1) Solve each rational inequality. Shown work must:

- Justify the borders/critical values that will determine the intervals to be tested.
- Identify the values used to test the x intervals
- Indicate the calculations and results that verify the conclusion for each interval.
- Clearly states the solution

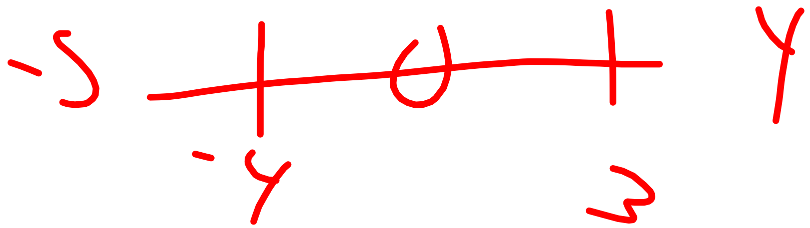
a) $12x^2+12x>144$ => $12x^2+12x-144>0$

$$12x^2+12x-144=12(x^2+x-12)=12(x+4)(x-3)$$

Borders: Zeros - X=-4,x=3

Test:-5,0,4 f(-5)=96 => true. f(0)=-144 => not true. f(4)=96 =>true

Solution: {x|x<-4 or 3<x}



Objectives:

- Learn proper terminology for elements of polynomial division
- Learn a theorem that gives a new way to calculate f(#)

When we divide, life is easy if the divisor goes into the dividend nicely.
But remainders happen.
Lets look at division a relationship of 4 quantities.

Different ways to express the same relationship:

Dividend → 13, Divisor → 3

$$\frac{13}{3} = \frac{4*3+1}{3} = 4 + \frac{1}{3}$$

1 ← Remainder

4 ← Quotient

OR

$$\frac{13}{3} = 4 + \frac{1}{3}$$

or

$$13 = 4*3+1$$

1 ← Remainder; 13 ← Dividend; 4 ← Quotient; 3 ← Divisor

Represent dividing 21 and 4 in this way

When we divide, life is easy if the divisor goes into the dividend nicely.
But remainders happen.
Lets look at division a relationship of 4 quantities.

If we were as familiar with polynomials as we are integers, we could see rational functions in this way:

$$\frac{3x^2-8x}{x-2}=\frac{3x^2-6x-2x}{x-2}=\frac{3x(x-2)-2x}{x-2}=3x-\frac{2x}{x-2}$$

$$\frac{3x^2-8x}{x-2}=3x-\frac{2x}{x-2}$$ OR (rewrite this one)

$$\frac{f(x)}{g(x)}=q(x)+\frac{r(x)}{g(x)} \quad \text{or} \quad f(x)=q(x)g(x)+r(x)$$

$f(x)$ ↑ dividend, $q(x)$ ↑ quotient, $g(x)$ ↑ divisor, $r(x)$ ↑ remainder

$$\frac{3x^2 - 8x}{x-2} = 3x - \frac{2x}{x-2}$$

$$3x^2 - 8x = 3x(x-2) - 2x$$

What is the remainder?

Division Algorithm for Polynomials

If $f(x)$ and $g(x)$ denote polynomial functions and if $g(x)$ is not the zero polynomial, then there are unique polynomial functions $q(x)$ and $r(x)$ such that

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad \underset{\text{dividend}}{\underset{\uparrow}{f(x)}} = \underset{\text{quotient}}{\underset{\uparrow}{q(x)}}\,\underset{\text{divisor}}{\underset{\uparrow}{g(x)}} + \underset{\text{remainder}}{\underset{\uparrow}{r(x)}} \qquad (1)$$

where $r(x)$ is either the zero polynomial or a polynomial of degree less than that of $g(x)$.

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)} \quad \text{or} \quad f(x) = q(x)g(x) + r(x)$$

dividend ↑ quotient ↑ divisor ↑ remainder ↑

If $f(x)=4x^2 - 12x$

What is f(3)?

(x-3)

$$\frac{4x^2-12x}{x-3} = \frac{4x(x-3)}{x-3} = 4x$$

$$4x^2 - 12x = 4x(x-3)$$

So, if c is a zero of f(x), then f(c) = 0. Well, yeah.
If c is a zero, then (x-c) is a factor of f(x). Well, yeah.
If c is a zero, then dividing by (x-c) has no remainder.

What if c is not a zero?

New Problem:

If $f(x) = 4x^2 - 12x + 4$

What is f(3)?

Go ahead.

$f(3) = 4x(x-3) + 4$

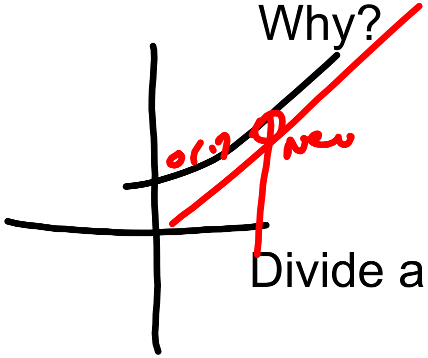
$$\frac{4x^2 - 12x + 4}{x - 3} = 4x + \frac{4}{x - 3}$$

$$4x^2 - 12x + 4 = 4x(x - 3) + 4$$

What???????

If I ask you, "What is f(#)?", then one way to find out is to:
- Divide by (x-#)
- Take the remainder

Why?



If $f(x)=4x^2 - 12x + 4$

What is $f(5)$?

$$
\begin{array}{r}
4x+8 \\
x-5\overline{)\,4x^2-12x+4} \\
4x^2-20x \\
8x+4 \\
8x-40 \\
+44
\end{array}
$$

Divide and discover:

$$f(x)=4x^2-12x+4=(4x+8)(x-5)+44$$

So, when we plug 5 in on the right:

$$f(x)=4(5)^2-12(5)+4=(28)(0)+44$$

We can either plug 5 in or divide to get the answer

If I ask you, "What is f(#)?", then one way to find out is to:
- Divide by (x-#)
- Take the remainder

What is f(2) for $3x^2-8x$?

Objective
- Learn rules to help find zeros
- Gain skill at using rules and synthetic division to quickly find zeros.

$F(X)=2X^3-2X^2-82X+210$

5? => (x-5) => to synthetic division

$$f(x)=2(x-5)(x-3)(x+7)$$

Switch signs

5 | 2	-2	-82	210
	10	40	-210
2	8	-42	0

(x-5)($2x^2+8x-42$) no remainder

$2x^2+8x-42=2x^2+14x-6x-42$

$=2x(x+7)-6(x+7)=(2x-6)(x+7)=2(x-3)(x+7)$

2*5

A) A biconditional:

1) If f(c)=0, then x-c is a factor of f(x)
2) If x-c is a factor of f(x) then f(c) =0

Use: If you find a zero, can be divided out to create a simpler polynomial for further study.

B) The degree of the polynomials tells you the maximum number of real zeros.
Use: You know when to stop.

C) IF A POLYNOMIAL FUNCTION HAS INTEGER COEFFICENTS
then make two lists:
1) factor the constant (call these p's)
2) factor the leading coefficient (call these q's)
3) make all the +/- fractions you can putting the p's/q's
ALL YOU REAL RATIONAL ZEROS ARE IN THIS LIST.
Use: It gives you all the possibilities.
- A useful list
- A guide on window size

D) IF THE LEADING COEFFICIENT OF A POLYNOMIAL IS 1
then all the zeros are between
-M to +M where M is
The smaller of
o The sum of the absolute value of the coefficients
and
o 1 + the coefficient with the biggest absolute value
Use: You know how big a window to set on the calculator.

E) End behavior and turning points are still useful.

F) Intermediate value theorem to find irrationals.

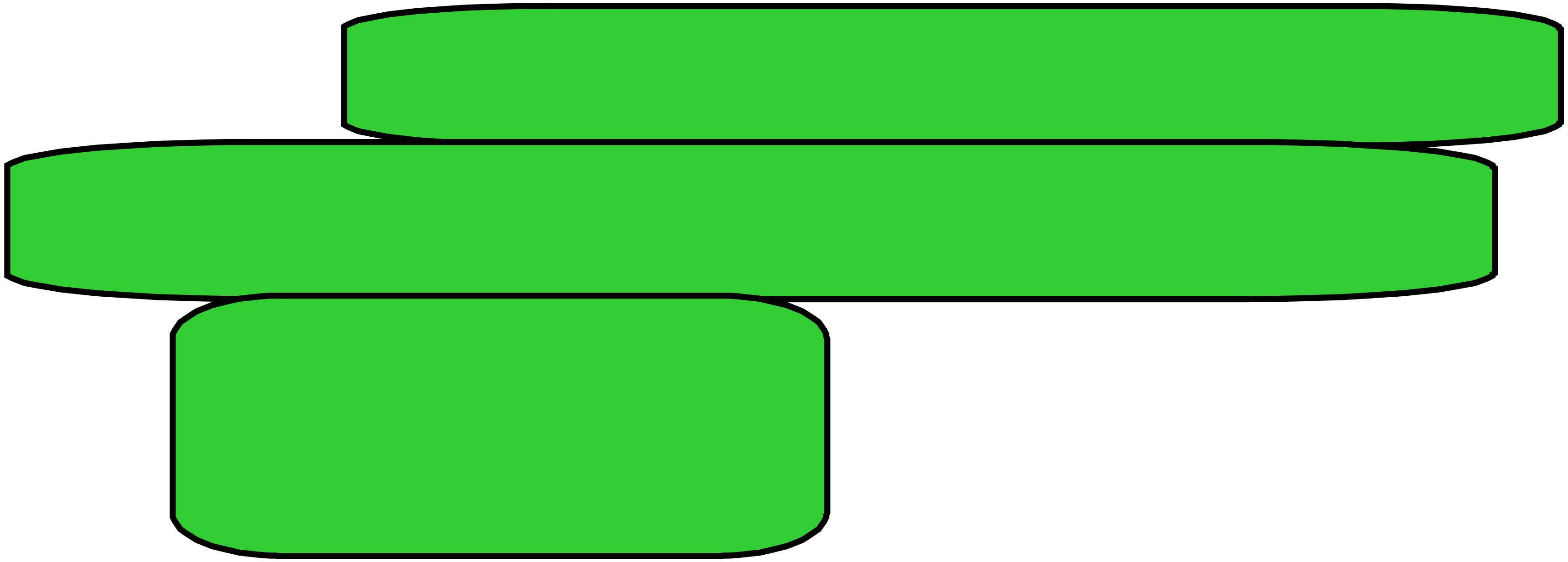
$f(x) = 2x^4+2x^3-14x^2-2x+12$

1

Try again.....

2

$$F(X)=2X^3-2X^2-82X+210$$



Find the k such that $x^4 - kx^3 + kx^2 + 1$ has a factor of $x+2$

$$\frac{6}{12}x$$

$$\frac{6}{12}x + \frac{12}{12}$$

$$0$$

$$x^3 + (-k-2)x^2$$

$$x+2\overline{)x^4 - kx^3 + kx^2 + 1}$$

$$\underline{x^3 + 2x^3}$$

$$(-k-2)x^3$$

$$\underline{(-k-2)x^3 + (-2k-4)x^2}$$

$$(3k+4)x^2$$

What are the zeros of:

x^2+1?

$x^2+2x+20$?

$x^3+3x^2+4x+12$?