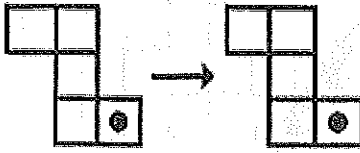


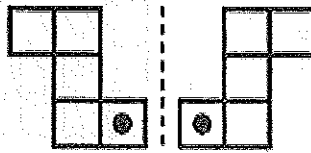
**RIGID TRANSFORMATIONS – (ITERATIONS)**

ISOMETRIES

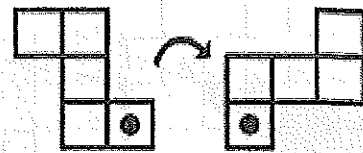
**Translations, Reflections, Rotations**



TRANSLATION

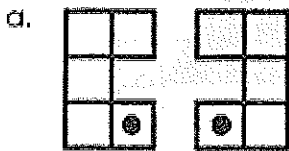


REFLECTION

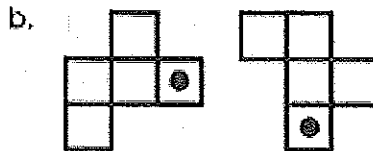


ROTATION

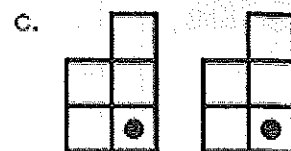
Tell how each figure was moved. Write translation, rotation or reflection.



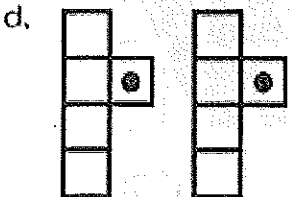
Reflection



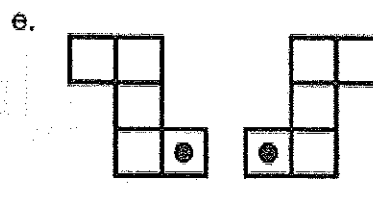
Rotation



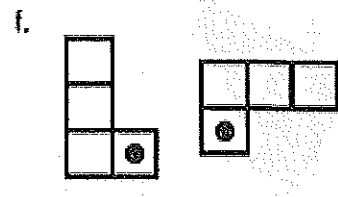
Translation



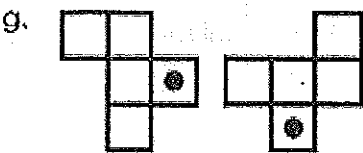
Trans



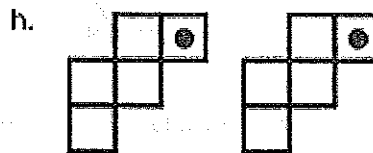
Ref.



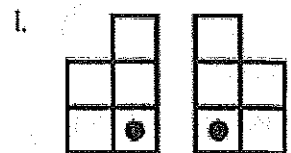
Rot.



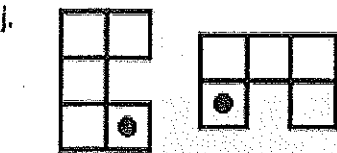
Rot.



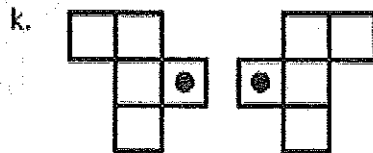
Transl.



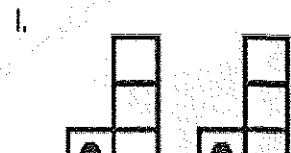
Ref.



Rot.



Ref.

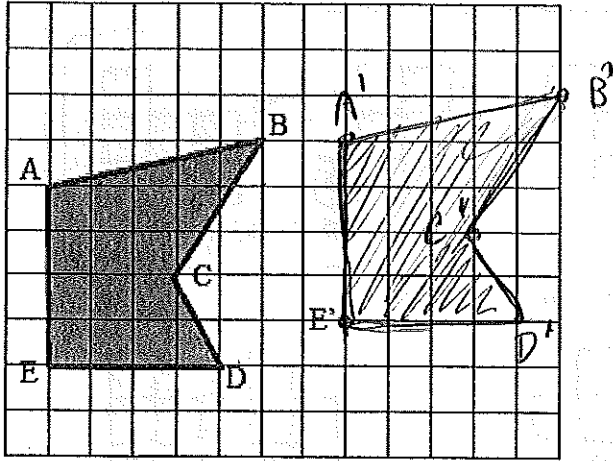


Transl.

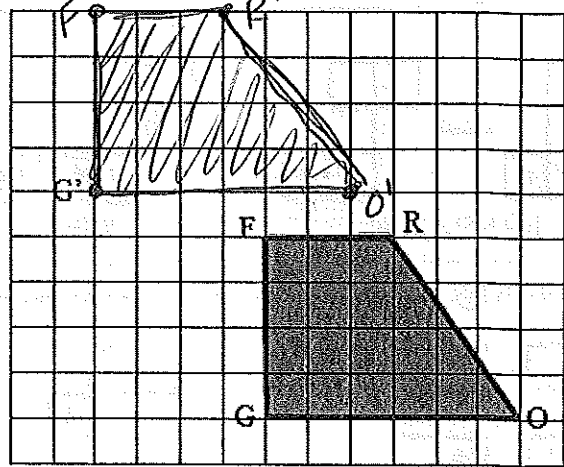
# TRANSLATIONS:

Directions: Perform each translation. You may use patty paper, geometry software, or any other tools or method that seems appropriate to help you.

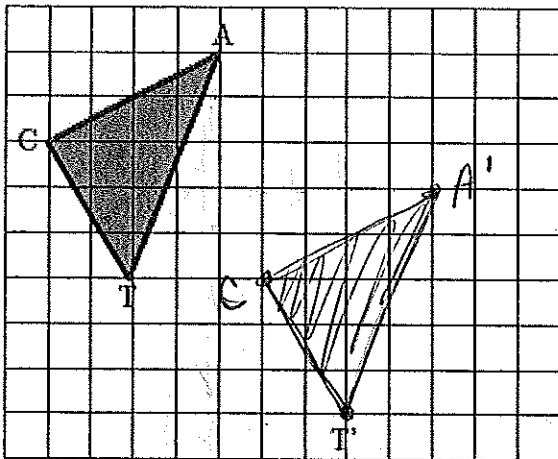
1. Translate  $ABCDE \rightarrow A'B'C'D'E'$



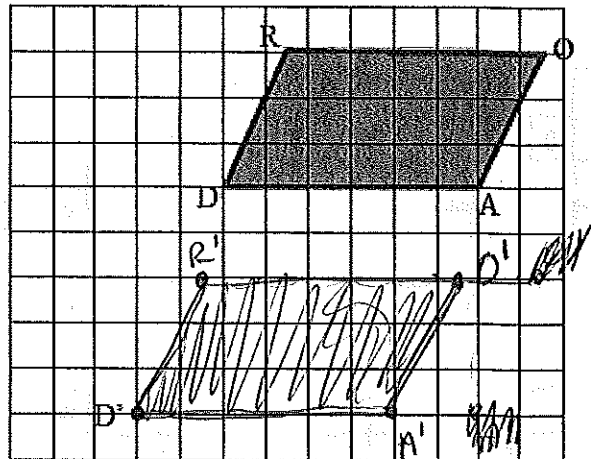
2. Translate  $FROG \rightarrow F'R'O'G'$



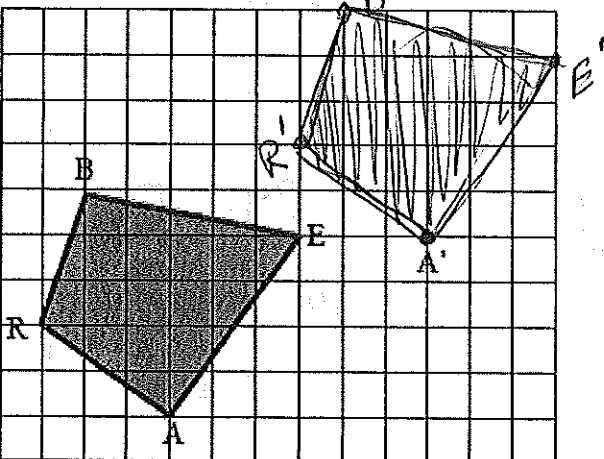
3. Translate  $CAT \rightarrow C'A'T'$



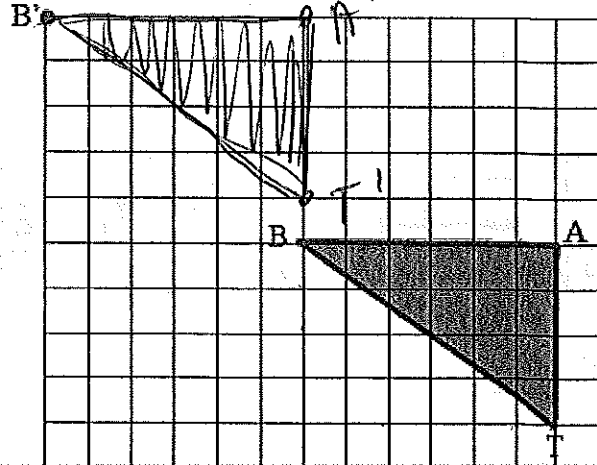
4. Translate  $ROAD \rightarrow R'O'A'D'$



5. Translate  $BEAR \rightarrow B'E'A'R'$



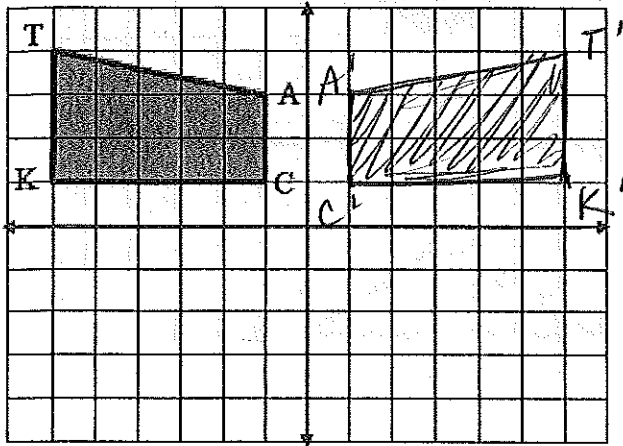
6. Translate  $BAT \rightarrow B'A'T'$



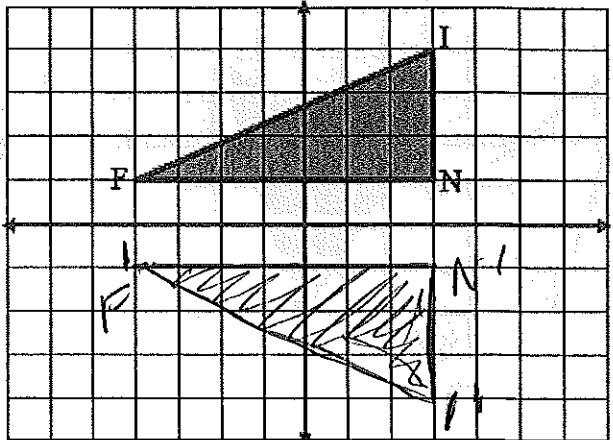
# REFLECTIONS:

Directions: Use patty paper, Geometry software, or any other method to reflect each figure as directed. Make sure to label your image figure correctly.

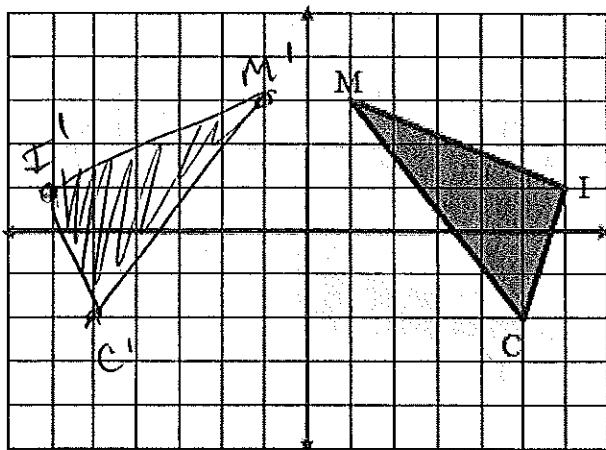
1. Reflect TACK through the y axis.  $R_{y \text{ axis}}$



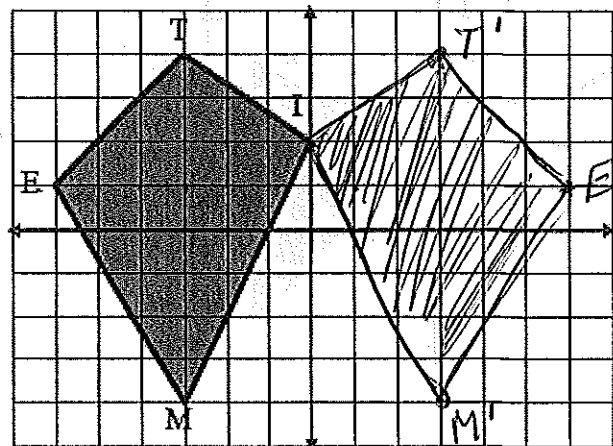
2. Reflect FIN through the x axis.  $R_{x \text{ axis}}$



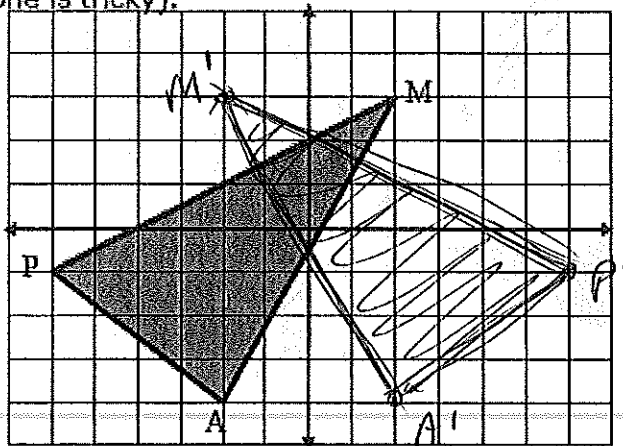
3. Reflect MIC through the y axis.  $R_{y \text{ axis}}$



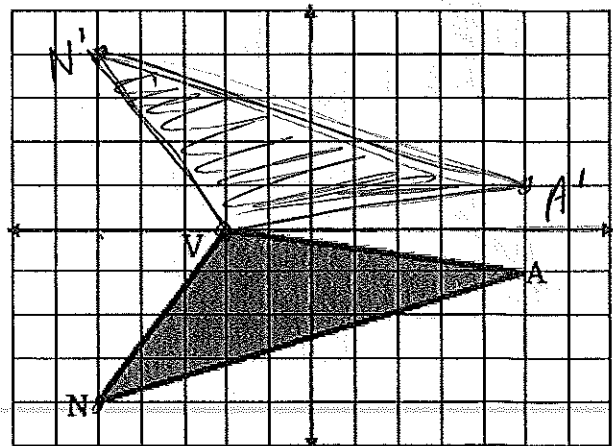
4. Reflect TIME through the y axis.  $R_{y \text{ axis}}$



5. Reflect MAP through the Y axis.  $R_{y \text{ axis}}$  (this one is tricky).

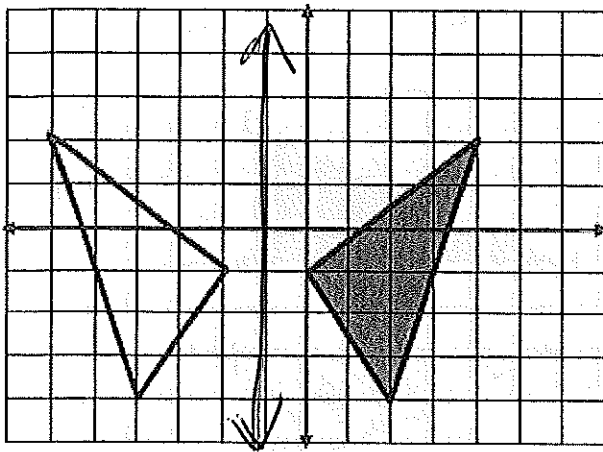


6. Reflect VAN through the x axis.  $R_{x \text{ axis}}$

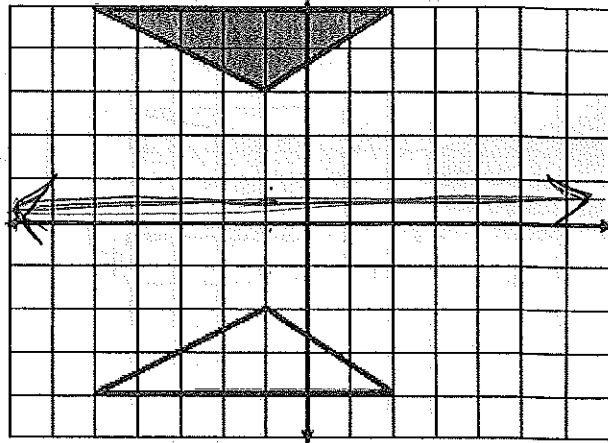


**Directions:** In each problem, a figure and its image are shown. Draw the line of reflection that will map the original onto its reflected image.

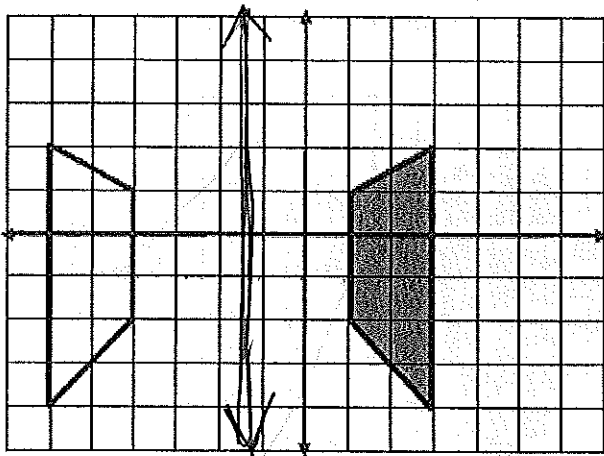
1



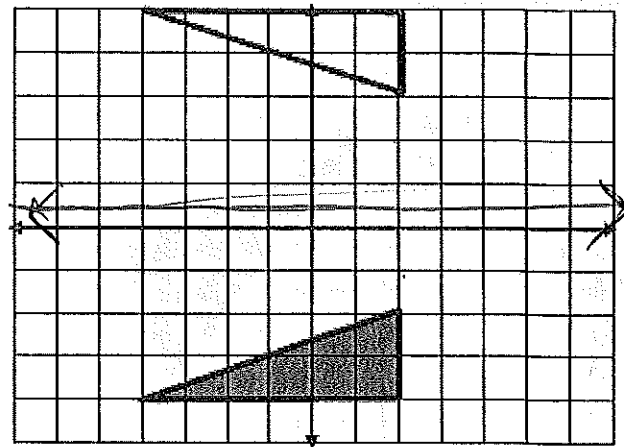
2



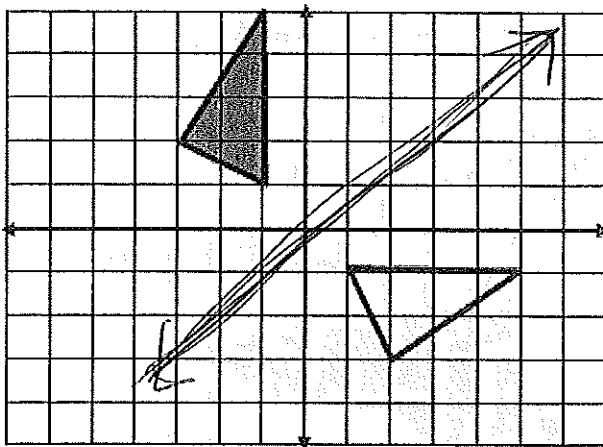
3



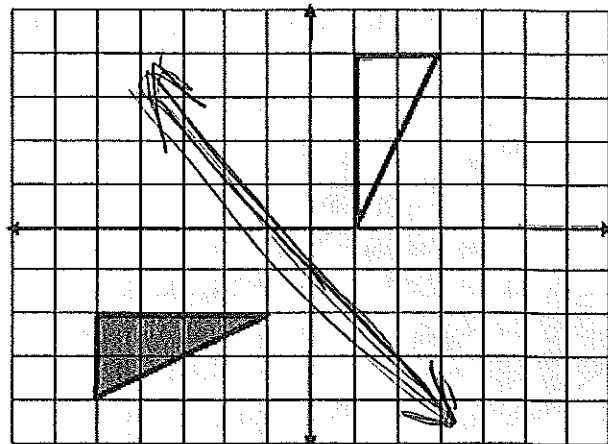
4



5



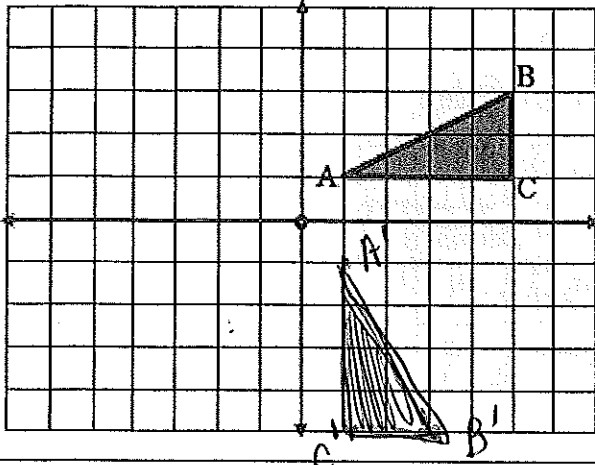
6



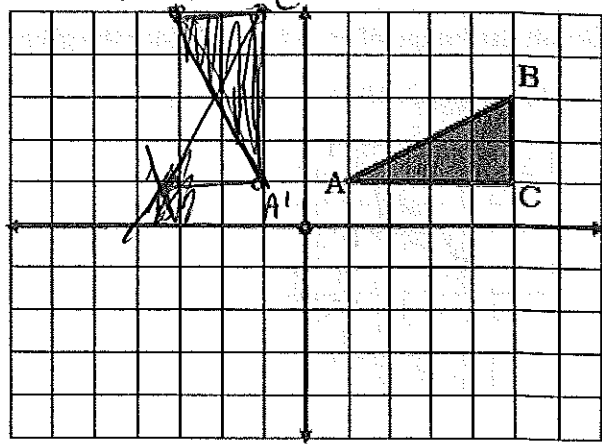
# ROTATIONS:

**Directions:** Use patty paper, Geometry software, or any other method available to you to rotate each figure as directed. Make sure to label your new figure.

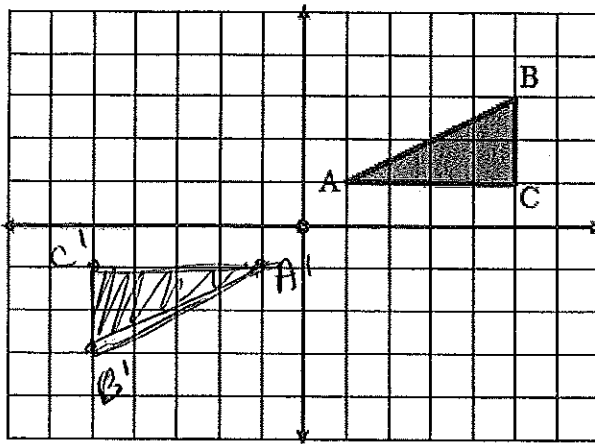
1. Rotate ABC 90° clockwise about the origin.  
 $R_{O,-90^\circ}$



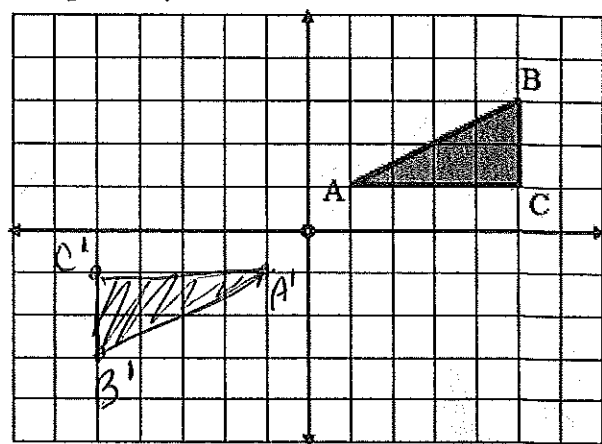
2. Rotate ABC 90° counter-clockwise about the origin.  
 $R_{O,90^\circ}$



3. Rotate ABC 180° clockwise about the origin.  
 $R_{O,-180^\circ}$



4. Rotate ABC 180° counter-clockwise about the origin.  
 $R_{O,180^\circ}$



5a. What do you think happens to the center point of rotation? Does it move?

No

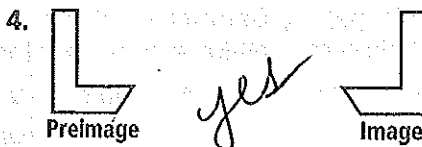
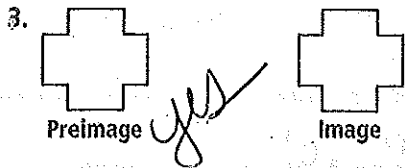
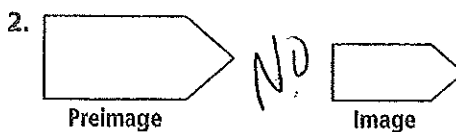
b. What do think... is each points in image the same distance from the center as in the original figure? Why?

yes  
 rotating about point  
 doesn't change  
 distance

6. Is a rotation an isometric transformation?

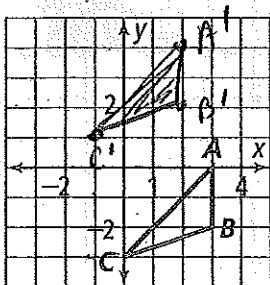
yes

Tell whether the transformation appears to be a rigid motion. Explain.

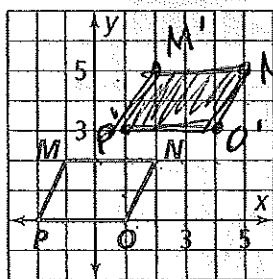


Graph the image of each figure under the given translation.

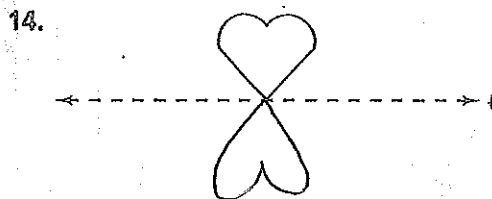
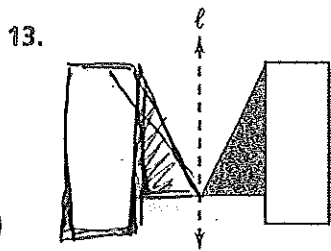
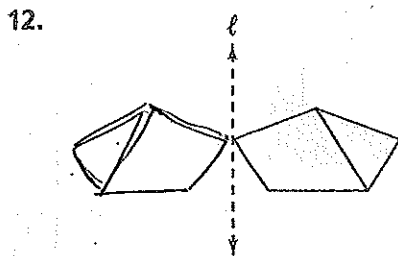
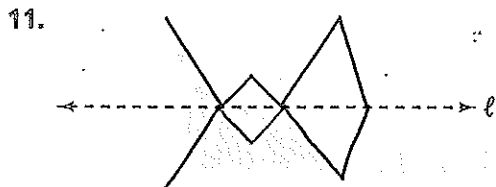
7.  $T_{\langle -1, 4 \rangle}(\triangle ABC)$



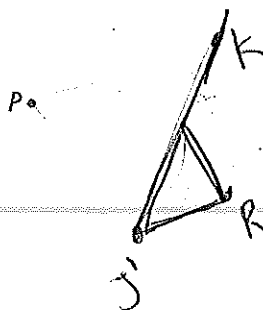
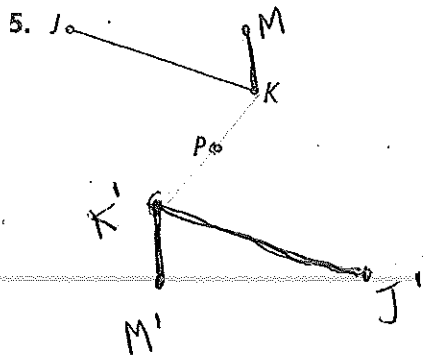
8.  $T_{\langle 3, 3 \rangle}(MNOP)$



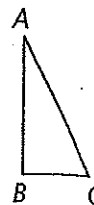
Copy each figure and line  $\ell$ . Draw each figure's reflection image across line  $\ell$ .



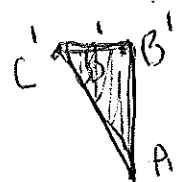
Copy each figure and point  $P$ . Then draw the image of  $\overline{JK}$  for a  $180^\circ$  rotation about  $P$ . Use prime notation to label the vertices of the image.



make this one  
7.  $90^\circ$



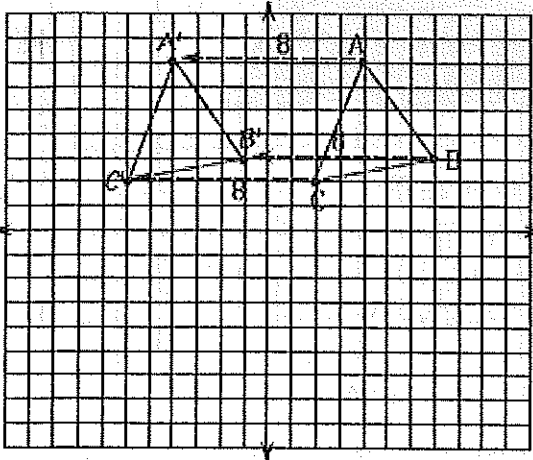
$P$



# Transformations on the Coordinate Plane

These are just some examples of how we can move transformations using coordinates and the x and y axes.

Ex. 1. Translate  $\triangle ABC$  8 units to the left.

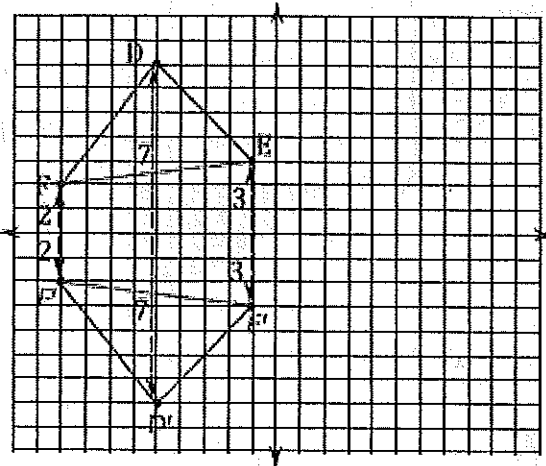


It's very easy to do this... First, start with A and count 8 units to the left and plot a point. Label this point A' ("A Prime"). Next, do this with all the other points. Last, connect them to form the new triangle.

The ' or "Prime" is simply a way to say "the new A" with less confusion and less words.

Sometimes these problems are written with a rule. This rule would be  $(x-8)$ . It just means "take all the x coordinates and subtract 8 from them." (Which is just counting 8 to the left.)

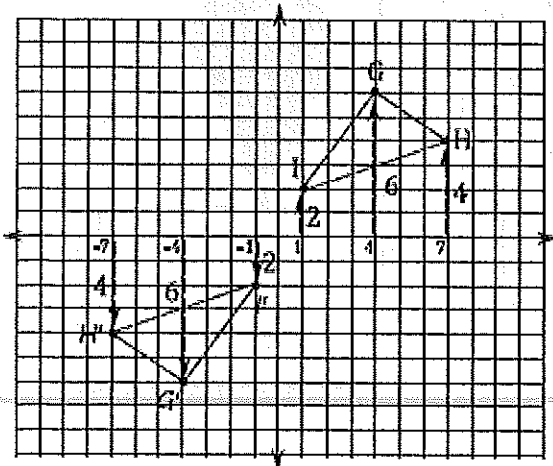
Ex. 2. Reflect  $\triangle DEF$  over the x axis.



To do this it's, also very easy... First, start with D and count how many units it is away from the x axis. Then count that many units on the other side and plot a point and label it D'. Next, do this with all the other points. Last, connect them to form the new triangle.

Be very careful on this type to reflect it over the correct axis. Remember, the y axis goes up and down, and the x axis goes left and right. Always double check which axis you are supposed to be reflecting over.

Ex. 3. Rotate  $\triangle GHI$   $180^\circ$  about the origin.



Now these get trickier... First, start with I. Count along the nearest axis the position of I, then go two axis to the left (or right for  $180^\circ$ ). Pretend this is the axis you were just using. Plot it in the same position. Label it I'. Next, do this with all the other points. Last, connect them to form the new triangle.

The tricky part is to figure out how far to go.

Another method is to actually turn your paper  $180^\circ$ , and then see where the triangle is and plot the points.

We will do  $90^\circ$  clockwise, and counterclockwise as well.

# TRANSLATIONS:

Directions: Use patty paper, Geometry Software, or any other method available to you, to translate each figure using the given function.

*change to*

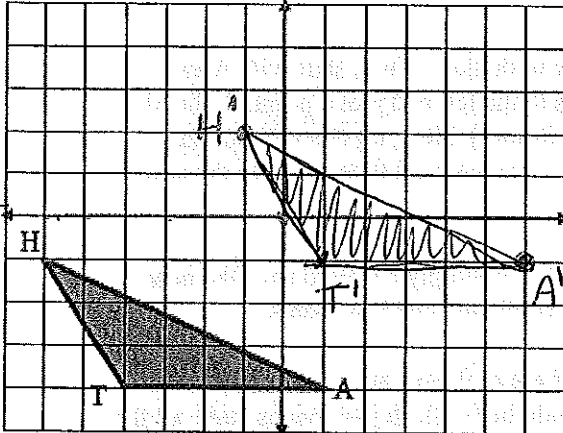
1. Use  $(x,y) \rightarrow (x+5, y+3)$

$\langle 5, 3 \rangle$

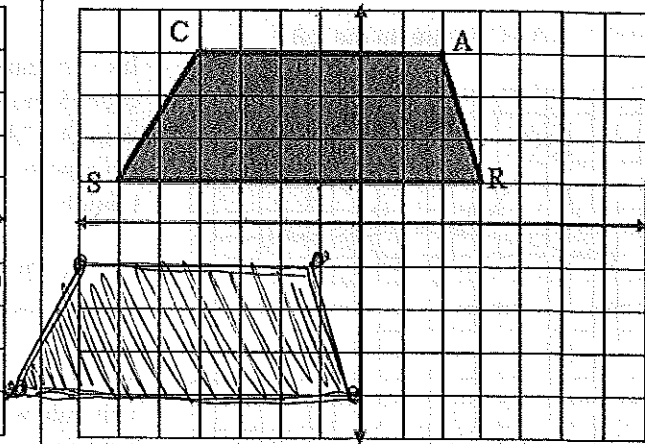
$+5 \ +3$   
 $H(-6, -1)$   
 $A(1, -4)$   
 $T(-4, -4)$   


---

 $H'(1, 2)$   
 $A'(6, -1)$   
 $T'(1, -1)$



2. Use  $(x,y) \rightarrow (x-3, y+5)$

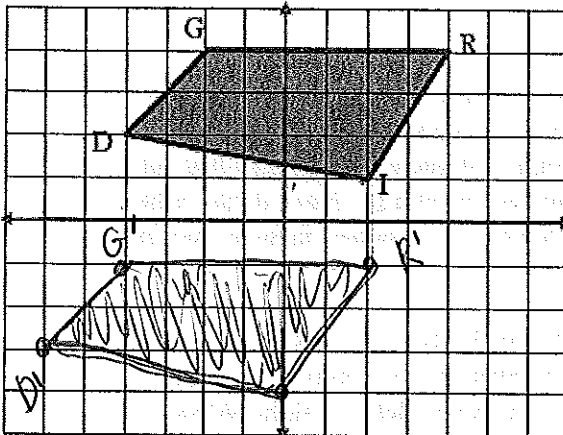


$a, 3 \times 5$   
 $A(2, 4)$   
 $R(3, 1)$   
 $S(-6, 1)$   
 $C(-4, 4)$   

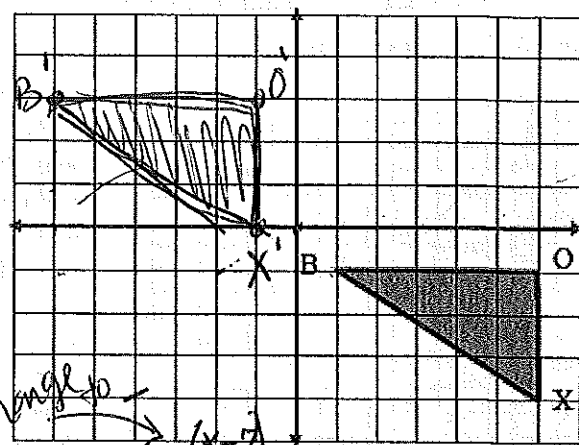

---

 $A'(-1, 9)$   
 $R'(0, 6)$   
 $S'(-9, 6)$   
 $C(-7, 9)$

3. Use  $(x,y) \rightarrow (x-2, y-5)$



4. Use  $(x,y) \rightarrow (x-7, y+4)$

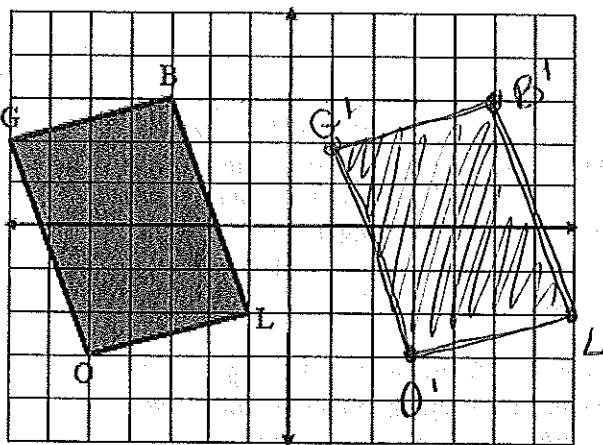


$-7 \ +4$   
 $B'(1, 1)$   
 $O(6, 1)$   
 $X(6, -4)$   

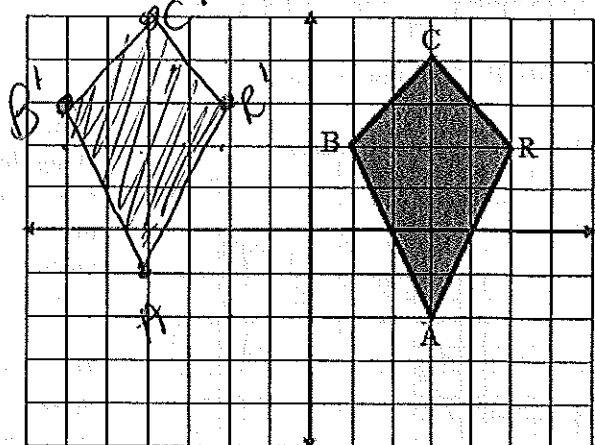

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 $B''(-6, 5)$   
 $O'(-1, 5)$   
 $X'(-1, 0)$

5. Use  $(x,y) \rightarrow (x+8, y)$

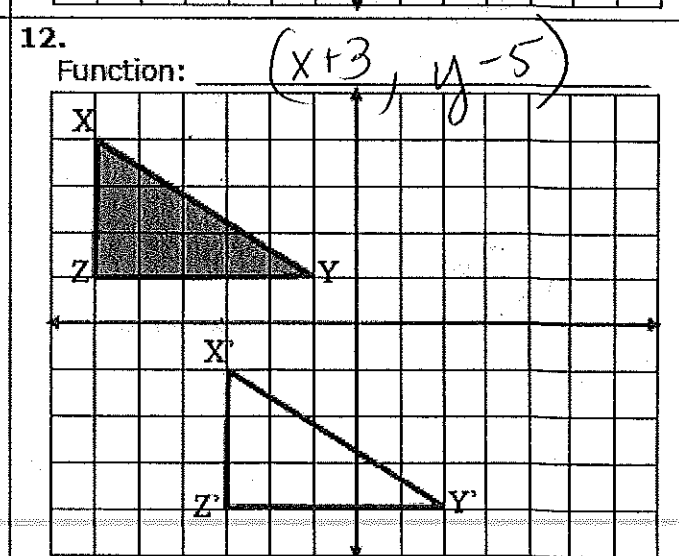
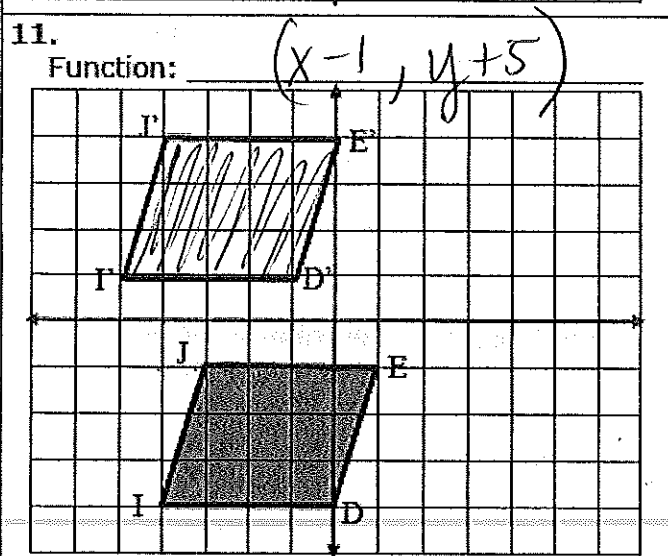
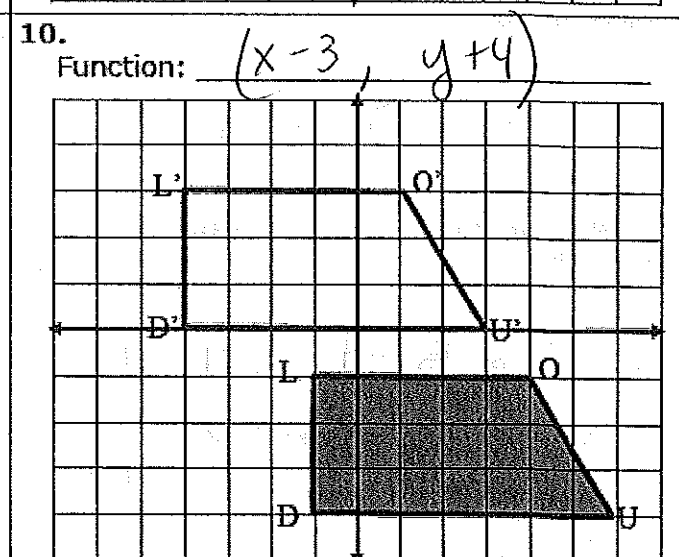
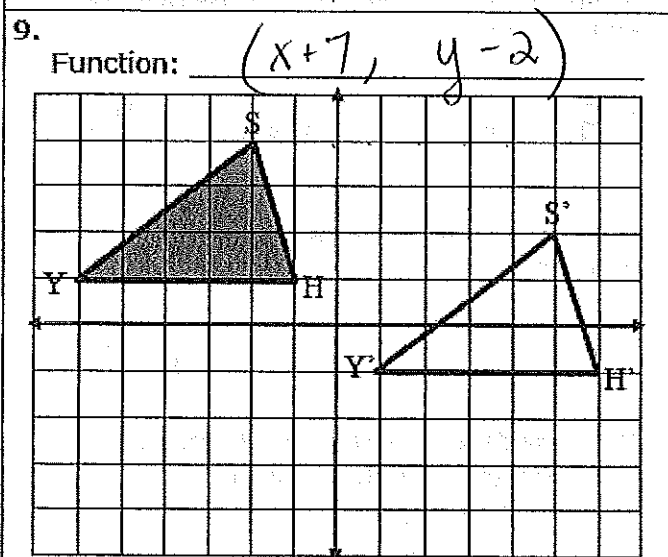
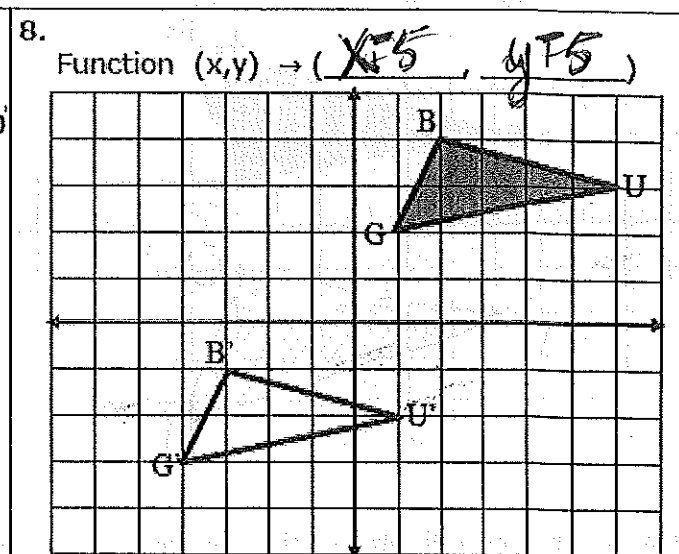
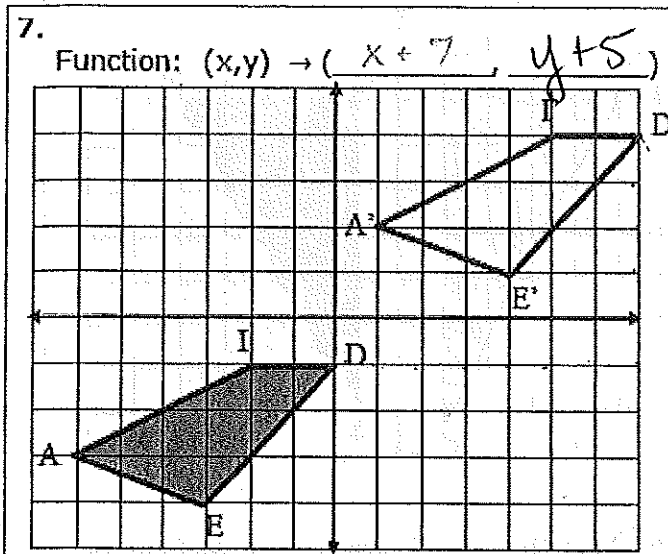


6. Use  $(x,y) \rightarrow (x+7, y+1)$



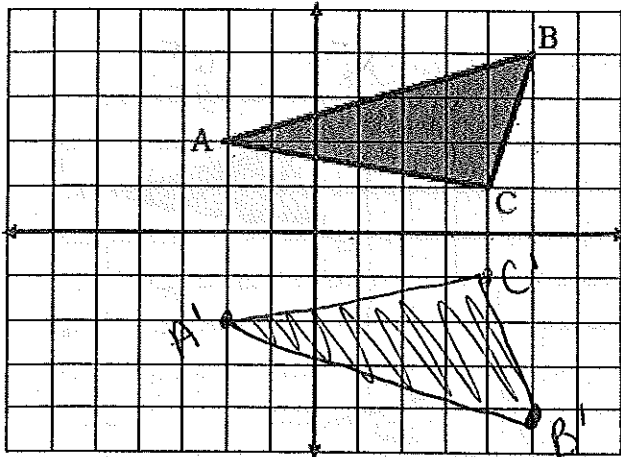


Directions: Write a geometric function that describes each translation.



# REFLECTIONS:

7. Reflect ABC through the x axis.



a. What are the coordinates of the vertices of the original figure?

$A(-2, 2)$   $B(5, 4)$   $C(4, 1)$

b. What are the coordinates of the vertices of A'B'C'?

$A'(-2, -2)$   $B'(5, -4)$   $C'(4, -1)$

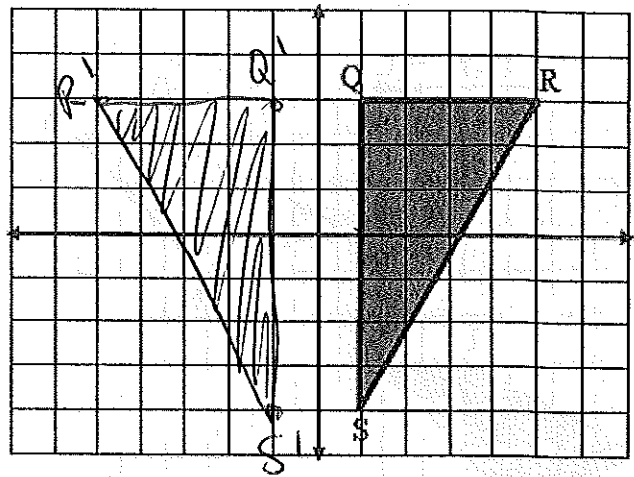
c. Explain in writing how the coordinates of ABC have been changed to create A'B'C' in this reflection through the x axis.

x stays same,  
y becomes opposite sign. (-)

d. Write a function that describes a reflection through the x axis.

$(x, y) \rightarrow (x, -y)$

8. Reflect QRS through the y axis.



a. What are the coordinates of the vertices of the original figure?

$Q(1, 3)$   $R(5, 3)$   $S(1, -4)$

b. What are the coordinates of the vertices of Q'R'S'?

$Q'(-1, 3)$   $R'(-5, 3)$   $S'(-1, -4)$

c. Explain in writing how the coordinates of QRS have been changed to create Q'R'S' in this reflection through the y axis.

x becomes opposite sign,  
y stays same

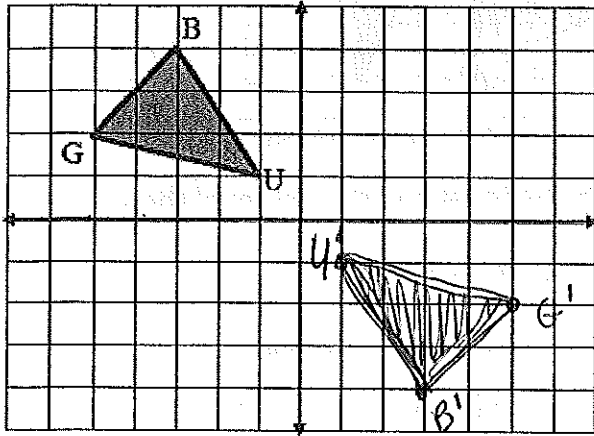
d. Write a function that describes a reflection through the y axis.

$(x, y) \rightarrow (-x, y)$

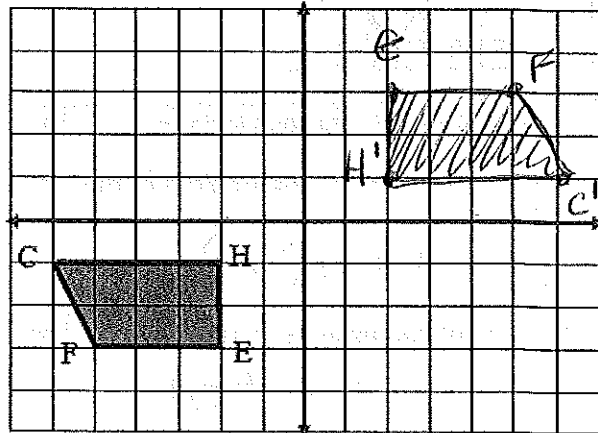
# ROTATIONS:

Directions: Use patty paper, Geometry software, or any other method to rotate each figure as directed. Make sure to label your image figure correctly.

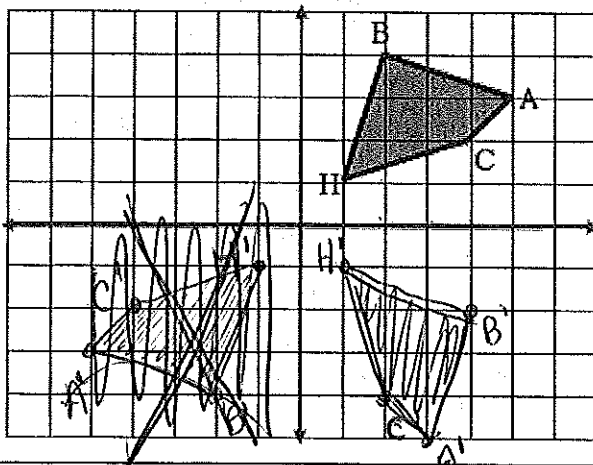
1. Rotate BUG 180° clockwise about the origin.  
 $R_{O,-180^\circ}$



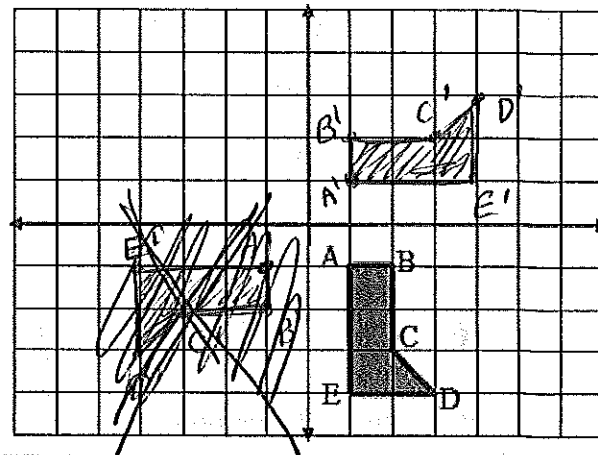
2. Rotate CHEF 180° counter-clockwise about the origin.  
 $R_{O,180^\circ}$



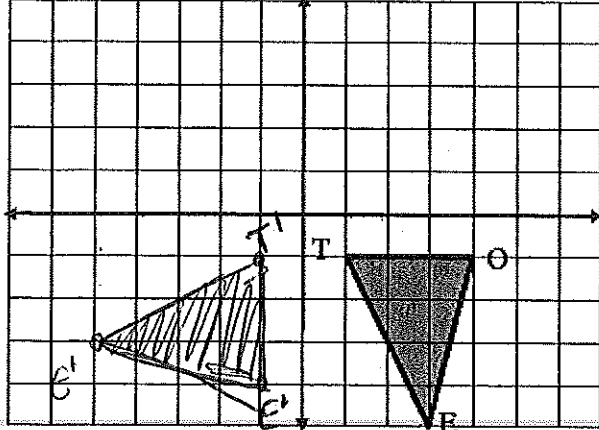
3. Rotate BACH 90° clockwise about the origin.  
 $R_{O,-90^\circ}$



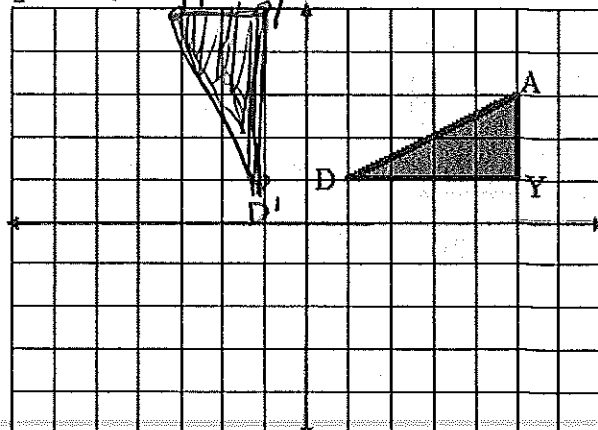
4. Rotate ABCDE 90° counter clockwise about the origin.  
 $R_{O,90^\circ}$



5. Rotate TOE 90° clockwise about the origin.  
 $R_{O,-90^\circ}$

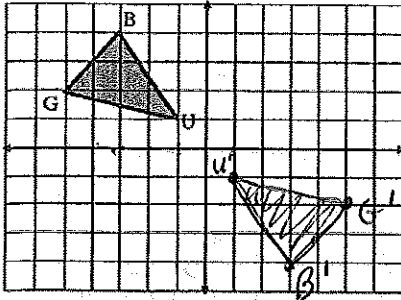


6. Rotate DAY 90° counter-clockwise about the origin.  
 $R_{O,90^\circ}$

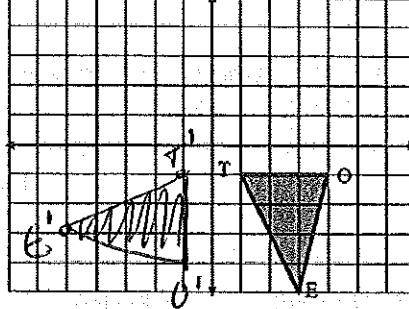


Use these problems from the previous page as a reference for the following questions.

1. Rotate BUG 180° clockwise about the origin.  
Ro. -180°



5. Rotate TOE 90° clockwise about the origin.  
Ro. -90°



7. For problem 1 (180° rotation clockwise)...

What are the coordinates of the vertices of the original figure?

B (-3, 4) U (-1, 1) G (-5, 2)

What are the coordinates in its image?

B' (3, -4) U' (1, -1) G' (5, -2)

Describe the relationship between the original and image coordinates in words.

both x ; y change  
to their  
oppos. signs

Describe the relationship with a function.

$(x, y) \rightarrow (-x, -y)$

9. For problem 6 (90° rotation counter clockwise)...

What are the coordinates of the vertices of the original figure?

D (1, 1) A (5, 3) Y (5, 1)

What are the coordinates in its image?

D' (-1, 1) A' (3, 5) Y' (-1, 5)

Describe the relationship between the original and image coordinates in words.

x moves to y position  
and y moves to x  
position ; becomes opposite  
sign

Describe the relationship with a function.

$(x, y) \rightarrow (-y, x)$

8. For problem 5 (90° rotation clockwise)...

What are the coordinates of the vertices of the original figure?

T (1, -1) O (4, -1) E (3, 5)

What are the coordinates in its image?

T' (-1, -1) O' (-1, -4) E' (-5, -3)

Describe the relationship between the original and image coordinates in words.

y moves to the x position  
and x moves to y position  
and becomes opposite  
sign

Describe the relationship with a function.

$(x, y) \rightarrow (y, -x)$

10. Bill says if you rotate a figure 180 clockwise or counterclockwise you will get the same image. Sally says you won't. Who is correct? Why?

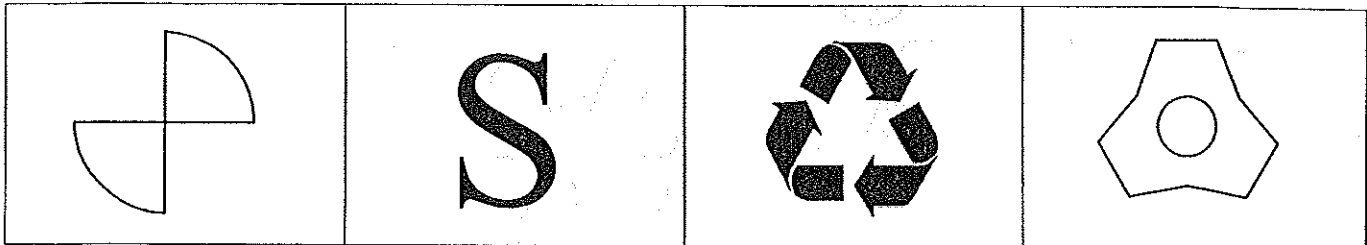
Bill is correct,  
b/c it is  
halfway  
around so either  
way you  
end in same  
spot.

## Rotational Symmetry

**Symmetry:** Symmetry is the property of a shape that allows it to be carried onto itself either by reflection, or rotation.

The second kind of symmetry is called rotational symmetry. Rotational symmetry occurs when a figure can be carried onto itself by turning it around a center point. Here are some examples...

**Rotational Symmetry:** A figure in the plane has rotational symmetry if and only if the figure can be mapped onto itself by a rotation about a point through any angle between  $0^\circ$  and  $360^\circ$ .



**Order of Rotational Symmetry:** The number of positions in which a figure can be carried onto itself is called the order of rotational symmetry (or simply rotation order).

order 1	order 2	order 3	order 4
 (No rotational symmetry)			

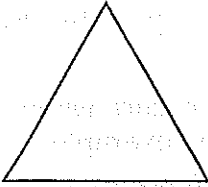
As you can see a rotational order of 1 actually means the figure has no rotational symmetry. This is because in order to carry the figure onto itself you must rotate it  $360^\circ$ , and because  $360^\circ$  is not BETWEEN  $0^\circ$  and  $360^\circ$  the figure does not have rotational symmetry. That makes sense visually because the shape of the lightning bolt can only appear as it does above in one and only one position.

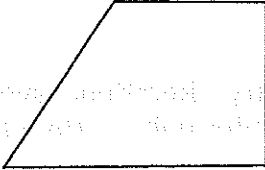
**Angle of rotation symmetry.** Another way to describe rotational symmetry is by determining the angle of rotation symmetry. In other words, how many degrees (or radians) around the circle you have to rotate a figure so that it is carried onto itself.

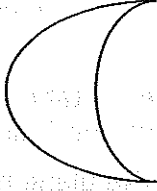
180° rotation	90° rotation	120° rotation	72° rotation

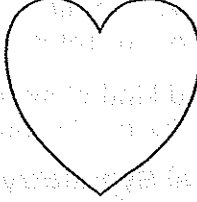
You will explore the relationship between rotational order, and the angle of rotational symmetry necessary to carry a figure onto itself as you practice.

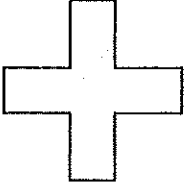
**Directions:** For each of the figures below, decide if the figure has rotational symmetry by circling yes or no.


1   
 Yes  NO

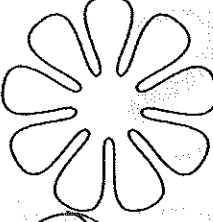
2   
 Yes  NO


3   
 Yes  NO

4   
 Yes  NO


5   
 Yes  NO

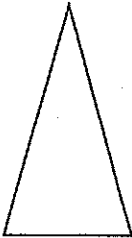
6   
 Yes  NO

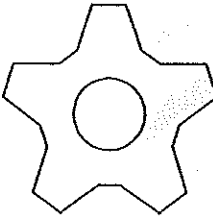
7   
 Yes  NO

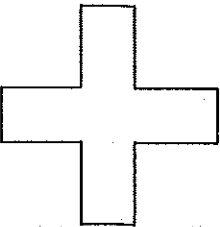
8   
 Yes  NO


**Directions:** for each of the figures below write the order of rotational symmetry. If the figure has no rotational symmetry write "no rotational symmetry" next to the figure.

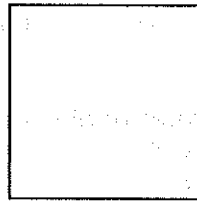
9   
 Order: 2

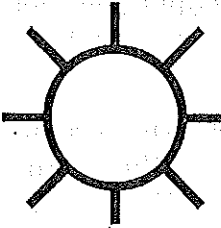
10   
 Order: none

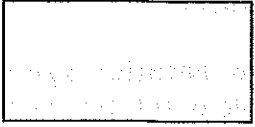
11   
 Order: 5

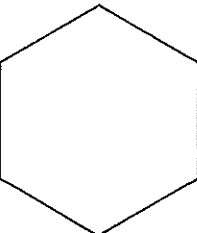
12   
 Order: 4


13   
 Order: none


14   
 Order: 4

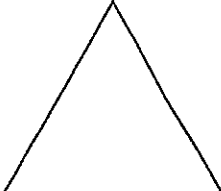
15   
 Order: 8

16   
 Order: 2

17   
 Order: 6

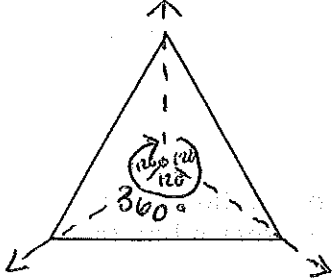
18   
 Order: none

19   
 Order: 2

20   
 Order: 3

**Directions:** For each regular polygon, record the number of sides the polygon has, and the rotational order of the polygon.  $\text{Degree of rotation} = \frac{360}{\text{order}}$

1 equilateral triangle

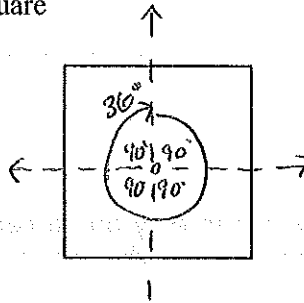


Sides: 3

Rotation Order: 3

Degree =  $\frac{360}{3} = 120^\circ$

2 square

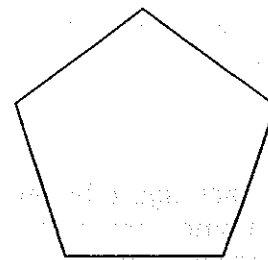


Sides: 4

Rotation Order: 4

Degree =  $\frac{360}{4} = 90^\circ$

3 regular pentagon

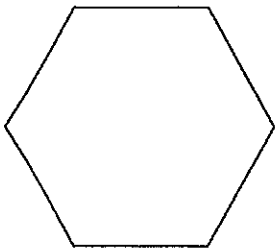


Sides: 5

Rotation Order: 5

Degree =  $\frac{360}{5} = 72^\circ$

4 regular hexagon

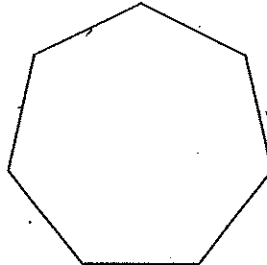


Sides: 6

Rotation Order: 6

Degree =  $\frac{360}{6} = 60^\circ$

5 regular heptagon

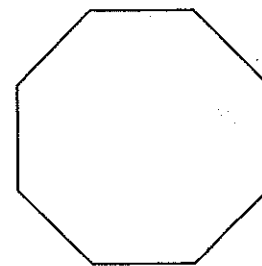


Sides: 7

Rotation Order: 7

Degree =  $\frac{360}{7} = 51.4^\circ$

6 regular octagon



Sides: 8

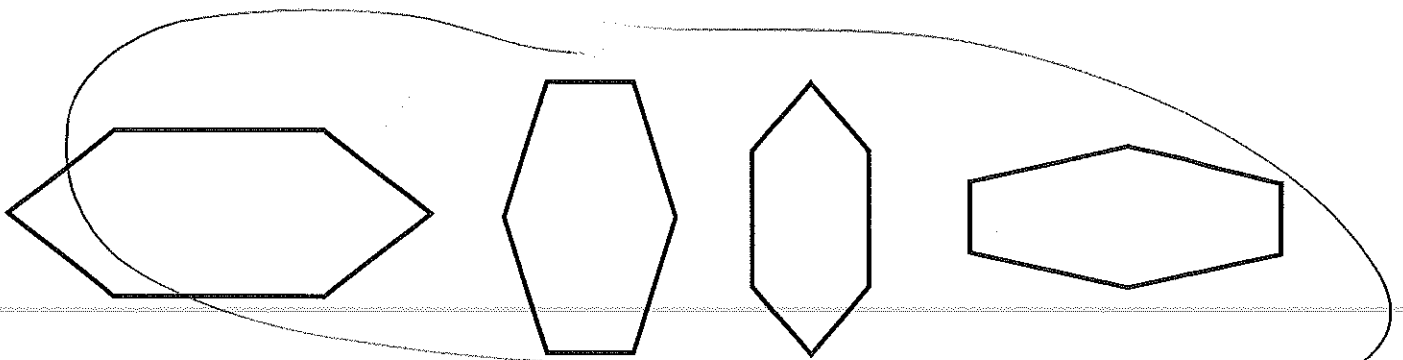
Rotation Order: 8

Degree =  $\frac{360}{8} = 45^\circ$

7. What is the relationship between the number of sides of a regular polygon and its rotational order?

*Same*

8. A regular hexagon has a rotational order of 6. Draw a hexagon that has a rotational order of only 2 in the space below.



6. A figure has a rotational order of 6. What is the minimum number of degrees the figure must be rotated to map it on top of itself? How did you figure it out?

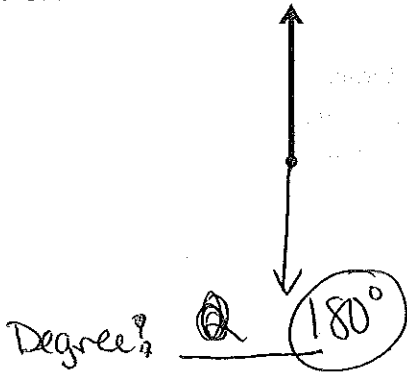
$$\frac{360}{6} = 60^\circ$$

7. A figure has a rotational order of 11. What is the minimum number of degrees the figure must be rotated to map it on top of itself?

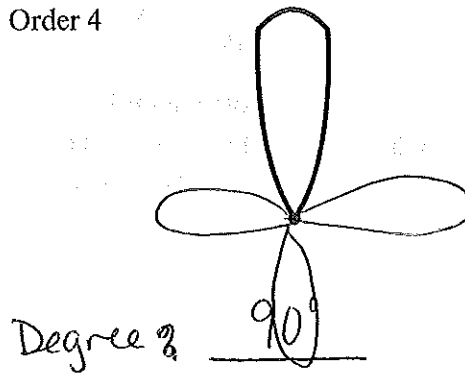
$$\frac{360}{11} = 32.7^\circ$$

**Directions:** Each figure below has rotational symmetry and a center point of rotation but is incomplete. Using patty paper or tracing paper, draw the rest of the figure so that it has the indicated rotational order.

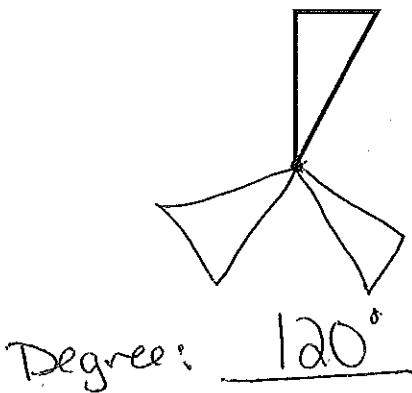
8 Order 2



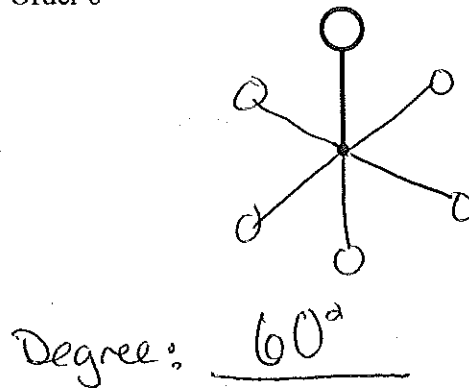
9 Order 4



10 Order 3



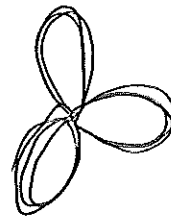
11 Order 6



7 Draw a figure that has a rotational order of 2.



7 Draw a figure that has a rotational order of 3.





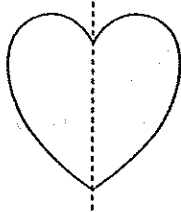
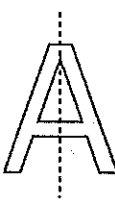
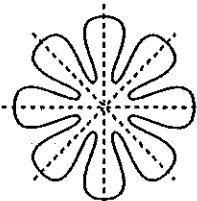
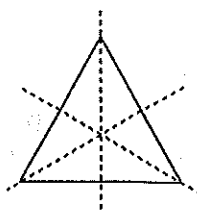
## Line Symmetry

**Symmetry:** Symmetry is the property of a shape that allows it to be carried onto itself either by reflection or rotation.

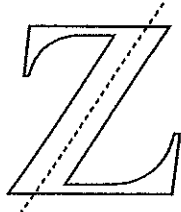
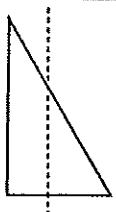
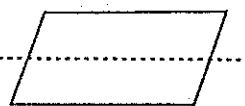
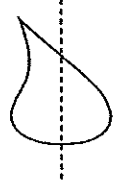
The first kind of symmetry is referred to as Line Symmetry or Linear Symmetry. Line Symmetry occurs when two halves of a shape can be reflected onto each other across a line. We call this line, a line of reflection. Sometimes it is also called a line of symmetry or the axis of symmetry.

**Line Symmetry:** A figure in the plane has line symmetry if and only if the figure can be mapped onto itself by a reflection through a line.

**Line of Reflection:** The line through which a figure can be reflected such that it is carried onto itself. This is often called a **Line of Symmetry** or the **Axis of Symmetry**.

one line of reflection	one line of reflection	4 lines of reflection	3 lines of reflection
			

The next few examples do not have line symmetry

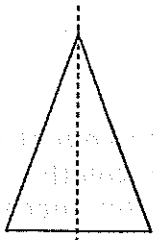
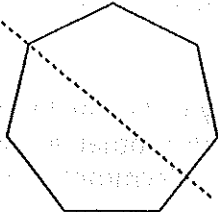
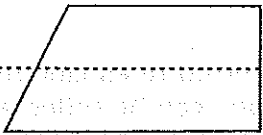
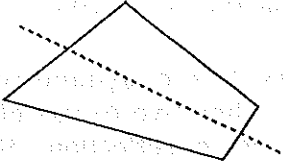
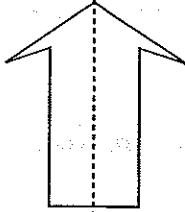
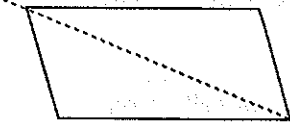
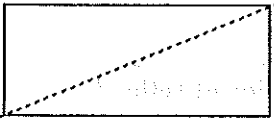
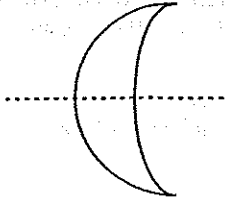
no line symmetry	no line symmetry	no line symmetry	no line symmetry
			

A nice way to decide if a shape has line symmetry is to trace the shape and the line of reflection on patty paper or tracing paper, and flip the paper over. When you match up the line of reflection if it doesn't match the original figure then it doesn't have line symmetry.

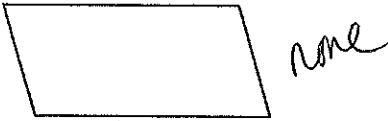
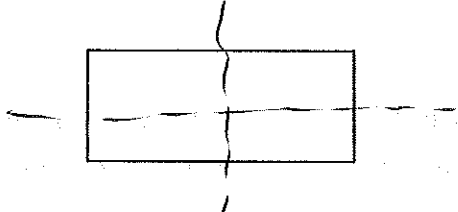
If you don't happen to have patty paper or tracing paper you can try folding your paper along the line of reflection to see if the two halves of the shape match up.

Go ahead and try one of these methods on the shapes above.

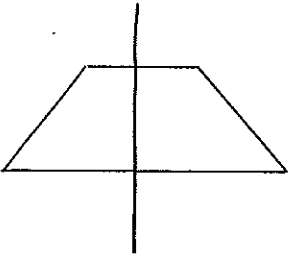
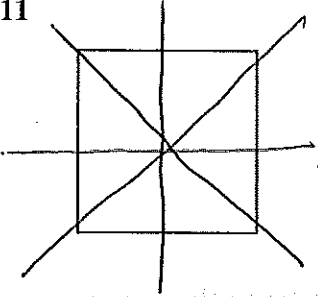
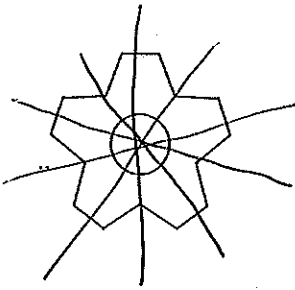
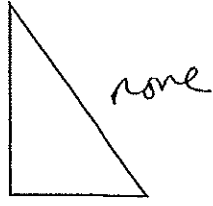
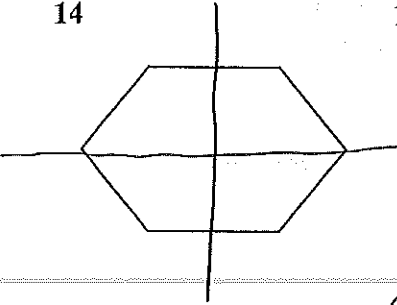
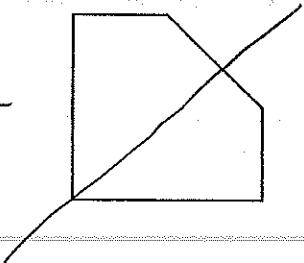
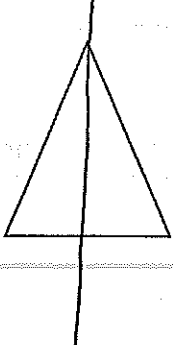
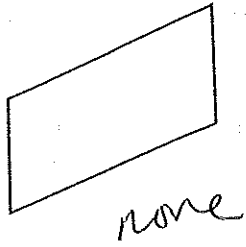
**Directions:** Look at the shape and the dotted line. Use patty paper or the folding technique to decide if the dotted line is a line of reflection and circle "yes" or "no".

1  <input checked="" type="radio"/> Yes <input type="radio"/> no	2  <input type="radio"/> Yes <input checked="" type="radio"/> no	3  <input type="radio"/> Yes <input checked="" type="radio"/> no	4  <input type="radio"/> Yes <input checked="" type="radio"/> no
5  <input checked="" type="radio"/> Yes <input type="radio"/> no	6  <input type="radio"/> Yes <input checked="" type="radio"/> no	7  <input type="radio"/> Yes <input checked="" type="radio"/> no	8  <input checked="" type="radio"/> Yes <input type="radio"/> no

9 Were you surprised by #6 and/or #7? Is there another way to draw lines of reflection for these shapes? Try to do it below and justify your answer by tracing or folding.

 none	
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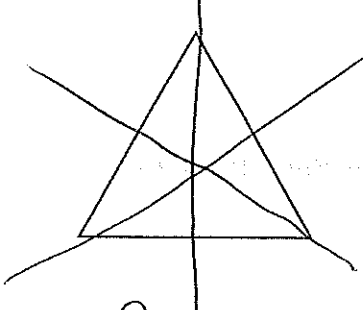
**Directions:** Decide if each figure below has line symmetry. If the figure does, draw all the possible lines of reflection. If it does not have line symmetry write "no line symmetry".

10 	11 	12 	13  none
14 	15 	16 	17  none

A **Regular Polygon** is a polygon that is both equilateral and equiangular. That is to say that all of its sides are the same length, and all of its angles are the same measure.

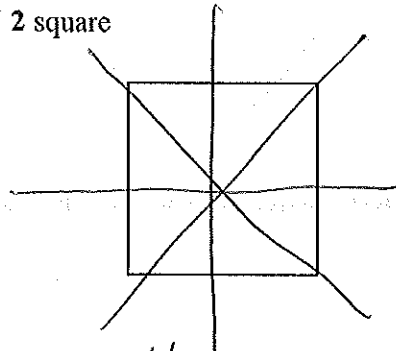
**Directions:** For each regular polygon, draw in all the possible lines of symmetry. Then, record the number of sides and lines of symmetry the figure has.

1 equilateral triangle



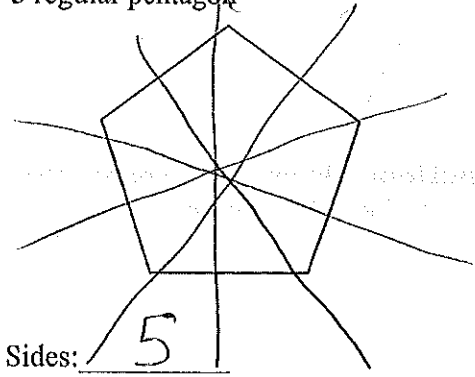
Sides: 3  
Lines of reflection: 3

2 square



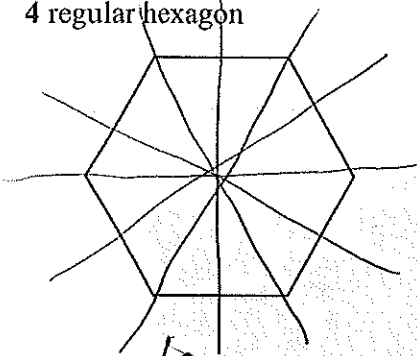
Sides: 4  
Lines of reflection: 4

3 regular pentagon



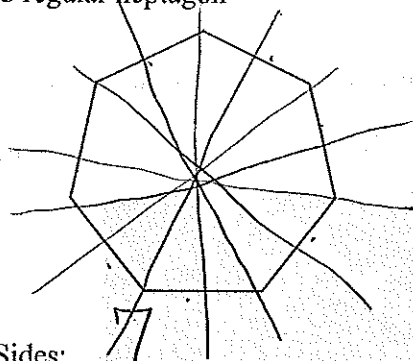
Sides: 5  
Lines of reflection: 5

4 regular hexagon



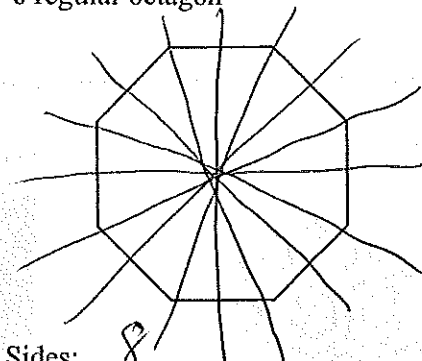
Sides: 6  
Lines of reflection: 6

5 regular heptagon



Sides: 7  
Lines of reflection: 7

6 regular octagon



Sides: 8  
Lines of reflection: 8

7 What is the relationship between the number of sides a regular polygon has and the maximum number of lines of reflection it can have?

same

only ↑

$$\# \text{ sides} = \# \text{ lines of symmetry}$$

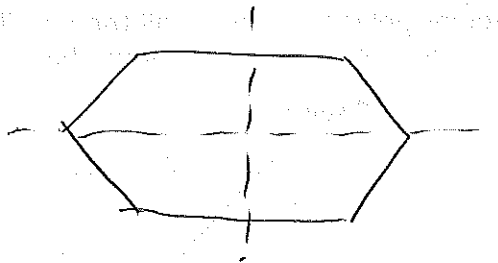
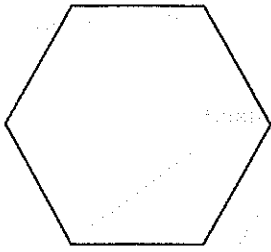
8 Use this idea to determine how many lines of reflection a regular decagon (ten sides) has without drawing one. Explain how you found your answer.

10

9 How many lines of reflection does a regular 38-gon have? Explain how you found your answer.

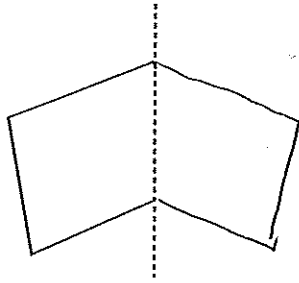
38

1 Look at the regular hexagon below. How can you alter the hexagon so that it only has two lines of reflection instead of 6? Draw the hexagon (you may need to make it irregular).

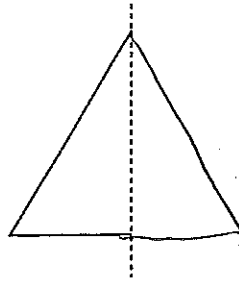


**Directions:** Below are 3 shapes that are half drawn and have a line of reflection. Draw the other half of each shape.

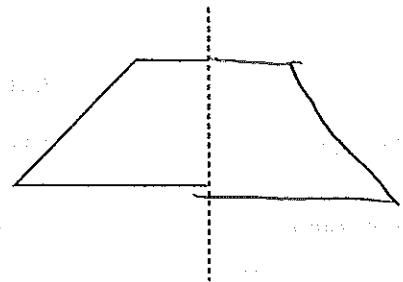
2



3

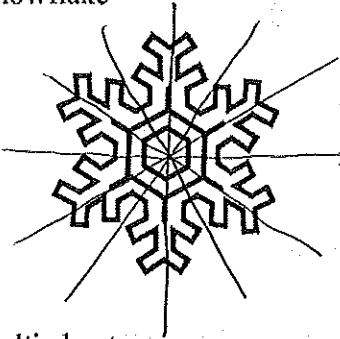


4

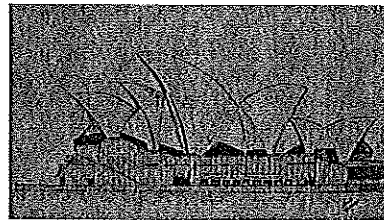


**Directions:** Which objects below have line symmetry? Draw in all possible lines of reflection for each.

5 snowflake

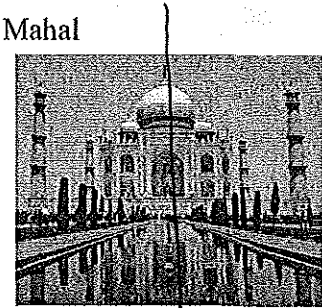


6 Sydney Opera House

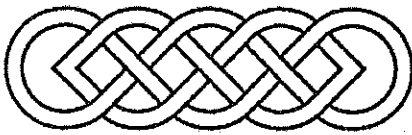


*no*

7 Taj Mahal



8 Celtic knot

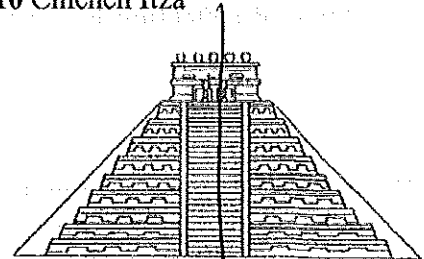


*not exactly*


9 Eiffel Tower



10 Chichen Itza



11 what are some objects around you that have line symmetry? List 3 of them below and describe their lines of reflection out loud to a partner (or to yourself if you are alone).

*ex. Vase* 

## 9-6 Dilations

A *dilation* is a **non-rigid** transformation in which a figure changes size. The preimage and image of a dilation are NOT congruent. The scale factor of the dilation is the same as the scale factor of these similar figures.

*not an isometry.*

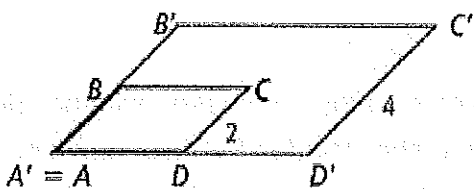
**Definition:** The scale factor of a dilation is the ratio of a length of the preimage to the corresponding length in the image, with the image length

always in the numerator.  $\left(\frac{\text{image}}{\text{pre-image}}\right)$  OR  $\text{preimage} \times \text{scale factor} = \text{image}$

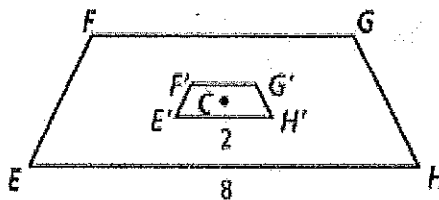
To find the scale factor, use the ratio of lengths of corresponding sides.

If the scale factor of a dilation is greater than 1, the dilation is an enlargement (expansion).

If it is less than 1, the dilation is a reduction (contraction).

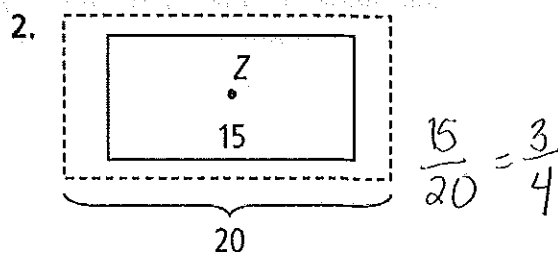
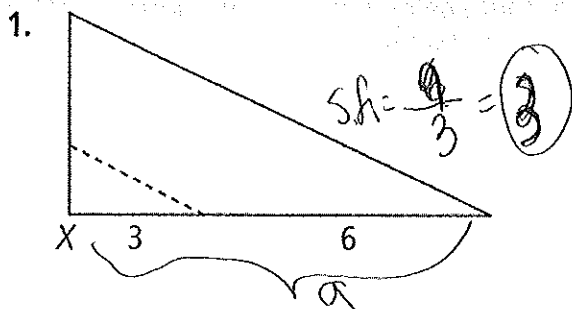


Enlargement  
center A, scale factor 2



Reduction  
center C, scale factor  $\frac{1}{4}$

The solid-line figure is a dilation of the dashed-line figure. The labeled point is the center of dilation. Find the scale factor for each dilation. Use whole numbers or decimals. Enter your responses on the grid provided.



3. The image of an eraser in a magnifying glass is three times the eraser's actual size and has a width of 14.4 cm. What is the actual width in cm?

$$3w = 14.4$$

4. A square on a transparency is 1.7 in. long. The square's image on the screen is 11.05 in. long. What is the scale factor of the dilation?

$$\frac{11.05}{1.7}$$

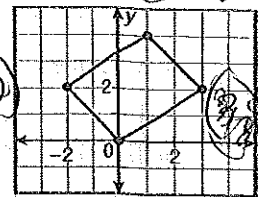
$$1.7(f) = 11.05$$

5. A dilation maps  $\triangle LMN$  to  $\triangle L'M'N'$ .  $MN = 14$  in. and  $M'N' = 9.8$  in.  
If  $LN = 13$  in., what is  $L'N'$ ?

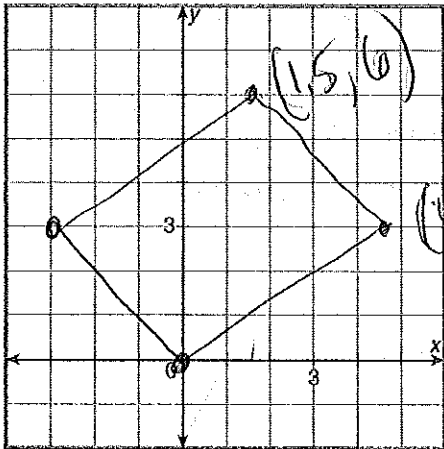
$$\frac{9.8}{14} =$$

Coord. Plane

6. A jeweler designs a setting that can hold a gem in the shape of a parallelogram. The figure shows the outline of the gem. The client, however, wants a gem and setting that is slightly larger.

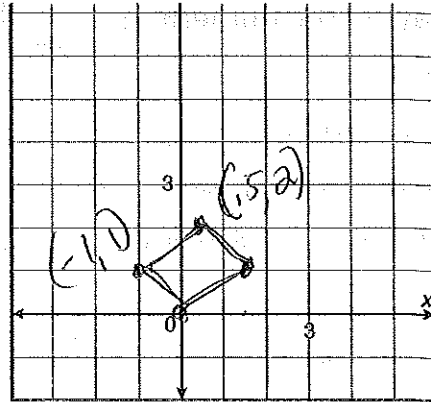


Every coordinate gets mult. by scale factor on coord plane



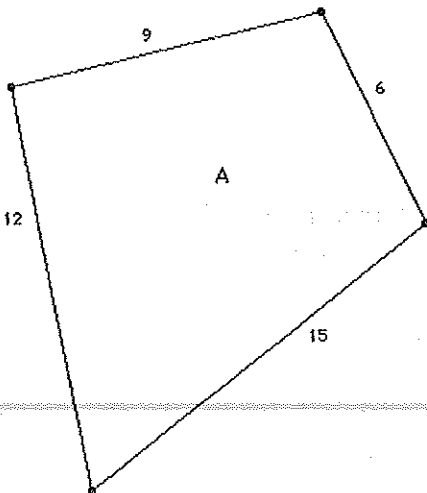
Draw the gem after a dilation with a scale factor of  $\frac{3}{2}$ .

$$(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right)$$



The client is so pleased with her ring that she decides to have matching but smaller earrings made using the same pattern. Draw the gem after a dilation from the original pattern with a scale factor of  $\frac{1}{2}$ .

7. Given the following two figures calculate the scale factor that scales figure A into figure B. Also, calculate the scale factor that scales figure B into figure A. What is the relationship you see between the two scale factors?



reciprocals

Scale Factor  $A \rightarrow B$   $\frac{3}{9} = \frac{1}{3}$  enlargement/reduction?

Scale Factor  $B \rightarrow A$   $\frac{9}{3} = 3$  enlargement/reduction?

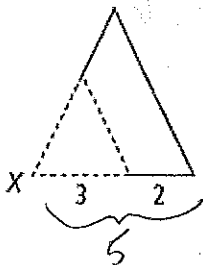
# 9-6 Practice

Form G

## Dilations

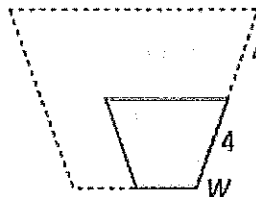
The solid-line figure is a dilation of the dashed-line figure. The labeled point is the center of dilation. Tell whether the dilation is an enlargement or a reduction. Then find the scale factor of the dilation.

1.



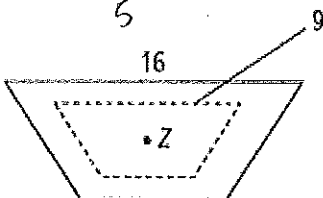
$\frac{5}{3} = 1\frac{2}{3}$   
enlargement

2.



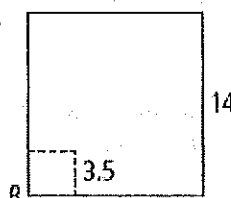
Reduction  
 $\frac{2}{4} = \frac{1}{2}$

3.



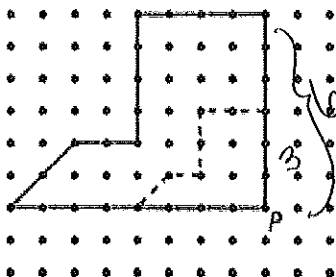
$\frac{16}{9} = 1\frac{7}{9}$   
enlarge.

4.



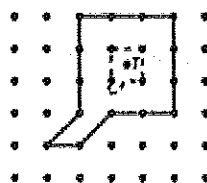
$\frac{14}{3.5} = 4$   
enlargement

5.



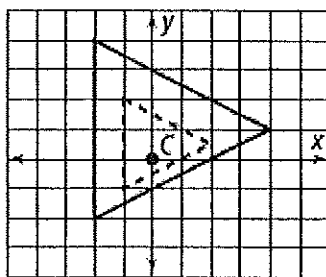
$\frac{6}{3} = 2$   
enlarge.

6.



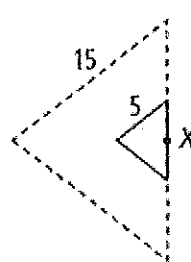
$\frac{3}{1} = 3$   
enlarge

7.



$\frac{6}{2} = 3$   
enlarge

8.



Reduction  
 $\frac{5}{15} = \frac{1}{3}$

You look at each object described in Exercises 9–11 under a magnifying glass. Find the actual dimension of each object.

9. The image of a ribbon is 10 times the ribbon's actual size and has a width of 1 cm.
10. The image of a caterpillar is three times the caterpillar's actual size and has a width of 4 in.
11. The image of a beetle is five times the beetle's actual size and has a length of 1.75 cm.
12.  $\Delta P'Q'R'$  is a dilation image of  $\Delta PQR$ . The scale factor for the dilation is 0.12. Is the dilation an enlargement or a reduction?

$\frac{1}{10}$  cm

$\frac{4}{3} = 1\frac{1}{3}$  in

$\frac{1.75}{5} = .35$  cm

reduction

# 9-6 Practice (continued)

Form G

## Dilations

A dilation has center  $(0, 0)$ . Find the image of each point for the given scale factor.

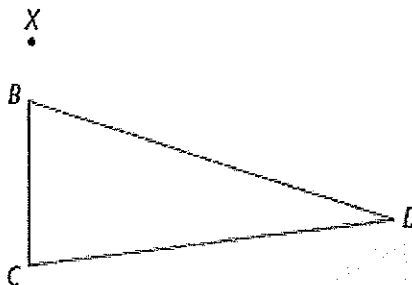
13.  $X(3, 4); D_7(X)$   $(21, 28)$
14.  $P(-3, 5); D_{1.2}(P)$   $(-3.6, 6)$
15.  $Q(0, 4); D_{3.4}(Q)$   $(0, 13.6)$
16.  $T(-2, -1); D_4(T)$   $(-8, -4)$
17.  $S(5, -6); D_{\frac{2}{3}}(S)$   $(\frac{10}{3}, -\frac{12}{3}) = (8\frac{1}{3}, -4)$
18.  $M(2, 2); D_5(M)$   $(10, 10)$

19. A square has 16-cm sides. Describe its image for a dilation with center at one of the vertices and scale factor 0.8.

20. Graph pentagon  $ABCDE$  and its image  $A'B'C'D'E'$  for a dilation with center  $(0, 0)$  and a scale factor of 1.5. The vertices of  $ABCDE$  are:  $A(0, 3)$ ,  $B(3, 3)$ ,  $C(3, 0)$ ,  $D(0, -3)$ ,  $E(-1, 0)$ .

Copy  $\triangle ABCD$  and point  $X$  for each of Exercises 21–23. Draw the dilation image  $\triangle B'C'D'$ .

21.  $D_{(1.5, X)}(\triangle ABCD)$



22.  $D_{(1.5, B)}(\triangle ABCD)$

23.  $D_{(0.8, C)}(\triangle ABCD)$



Vocabulary; define each of the following;

- Pre-image - original figure
- Image - figure after transformation
- Isometry - transformation whose preimage  $\cong$  image (Ref., Rot., Transl.)
- Reflection - flip across mirror line
- Rotation - turn about a point
- Translation - slide
- Dilation - changes size
- Transformation - movement of a figure
- Scale factor - ratio of side lengths of preimage; image in a dilation
- Reflection line - line of symmetry
- Regular polygon - all sides  $\cong$ , all  $\angle$ 's  $\cong$
- Line symmetry - figure can be folded upon itself; match up
- Rotational symmetry - figure can be rotated onto itself; match up.
- Order of rotation - # of times a figure can rotate onto itself.
- Degree of rotation - # of degrees each rotation is,  $\frac{360}{\text{order}}$

Tell what type of TRANSFORMATION is shown in each diagram;

1)

a.

Ref.

b.

transl.

c.

rotat.

d.

Rotat.

e.

Refle.

f.

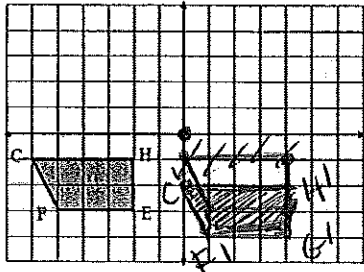
transl.

True or False?

- 2) The pre-image is congruent to the image for any transformation. False (dilation)
- 3) If triangle XYZ is reflected over the x-axis, then point Y and Y' are both the same distance from the x-axis. True
- 4) If triangle XYZ is rotated about the origin, then point Y and Y' are both the same distance from the origin. True
- 5) In a reflection, if each image point is connected to its pre-image point, then the reflection line is the perpendicular bisector of the segment formed. True

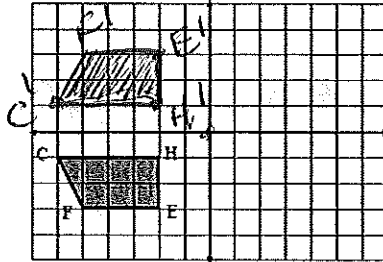
Perform each transformation. (you may use patty paper); Give the ordered pairs of the pre-image and the image; Then write the transformation RULE for #11-15.

10)  $(x, y) \rightarrow (x + 6, y - 1)$



- C(-6, -1)    C'(0, -2)  
 H(-2, -1)    H'(4, -2)  
 E(-2, -3)    E'(4, -4)  
 F(-5, -3)    F'(1, -4)

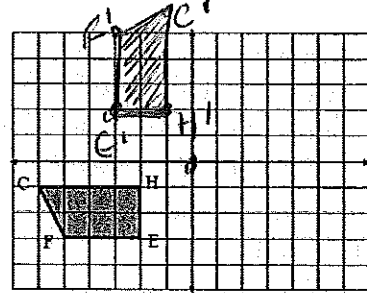
11) Reflect over the x-axis



- C'(-6, 1)  
 H'(-2, 1)  
 E'(-2, 3)  
 F'(-5, 3)

Rule?  $(x, y) \rightarrow (x, -y)$

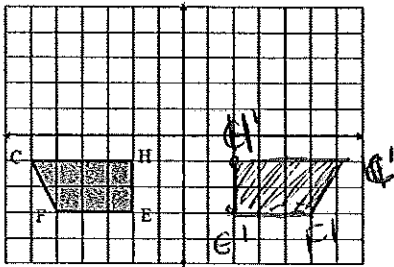
12) Rotate 90° clockwise about the origin



- C'(-1, 6)  
 H'(-1, 2)  
 E'(-3, 2)  
 F'(-3, 5)

Rule?  $(x, y) \rightarrow (y, -x)$

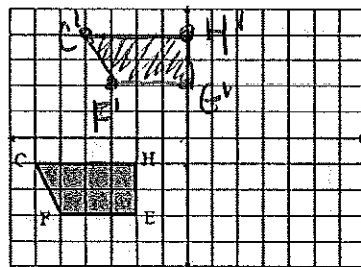
13)  $R_{y\text{-axis}}$



- C(-6, -1)    C'(6, -1)  
 H(-2, -1)    H'(2, -1)  
 E(-2, -3)    E'(2, -3)  
 F(-5, -3)    F'(5, -3)

Rule?  $(x, y) \rightarrow (-x, y)$

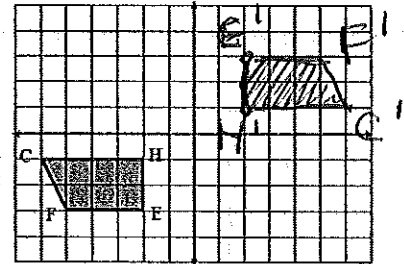
14)  $T_{\langle 2, 5 \rangle}$



- C'(-4, 4)  
 H'(0, 4)  
 E'(0, 2)  
 F'(-3, 2)

Rule?  $(x, y) \rightarrow (x+2, y+5)$

15)  $R_{180^\circ}$  clockwise



- C'(6, 1)  
 H'(2, 1)  
 E'(2, 3)  
 F'(5, 3)

Rule?  $(x, y) \rightarrow (-x, -y)$

Decide if each figure below has rotational symmetry, then, if YES, give the order and degree of rotation.

25) equilateral triangle



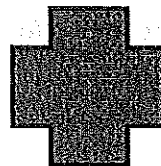
Circle YES or NO  
 Order 3  
 Degree 120°

26)



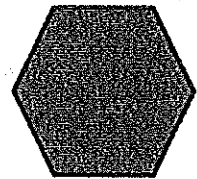
Circle YES or NO  
 Order 1  
 Degree 360°

27)



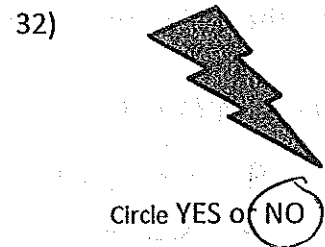
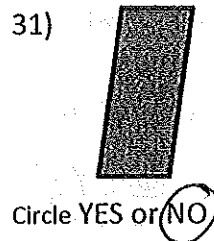
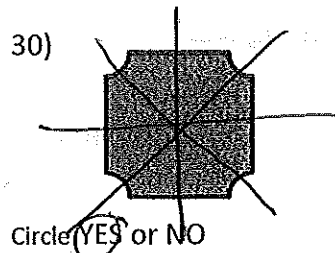
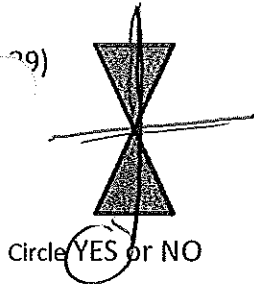
Circle YES or NO  
 Order 4  
 Degree 90°

28) regular hexagon



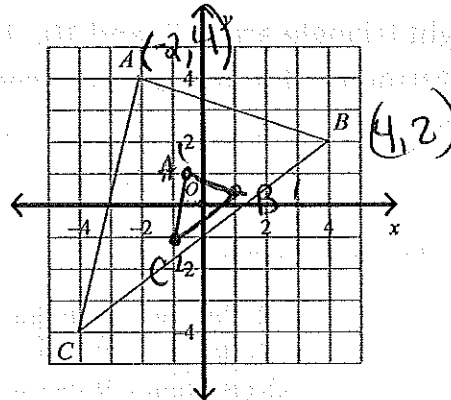
Circle YES or NO  
 Order 6  
 Degree 60°

Decide if each figure below has reflectional symmetry, then, if YES, draw all lines of symmetry.



33) Dilate  $\triangle ABC$  by a scale factor of  $\frac{1}{4}$ .

$A'(-\frac{1}{2}, 1)$   
 $B'(1, \frac{1}{2})$   
 $C'(-\frac{1}{2}, -1)$

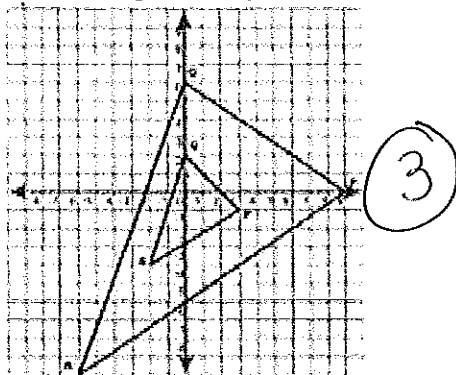


34) Given the scale factor, tell whether each dilation is an enlargement or a reduction;

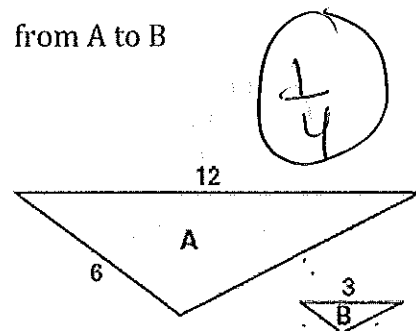
- a) Scale factor = 5 enlargement
- b) Scale factor =  $\frac{1}{2}$  reduction
- c) Scale factor = -2 (neither)
- d) Scale factor =  $\frac{7}{2}$  enlargement
- e) Scale factor = 2.3 enlargement

35) What is the scale factor of the dilations shown below?

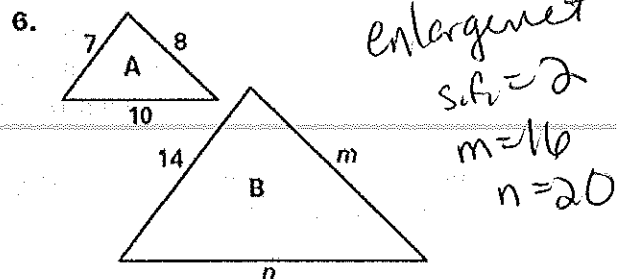
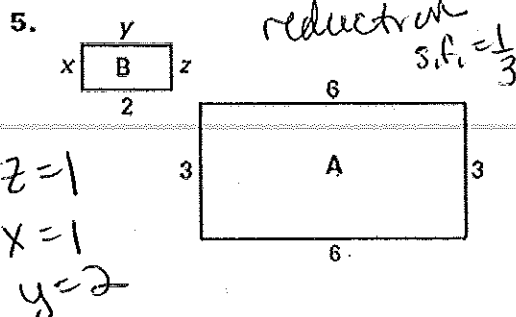
a) small is pre-image



b) from A to B



Determine whether the dilation from Figure A to Figure B is a *reduction* or an *enlargement*. Then, find the values of the variables.



Geometry 21 supplement

36) Given the point and its image, determine the scale factor.

a)  $A(3,6) \rightarrow A'(4.5, 9)$

$\frac{9}{6} = \frac{3}{2}$  or 1.5

b)  $G'(3,6) \rightarrow G(1.5,3)$

$\frac{6}{3} = 2$

c)  $B(2,5) \rightarrow B'(1,2.5)$

$\frac{1}{2}$

37) The sides of one right triangle are 6, 8, and 10. The sides of another right triangle are 10, 24, and 26. Determine if the triangles the second one is a dilation of the first. If so, what is the scale factor?

$\frac{6}{10} = \frac{3}{5}$     $\frac{8}{24} = \frac{1}{3}$     $\frac{10}{26} = \frac{5}{13}$

NOT a dilation

38) Circle the transformation that matches the rule.

a) $G(x,y) \rightarrow (-y,x)$	Reflection	<u>Rotation</u>	Translation	Dilation
b) $H(x,y) \rightarrow (x+2,y+10)$	Reflection	Rotation	<u>Translation</u>	Dilation
c) $F(x,y) \rightarrow (-x,y)$	<u>Reflection</u>	Rotation	Translation	Dilation
d) $N(x,y) \rightarrow (3x,3y)$	Reflection	Rotation	Translation	<u>Dilation</u>
e) $W(x,y) \rightarrow (-x,-y)$	Reflection	<u>Rotation</u>	Translation	Dilation
f) $Z(x,y) \rightarrow (y,x)$	Reflection	<u>Rotation</u>	Translation	Dilation

39) Find the treasure.....

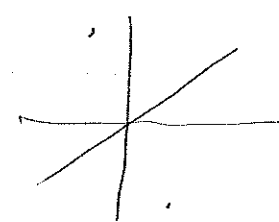
-- You start at the  $(-3,5)$ . The treasure map has the following instructions.

- (1) Translate  $(x,y) \rightarrow (x-6, y-3)$       (2) Reflect over the x axis

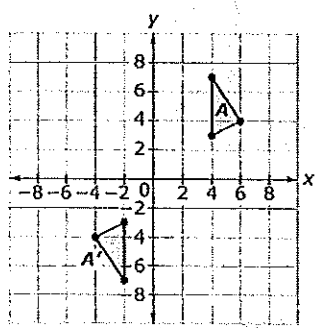
$(-9, 2) \rightarrow (-9, -2)$

- (3) Rotate it  $90^\circ$  about the origin *clockwise*      (4) Reflect it over the  $y=x$  line

$(-2, 9)$   
Where is the treasure?  $(2, -9)$



40) Mia and Brittany are studying geometric transformations.



Mia is able to move triangle A to triangle A' using the following sequence of basic transformations:

1. Reflection across the x-axis
2. Reflection across the y-axis
3. Translation two units to the right

Brittany claims that the same three transformations, done in any order, will always produce the same result. Explain why Brittany's claim is incorrect.

if translate first, then the reflections will end up in diff. place

Please apply the given rules to each set of pre-image ordered pairs and write the new coordinates for each image. Then fill what type of transformation it is.

<u>Rule</u>	<u>Pre-image coordinates</u>	<u>New image coordinates</u>	<u>Describe this transformation;</u>
$(x, y) \rightarrow (-x, y)$	(3, -7)	(-3, -7)	Reflect over y-axis
	(10, 8)	(-10, 8)	
	(-2, -5)	(2, -5)	
$(x, y) \rightarrow (-x, -y)$	(6, -4)	(-6, 4)	Rotates 180°
	(1, 28)	(-1, -28)	
	(-12, -3)	(12, 3)	
$(x, y) \rightarrow (y, -x)$	(5, -2)	(-2, -5)	Rotates 90° clockwise
	(8, 1)	(1, -8)	
	(-9, -2)	(-2, 9)	
$(x, y) \rightarrow (x+2, y-4)$	(3, -7)	(5, -11)	Translates right 2 down 4
	(10, 8)	(12, 4)	
	(-2, -5)	(0, -9)	
$(x, y) \rightarrow (x, y+6)$	(6, -4)	(6, 2)	Translates up 6
	(1, 28)	(1, 34)	
	(-12, -3)	(-12, 3)	
$(x, y) \rightarrow (x-3, y)$	(5, -2)	(2, -2)	translates left 3
	(8, 1)	(5, 1)	
	(-9, -2)	(-12, -2)	
$(x, y) \rightarrow (3x, 3y)$	(3, -7)	(9, -21)	Dilation, Enlarges w/ scale factor 3
	(10, 8)	(30, 24)	
	(-2, -5)	(-6, -15)	
$(x, y) \rightarrow (.5x, .5y)$	(6, -4)	(3, -2)	Dilation Reduction w/ scale factor 1/2
	(2, 28)	(1, 14)	
	(-12, -10)	(-6, -5)	
$(x, y) \rightarrow (-(x+1), (y-4))$	(5, -2)	(-6, -6)	Transformation Sequence: • right 1, down 4 translation • Reflected over y-axis
	(8, 1)	(-9, -3)	
	(-9, -2)	(8, -6)	
$(x, y) \rightarrow (-(x-6), -(y+3))$	(3, -7)	(3, 4)	Transformation Sequence: • translated left 6, up 3 • Rotated 180°
	(10, 8)	(-4, -11)	
	(-2, -5)	(8, 2)	

## Transformations Cheat Sheet

Transformation	Preserves Size	Preserves Angle Measure	Rigid Motion
Reflection	Yes	Yes	Yes
Translation	Yes	Yes	Yes
Rotation	Yes	Yes	Yes
Dilation	No	Yes	No

A rigid motion must preserve both size and angle measures

### Reflections

A reflection is a flip. The size and the angle measure stay the same.

Reflection over the x-axis	<p>When you reflect a point across the x-axis the x-coordinate remains the same, but the y-coordinate is transformed into its opposite.</p> $P(x, y) \rightarrow P'(x, -y) \text{ or } r_{x\text{-axis}}(x, y) = (x, -y)$
Reflection in the y-axis	<p>When you reflect a point across the y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite.</p> $P(x, y) \rightarrow P'(-x, y) \text{ or } r_{y\text{-axis}}(x, y) = (-x, y)$

### Rotations

A rotation turns a figure through an angle about a fixed point called the center. The figure will rotate counterclockwise when we are given an angle that is positive.

Rotation of $90^\circ$ counter cw	$R_{90^\circ}(x, y) = (-y, x)$
Rotation of $90^\circ$ clockwise	$R_{-90^\circ}(x, y) = (y, -x)$
Rotation of $180^\circ$	$R_{180^\circ}(x, y) = (-x, -y)$