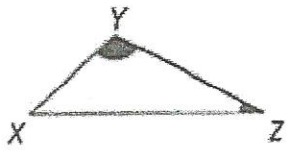
ANSWERS

Lesson 5.6: Inequalities in One Triangle

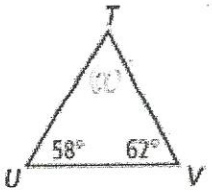
Objective: To use inequalities involving angles and sides of triangles.

Theorem 5-10		
Theorem	**If...**	**Then...**
If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.	$XZ > XY$ 	$m\angle Y > m\angle Z$

The converse is also true…

Theorem 5-11		
Theorem	**If...**	**Then...**
If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.	$m\angle A > m\angle B$ 	$BC > AC$

1. Using the information below, order the sides of $\triangle TUV$ from shortest to longest.

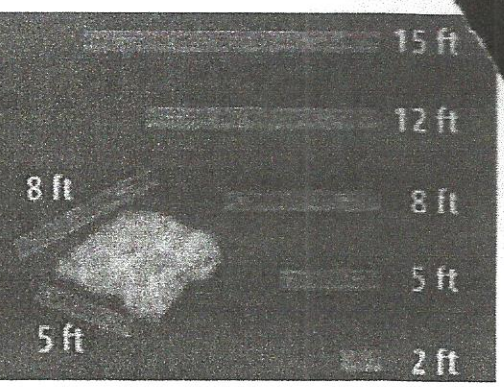


$\overline{TV}, \overline{UV}, \overline{TU}$

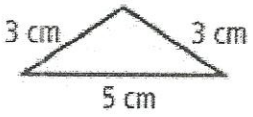
2. In the figure below, $m\angle S = 24$ and $m\angle O = 130$. Which side of $\triangle SOX$ is the shortest side? Why?

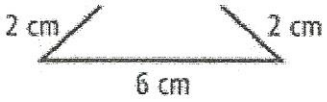
$\overline{OX}$ is shortest b/c across from smallest angle ($\angle S$)

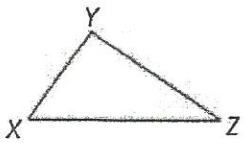
For a neighborhood improvement project, you volunteer to help build a new sandbox at the town playground. You have two boards that will make up two sides of the triangular sandbox. One is 5 ft long and the other is 8 ft long. Boards come in the lengths shown. Which boards can you use for the third side of the sandbox? Explain.



For three segments to form a triangle, their length must be related in a certain way. Notice that only of the sets of segments below can form a triangle. The sum of the smallest two lengths must be greater than the greatest length.





Theorem 5-12 Triangle Inequality Theorem		
Theorem		
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.	$XY + YZ > XZ$ $YZ + XZ > XY$ $XZ + XY > YZ$	

*Can the following segment lengths form a triangle? Why or why not?? (**hint:** add the 2 smaller sides and check to see if that sum is GREATER than the 3rd side. If it is GREATER than it CAN be a triangle)*

7, 4, 15 7+4 = 11 < 15 sum is less, so NOT a Δ	*3, 20, 21* 3+20 =23 > 21 sum is greater, so yes Δ
3, 7, 8 3+7 = 10 > 8 sum is greater, so yes a Δ	*5, 10, 15* 5+10 = 15 = 15 sum is = to 3rd side, so NOT a Δ

A triangle has side lengths of 5 ft. and 8 ft. What is the range of the possible lengths for the third side? ***(hint:*** *SUBTRACT for the lower end of range, and ADD for upper end of range)*

5, 8, x

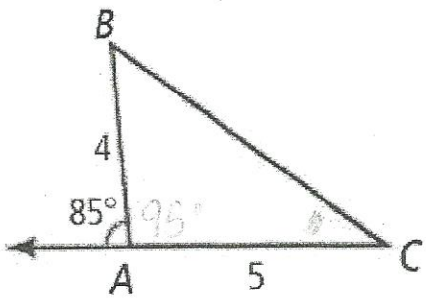
$3 < x < 11$

A triangle has side lengths of 4 in. and 7 in. What is the range of the possible lengths for the third side?

4, 7, x

$3 < x < 11$

Use $\triangle ABC$ for Exercises 1 and 2.



1. Which side is the longest? $\overline{BC}$

2. Which angle is the smallest? $\angle B$

3. Can a triangle have sides of lengths 4, 5, and 10? Explain.

No, b/c 4+5=9 which is less than 10.

4. A friend tells you that she drew a triangle with perimeter 16 in. and one side of length 8 in. How do you know she made and error in her drawing?

If perimeter was 16, then the other 2 sides would have to add up to 8 as well but 2 sides have to add to more than 3rd side.

Practice with FACTORING…solve by factoring.

1) $k^2 + 8k + 12 = 0$

$(k+2)(k+6)=0$

$k=-2, -6$

2) $3x^2 + 14x + 16 = 0$

8	$8x$	16
$3x$	$3x^2$	$6x$
	x	2

$24x^2$, $8x$, $6x$, $14x$

$(x+2)(3x+8)=0$

$x=-2, -8/3$

3) $2x^2 + 7x - 15 = 0$

-3	$-3x$	-15
$2x$	$2x^2$	$10x$
	x	5

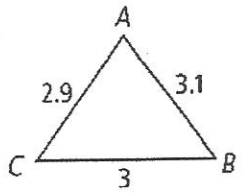
$-30x^2$, $-3x$, $10x$, $7x$

$(x+5)(2x-3)=0$

$x=-5, 3/2$

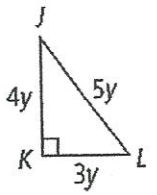
For Exercises 1–3, list the angles of each triangle in order from smallest to largest.

1.



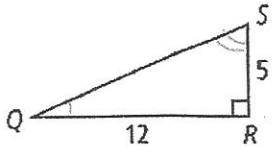
∠B, ∠A, ∠C

2.



∠J, ∠L, ∠K

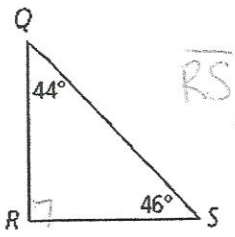
3.



∠Q, ∠S, ∠R

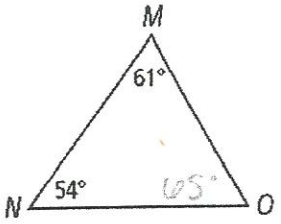
For Exercises 4–7, list the sides of each triangle in order from shortest to longest.

4.



$\overline{RS}, \overline{QR}, \overline{QS}$

5.



$\overline{MO}, \overline{NO}, \overline{MN}$

6. $\triangle ABC$, with $m\angle A = 99°$, $m\angle B = 44°$, and $m\angle C = 37°$

$\overline{AB}, \overline{AC}, \overline{BC}$

7. $\triangle ABC$, with $m\angle A = 122°$, $m\angle B = 22°$, and $m\angle C = 36°$

$\overline{AC}, \overline{AB}, \overline{BC}$

Can a triangle have sides with the given lengths? Explain.

8. 8 cm, 7 cm, 9 cm

yes $8+7>9$

9. 7 ft, 13 ft, 6 ft $7+6=13$ NO

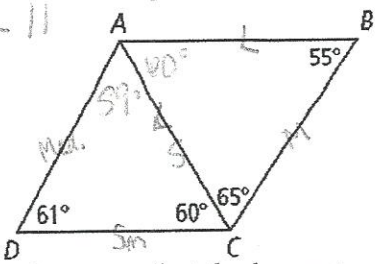
10. 20 in., 18 in., 16 in.

$18+16$
$34 > 20$
yes

11. 3 m, 11 m, 7 m

$3+7$
$10 < 11$
NO

12. **Determine which side is shortest in the diagram to the right.**



$\overline{DC}$

13. **Error Analysis** A student draws a triangle with a perimeter 36 cm. The student says that the longest side measures 18 cm. How do you know that the student is incorrect? Explain.

the other 2 sides would add up to 18, but should be greater than 3rd side.