = \frac{2x^4 + 6}{x^3 + 1}$$

$$h(x) = \frac{x^4 - 2x^3 - 7x^2 - 2x - 8}{x^3 + 4x^2 + x - 6}$$

$$(x-4)(x+2)(x^2+1)$$

(x+3)(x+2)(x-1)

Create a function with:

- Vertical asymptotes of $x=4$ and $x=-2$
- A horizontal asymptote at $y=3$
- and
- Zeros at -2 and 6

Attachments

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