

These problems are designed both to exercise your skills with rational functions and recall skills in analyzing the polynomials in the numerator and denominator. Except for inspecting range and solving the last problem on this page, you should be able to complete all analysis without a calculator.

Describe domain, range, all intercepts, and all asymptotes:

$$d(x) = \frac{x^2 - 2x}{x - 3}$$

Asymptotes: Vertical at $x=3$. Long division result: $(x+1) + 3/(x-3) \Rightarrow$ Oblique $y=x+1$
 Zeros: 0 and 2. X-intercepts: $(0,0), (2,0)$, y-intercept: $(0,0)$
 Domain: $-\infty < x < 3$ or $3 < x < \infty$. Range: $-\infty < y \leq -5.5359$ or $7.4641 < y < \infty$ (by inspection of graph)

$$e(x) = \frac{2x^2 - 32}{x^2 - 4x - 21}$$

Asymptotes: Vertical at $x=7$, $x=-3$. Horizontal at $y=2$. Zeros: 4, -4
 x-intercepts $(4,0), (-4,0)$, y-intercepts = $(0, 32/21)$ Domain: All reals except 7 and -3. Range: All reals

$$f(x) = \frac{x^4 - 6x^2 + 8}{x^2 - x - 6}$$

Asymptotes: Vertical at $x=3$. Note: A hole at $(-2,0)$ Zeros: $\pm\sqrt{2}$ and 2. x-intercepts $(-\sqrt{2},0), (+\sqrt{2},0), (2,0)$, y-intercept: $(0, -4/3)$. Domain: All reals except 3 and -2. Range: All reals

$$g(x) = \frac{2x^4 + 6}{x^3 + 1}$$

Asymptotes: Vertical at $x=-1$, Long division result: $2x + (-2x+6)/(x^3+1)$, Oblique asymptote of $y=2x$. y-intercept: $(0,6)$ Domain: All reals except -1. Range: $-\infty < y \leq -5.4258$ or $3.6528 < y < \infty$ (by inspection of graph)

Requires use of calculator, but use minimally.

$$h(x) = \frac{x^4 - 2x^3 - 7x^2 - 2x - 8}{x^3 + 4x^2 + x - 6}$$

Vertical asymptotes: $x=-3, x=1$ Oblique asymptote: $y=x-6$ by long division Domain: All reals except -3, -2, 1
 Zeros: $x=4$ x-intercepts $(4,0)$, y-intercept: $(0, 4/3)$ Range: All reals

Create a rational function with:

Vertical asymptotes of $x=3$ and $x=-2$

A horizontal asymptote of $y=5$

And

Zeros of 2 and 4.

$$5 \cdot \frac{(x-2)(x-4)}{(x-3)(x+2)} = \frac{5x^2 - 30x + 40}{x^2 - x - 6}$$

Create a rational function which could correspond to this graph.

$$2 \cdot \frac{(x-1)(x+3)}{(x+2)(x-3)} = \frac{2x^2 + 4x - 6}{x^2 - x - 6}$$

