This is a take home assignment. This means you have time to do work carefully, neatly, and completely. Features of complete work include thorough, well organized work, units, and, where relevant, complete sentences. All graphs should have reasonable proportions and clear scales. Polynomials can be expressed in standard form or factored form, unless specified otherwise. Do not feel constrained to present all your work on these pages.

For items 1-4 use the polynomial $f(x)=(x-8)(x+2)(5 x-3)$

1) What is the degree of the polynomial?
2) At how many points does the graph intersect the $x$-axis?
3) $\qquad$
4) What are the zeros of the function? Show the algebraic solution for each. Leave all answers in exact form.
5) Make a sketch of the graph to the right.

Adjust scales of grid as required.
List below and identify any features you plotted.
3)

5) The zeros of a function are $-6,3$, and 5 . Find two such functions and sketch their graphs below. Adjust scales of grid as required.

Function a) $\qquad$ Function b)
a)

b)


For items 6-8: Graph the polynomial $p(x)=(x+3)(x-2)\left(x^{2}+4\right)$
6) What are the $x$-intercepts?

Intercepts $\qquad$
7) Find all the zeros of the polynomial. Show work below.


Zeros $\qquad$
8) Explain why the number of zeros does not match the number of $x$-intercepts.
9) a) Using your calculator, graph $y=x^{3}+3 x^{2}-22 x-24$
b) Identify its zeros from the graph
b) $\qquad$
c) Rewrite the polynomial as a product of linear factors
c) $\qquad$
10) Find a polynomial of lowest possible degree that touches the $x$-axis at $x=-3$, and crosses it at 4 . Make a sketch.

Polynomial $\qquad$
10)

11)

11) Using the grid above and to the right, show a polynomial that touches the $x$-axis at -1 and 3 with no other $x$ intercepts. Write the polynomial below.
12) By approximating a beverage can as a cylinder you can estimate its surface area and volume. Since the surface area affects the amount of aluminum in a can, manufacturers might want to maximize volume for a given surface area. You can test whether or not this is what they have actually done. A diameter of 2.56 inches and a height of 4.5 inches are typical dimensions of most beverage cans.

The surface area of a cylinder is twice the area of the base plus the area of the sides (the circumference times the height), or $A=2 \pi r^{2}+2 \pi r h$. The volume is the area of the base times the height, or $V=\pi r^{2} h$
a) Calculate the surface area, A, of a typical soda can to the fourth decimal place
b) Assume we want to keep the typical surface area, A, but want to consider whether a different radius could achieve a greater volume with this same surface area. To find the $r$ that maximizes volume we want to write volume as a function of $r$ alone, rather than both $r$ and $h$. Assume the same surface area as in part a, keep $r$ and $h$ as variables, and solve the surface area formula for $h$. Make a new function by substituting this expression for $h$ into the volume formula. Show your work below.
c) Analyze the function found in step b. Sketch it to the right. - What do the values of $x$ represent in this graph?

- In the context of the soda can, what do the x-intercepts mean?

- In the context of making real soda cans, what are the meaningful range and domain of the function?

Range $\qquad$
Domain $\qquad$

- What radius maximizes the volume of the soda can?

Radius

- Given this function as defined above, what would be the height of the can at the maximum volume? Show work.

Hieght $\qquad$

- Compare the height and radius that you found to the typical soda can described above.

13) You have been asked to design a grain silo in the shape of a cylinder with a hemispherical top. The total height of silo, cylinder plus hemisphere, must be 60 feet tall. Recall that the volume of a cylinder with a base with a radius $r$ and height $h$ is $V=\pi r^{2} h$ and the volume of a sphere is $v=\frac{4 \pi r^{3}}{3}$. Use $r$ for the radius of the base of the silo and h for the height of the cylinder.
a) Write an equation to express the total height of the silo based on its parts, using the variables stated above.
b) Assuming a total height of 60 feet, express the total volume of the silo as a function of the radius of the base of the silo.
b)
c) Is the function you have written a polynomial? Explain why or why not.
d) What is the degree of the polynomial?
d) $\qquad$
e) Sketch the graph to the right, including all significant features.

- What do the values of x represent in this graph?
- In the context of the silo, what would the $x$-intercepts represent?

f) What interval of values for the radius give the silo a volume of at least $20,000 \mathrm{ft}^{3}$ ?
f) $\qquad$
g) What radius would give the maximum volume? Answer to the nearest tenth?

Radius $\qquad$
Volume $\qquad$
h) Find the total height of the silo associated with this radius and volume. Discuss any issues that arise and how the silo builder might chose to use the volume function given this issue.

