Decomposition of fractions:

Doing something backwards that we almost always do forwards.

What do you do with this?

$$\frac{2}{5}+\frac{5}{6}=\frac{2*6}{5*6}+\frac{5*5}{6*5}=\frac{12}{30}+\frac{25}{30}=\frac{27}{30}=\frac{9}{10}$$

Can you do that backwards?

$$\frac{11}{14}$$

1) 14 is 7*2

2) The we can find some a and b such that:

$$\frac{a}{2}+\frac{b}{7}=\frac{11}{14}$$

3) Multiply through by 14

$$7a+2b=11$$ Pick a=1 and b=2

LIKE FACTORING: You are mentally working backwards.

Like factoring, we will have an algorithm or procedure to help us see the result.

$$\frac{2x+3y}{xy}$$

1) xy factors into x and y

2) $$\frac{2x+3y}{xy} = \frac{a}{x} + \frac{b}{y}$$

3) Multiply through by xy

$$2x+3y = ay + bx$$

a=3,b=2

$$\frac{7}{12} \qquad \frac{2x+3y}{xy}$$

- Factor the denominator.
- make an a fraction and a b fraction
- multiply through by the denominator
- choose the a and b that works ← This step gets trickier

Let's upgrade the problems.

$$\frac{5x-4}{x^2-3x-10}$$

$$(x-5)(x+2)=x^2-3x-10 \qquad \frac{5x-4}{x^2-3x-10}=\frac{Something}{x-5}+\frac{Something}{x+2}$$

What is something going to look like?

$$\frac{5x-4}{x^2-3x-10}=\frac{A}{x-5}+\frac{B}{x+2}$$

First trick: Mulitply through by the denominator (IN FACTORED FORM!)

$$5x-4=A(x+2)+B(x-5)$$

Don't think of this as a pattern of rewriting, "do" the multiplication.

How do we "pick" A and B?

2 methods:

Method 1, use correspondence and simultaneous equations.

$$5x-4=Ax+2A+Bx-5B$$

$$5x-4=(A+B)x+(2A-5B)$$

$$A+B=5$$

$$2A-5B=-4$$

Solve (Substitution, linear combination, matrices)

Method 2, pick clever values for X and solve:

$$5x-4=A(x+2)+B(x-5)$$

Pick X = 5

$$5*5-4=A(5+2)+B(5-5)$$

Why did we pick 5?

$$21=A(5+2)$$

A=3

$$5x-4=3(x+2)+B(x-5)$$

$$5x-4=3x+6+Bx-B5$$

$$2x-10=Bx-B5$$

$$2(x-5)=B(x-5)$$

$$B=2$$

Why not this?

$$\frac{x^3+4x^2-10x}{x^2-3x-10}$$

We will only do this for proper fractions.
The technique can be used in analyzing
the remainder of this rational function
but we won't be doing that just to save time.

$$\frac{5x-31}{x^2+3x-10}$$

$$\frac{11x+52}{x^2+5x-6}$$

One step trickier problems.

They need a further technique.

(I am going to show you 3 ways to solve the next problem)

$$\frac{x^2}{(x-1)^2(x+1)}$$

$(x+1),\ (x-1),\ (x-1)^2$

$$\frac{11}{14} = \frac{A}{7} + \frac{B}{2}$$

$8.6 \quad 3.2$

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$x^2 = A(x-1)^2 + B(x-1)(x+1) + C(x+1)$$

$$x^2 + 0x + 0 = A(x^2 - 2x + 1) + B(x^2 - 1) + C(x+1)$$

$-2Ax \qquad Cx$

	A	B	C	
x^2:	1	1	0	= 1
x:	-2	0	1	= 0
consts:	1	-1	1	= 0

$[A]^{-1}[B]$

Trickier:
If the factors of the denominator have factors,
we must allow for all of them possibly having a term.

Factor: $x^4 + 8x^3 + 16x^2 = x^2(x+4)^2$ ⟵ We will use this.

List ALL the factors: $x, x^2, (x+4), (x+4)^2$ ⟵ We also use this, as we decompose across all factors.

1) $$\frac{8x^2+32}{x^4+8x^3+16x^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4} + \frac{D}{(x+4)^2}$$

$x^2(x+4)^2$

⟵ A list of all the forms of the factors.

2) Mulitply through by the denominator (IN FACTORED FORM!)

Make sure this makes sense ⟶ $$8x^2 + 32 = Ax(x+4)^2 + B(x+4)^2 + Cx^2(x+4) + Dx^2$$

How do we "pick" A, B, C, D? Lots of ways, learn what you like.

Do this: $$8x^2+32=Ax(x+4)^2+B(x+4)^2+Cx^2(x+4)+Dx^2$$

Phase 1: Make stuff go away by picking clever numbers

Assume x=-4

$$8(-4)^2+32=D(-4)^2$$

$$10=D$$

$$8x^2+32=Ax(x+4)^2+B(x+4)^2+Cx^2(x+4)+10x^2$$

Assume x=0 (Why?)

$$32=B(0+4)^2$$

$$2=B$$

$$8x^2+32=Ax(x+4)^2+2(x+4)^2+Cx^2(x+4)+10x^2$$

No clever assumptions left. (Why?)

Phase 2: Expand everything & collect like terms

$$8x^2+32=A(x^3+8x^2+16x)+2(x^2+8x+16)+C(x^3+4x^2)+10x^2$$

$$8x^2+32=Ax^3+8Ax^2+16Ax+2x^2+16x+32+Cx^3+4Cx^2+10x^2$$

$$8x^2+32=(A+C)x^3+(8A+2+4C+10)x^2+(16A+16)x+32$$

Use correspondence to set up equations and solve.

0=A+C
8=8A+4C+12
0=16A+16

A=-1
C=1

$$\frac{8x^2+32}{x^4+8x^3+16x^2}=\frac{-1}{x}+\frac{2}{x^2}+\frac{1}{x+4}+\frac{10}{(x+4)^2}$$

$$8x^2+32=Ax\left(x+4\right)^2+B\left(x+4\right)^2+Cx^2\left(x+4\right)+Dx^2$$

$$8x^2+32=Ax(x^2+8x+16)+B(x^2+8x+16)+Cx^3+4Cx^2+Dx^2$$

$$8x^2+32=Ax^3+8Ax^2+16Ax+Bx^2+8Bx+16B+Cx^3+4Cx^2+Dx^2$$

$$8x^2+32=\left(A+C\right)x^3+\left(8A+B+4C+D\right)x^2+\left(16A+8B\right)x+16B$$

	A	B	C	D
x^3: 0 =	1	0	1	0
x^2: 8 =	8	1	4	1
x: 0 =	16	8	0	0
c: 32 =	0	16	0	0

$$[A]=\begin{bmatrix}0\\8\\0\\32\end{bmatrix}\qquad [B]=\begin{bmatrix}1&0&1&0\\8&1&4&1\\16&8&0&0\\0&16&0&0\end{bmatrix}$$

$$[B]^{-1}[A]=\begin{bmatrix}-1\\2\\1\\10\end{bmatrix}$$

A=-1,B=2,C=1,D=10

$$\frac{8x^2+32}{x^4+8x^3+16x^2}=\frac{-1}{x}+\frac{2}{x^2}+\frac{1}{x+4}+\frac{10}{\left(x+4\right)^2}$$

$$8x^2+32=Ax(x+4)^2+B(x+4)^2+Cx^2(x+4)+Dx^2$$

$$8x^2+32=A\left(x^3+8x^2+16x\right)+B\left(x^2+8x+16\right)+C\left(x^3+4x^2\right)+Dx^2$$

	A	B	C	D
x^3: 0 =	1	0	1	0
x^2: 8 =	8	1	4	1
x: 0 =	16	8	0	0
c: 32 =	0	16	0	0

A=-1,B=2,C=1,D=10

$$\frac{8x^2+32}{x^4+8x^3+16x^2}=\frac{-1}{x}+\frac{2}{x^2}+\frac{1}{x+4}+\frac{10}{(x+4)^2}$$

And it still gets trickier....

$$\frac{2x+3}{x^3+x^2-x+15}$$

$(x-1)(x^2+x+1)$

Cases: Every polynomial with real coefficients can be factored into
- the product of linear factors

and/or

- irreducible quadratic factors

"List all the factors" some of them will be a quadratic.

x^3+x^2-x+15

$x^3+x^2-x+15 = (3+x)(x^2-2x+5)$

When there is a irreducible quadratic in the denominator,
you have a linear expression in the numerator

$$\frac{2x+3}{x^3+x^2-x+15} = \frac{A}{(3+x)} + \frac{Bx+C}{\left(x^2-2x+5\right)}$$

So,every quadratic denominator generates at least two unknowns.

$$\frac{5x^2}{x^3-2x-4}$$

Factor: $(x-2)(x^2+2x+2)$

List all factors: $(x-2),(x^2+2x+2)$

$$\frac{5x^2}{(x-2)(x^2+2x+2)}=\frac{A}{x-2}+\frac{Bx+C}{(x^2+2x+2)}$$

$$5x^2=A(x^2+2x+2)+(Bx+C)(x-2)$$

$$5x^2=Ax^2+2Ax+2A+Bx^2+Cx-2Bx-2C$$

$$5x^2=(Ax^2+Bx^2)+(2Ax-2Bx+Cx)+2A-2C$$

5=A+B
0=2A-2B+C
0=2A-2C

10=2A+2B
0=2A-2B+C

10=4A+C

0=2A-2(10-4A)
0=10A-20
2=A
3=B
2=C

$$\frac{5x^2}{(x-2)(x^2+2x+2)}=\frac{2}{x-2}+\frac{3x+2}{(x^2+2x+2)}$$

5=A+B
0=2A-2B+C
0=2A-2C

1	1	0	5
2	-2	1	0
2	0	-2	0

$$\frac{x^2 - x - 8}{(x+1)(x^2+5x+6)} =$$

$$= \frac{-3}{x+1} + \frac{2}{x+3} + \frac{2}{x+2}$$

(x^2+5x+6) x^2+3x+2 x^2+4x+3

$-15x + 6x + 8x$

$-18 + 4 + 6$

$$\frac{8x^2+32}{x^4+8x^3+16x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+4}+\frac{D}{(x+4)^2}$$

$$\frac{4x^2+3}{x^6+2x^4+x^2}$$

Factor: $x^2(x^2+1)^2$

List all the factors: $x, x^2, (x^2+1), (x^2+1)^2$

$$\frac{4x^2+3}{x^6+2x^4+x^2}=\frac{4x^2+3}{x^2\left(x^2+1\right)^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{Cx+D}{\left(x^2+1\right)}+\frac{Ex+F}{\left(x^2+1\right)^2}$$

$$4x^2+3=Ax\left(x^2+1\right)^2+B\left(x^2+1\right)^2+\left(Cx+D\right)x^2\left(x^2+1\right)+\left(Ex+F\right)x^2$$

	A	B	C	D	E	F
x^5:0=	1	0	1	0	0	0
x^4:0=	0	1	0	1	0	0
x^3:0=	2	0	1	0	1	0
x^2:4=	0	2	0	1	0	1
x:0=	1	0	0	0	0	0
c:3=	0	1	0	0	0	0

A=0
B=3
C=0
D=-3
E=0
F=1

$$\frac{4x^2+3}{x^6+2x^4+x^2}=\frac{3}{x^2}+\frac{-3}{\left(x^2+1\right)}+\frac{1}{\left(x^2+1\right)^2}$$

$$x = 3$$

$$x(t) = v_0 \cos(\theta) t$$

$90°$

0

$$y(t) = -\frac{1}{2} g t^2 + v_0 \sin(\theta) t + h$$

$$y(t) = -\frac{1}{2} 32 t^2 + 50 \sin(90) t + 6$$

$$\frac{x}{(3x-2)(2x+1)} = \frac{A}{(3x-2)} + \frac{B}{(2x+1)}$$

$$x = A(2x+1) + B(3x-2)$$

$$\frac{x+4}{x^2(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$$

$A = \frac{1}{4}$, $B = 1$, $Cx + D = -\frac{1}{4}x - 1$

$$\frac{x^2 \quad (x^2+4)}{x,\ x^2,\ x^2+4}$$

$$x+4 = A\,x(x^2+4) + B(x^2+4) + (Cx+D)\,x^2$$

$Ax^3 + 4Ax$

	A	B	C	D	
x^3	1	0	1	0	0
x^2	0	1	0	1	0
x	4	0	0	0	1
1	0	4	0	0	4

$B = 1$

$A = \frac{1}{4}$

$D = -1$

$C = -\frac{1}{4}$