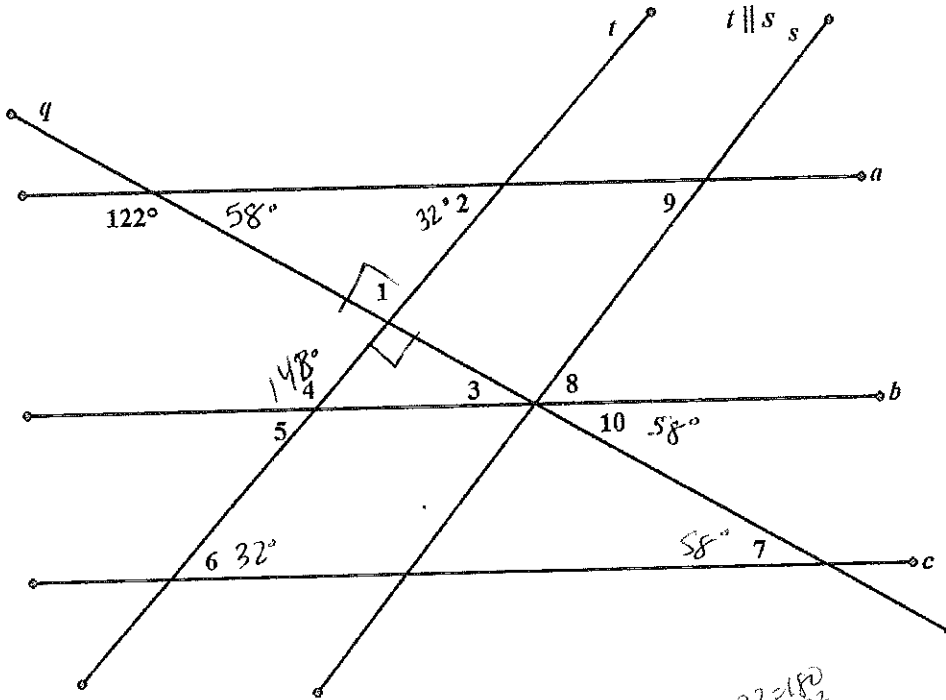


Remember to mark your diagrams or DRAW a diagram to help you solve when necessary.

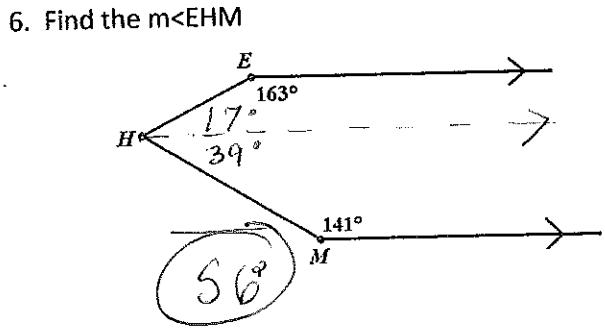
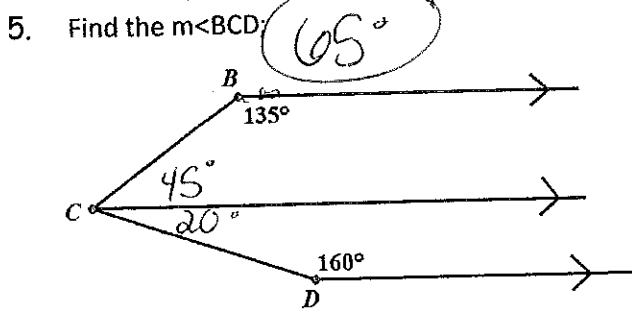
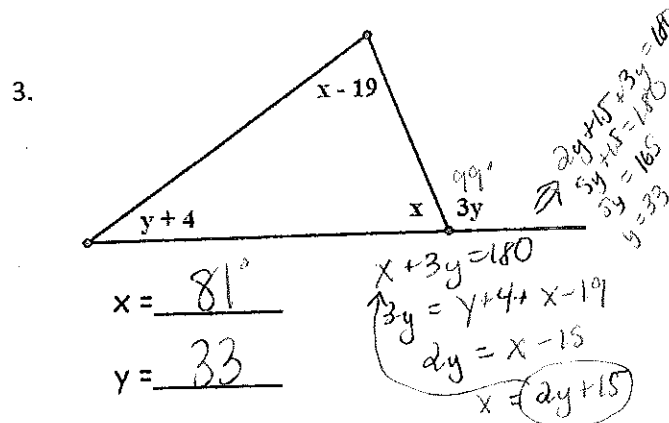
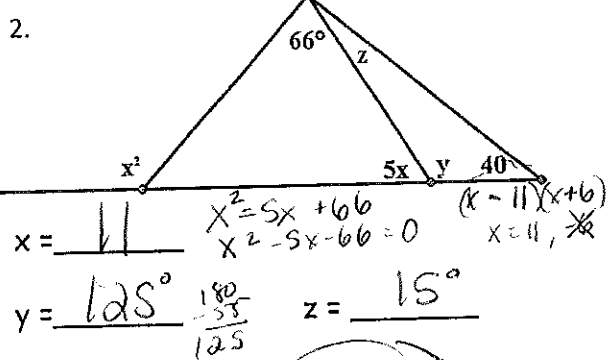
$a \parallel b \parallel c$

1. Find the measure of each numbered angle in the figure below.  $t \perp q$

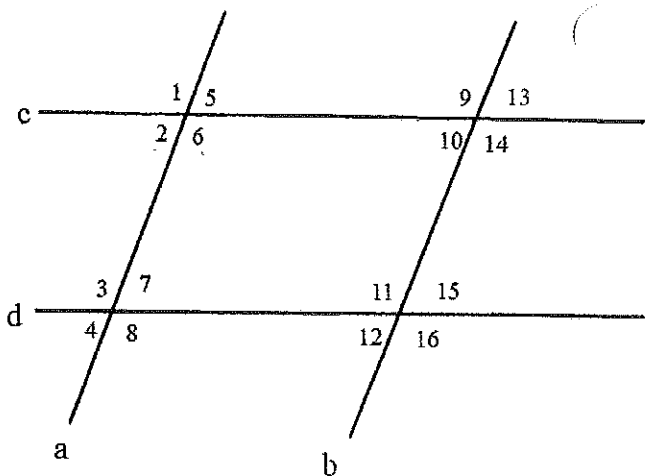


- $m\angle 1 = 90^\circ$       $m\angle 2 = 32^\circ$       $m\angle 3 = 58^\circ$       $m\angle 4 = 148^\circ$       $m\angle 5 = 32^\circ$   
 $m\angle 6 = 32^\circ$       $m\angle 7 = 58^\circ$       $m\angle 8 = 32^\circ$       $m\angle 9 = 32^\circ$       $m\angle 10 = 58^\circ$

Find the value of each variable:



Refer to the diagram at the right. Use the given information to determine which lines, if any, must be parallel. If any lines are parallel, use a theorem or postulate to tell why. If no lines are parallel write "none".



7.  $\angle 6$  is supplementary to  $\angle 2$ .

~~\_\_\_\_\_~~  $\parallel$  \_\_\_\_\_; thm: \_\_\_\_\_

8.  $\angle 7 \cong \angle 15$

$a \parallel b$ ; thm: converse of Corresp.  $\angle$  thm

9.  $\angle 14 \cong \angle 11$

$c \parallel d$ ; thm: conv. of alt. int.  $\angle$  thm

10.  $\angle 4 \cong \angle 7$

~~\_\_\_\_\_~~  $\parallel$  \_\_\_\_\_; thm: \_\_\_\_\_

11.  $\angle 1 \cong \angle 8$

$c \parallel d$ ; thm: conv. of alt. ext.  $\angle$  thm

12.  $\angle 14 \cong \angle 8$

~~\_\_\_\_\_~~  $\parallel$  \_\_\_\_\_; thm: \_\_\_\_\_

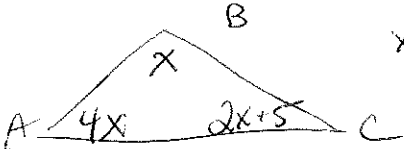
13.  $\angle 7$  is supplementary to  $\angle 11$ .

$a \parallel b$ ; thm: conv. of SSII  $\angle$  thm

14.  $\angle 15 \cong \angle 9$

~~\_\_\_\_\_~~  $\parallel$  \_\_\_\_\_; thm: \_\_\_\_\_

15. In a triangle,  $m\angle A$  is 4 times the  $m\angle B$ .  $m\angle C$  is 5 more than twice the  $m\angle B$ . Find the measure of each angle.



$$\begin{aligned}
 x + 4x + 2x + 5 &= 180 \\
 7x + 5 &= 180 \\
 7x &= 175 \\
 x &= 25 \\
 m\angle A &= 100^\circ \quad m\angle B = 25^\circ \quad m\angle C = 55^\circ
 \end{aligned}$$

PRACTICE solving systems of equations;

16)  $7x + 2y = -19$

$-x + 2y = 21$

$x = 2y - 21$

$7(2y - 21) + 2y = -19$

$14y - 147 + 2y = -19$

$16y = 128$

$y = 8$

17)

$x + 7y = 0$

$2x - 8y = 22$

$x = -7y$

$x = 7$

$2(-7y) - 8y = 22$

$-14y - 8y = 22$

$-22y = 22$

$y = -1$

18)  $4x - y = 20 \Rightarrow y = 4x - 20$

$-2x - 2y = 10$

$-2x - 2(4x - 20) = 10$

$-2x - 8x + 40 = 10$

$-10x = -30$

$x = 3$

PRACTICE solving for a variable by factoring; (hint: don't forget to divide by the GCF)

19)  $7v^2 - 42 = -35v$

$v^2 - 6 = -5v$

$v^2 + 5v - 6 = 0$

$(v - 1)(v + 6) = 0$

$v = 1, -6$

20)  $-4n^2 + 6n - 16 = -5n^2$

$n^2 + 6n - 16 = 0$

$(n - 2)(n + 8) = 0$

$n = 2, -8$

21)  $10n^2 - 35 = 65n$

$2n^2 - 7 = 13n$

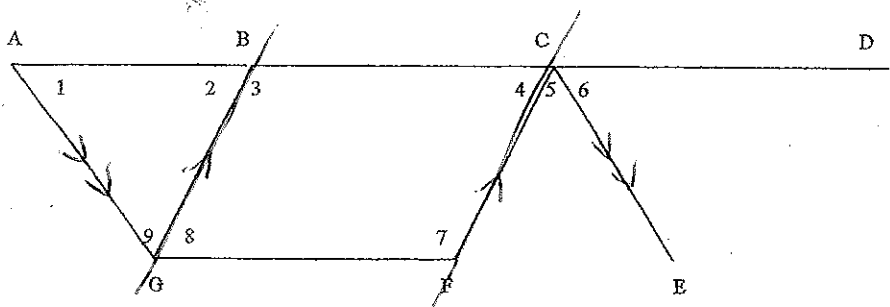
$2n^2 - 13n - 7 = 0$

$(n - 7)(2n + 1) = 0$

$n = 7, n = -1/2$

Worksheet on Parallel Lines

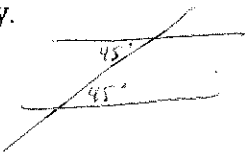
Use the figures on the right. True or False. Given  $\overline{BG} \parallel \overline{CF}$  and  $\overline{AG} \parallel \overline{CE}$



- 1.  $\angle 1 \cong \angle 5$  F
- 2.  $\angle 8 \cong \angle 2$  F never told  $\overline{AB} \parallel \overline{GF}$
- 3.  $\angle 3 \cong \angle FCD$  T corresp.
- 4.  $m\angle 2 + m\angle 4 = 180^\circ$  F corresp. so  $\angle 2 \cong \angle 4$
- 5.  $\angle 2 \cong \angle 4$  T corresp.
- 6.  $\angle 9 \cong \angle 3$  F not ||
- 7.  $\angle 1$  and  $\angle BCE$  are supplementary T SSI  $\angle$ 's
- 8.  $\angle 8$  and  $\angle 3$  are supplementary F never told  $\overline{AB} \parallel \overline{GF}$
- 9.  $\angle 7 \cong \angle 5$  F not ||
- 10.  $\angle 4 \cong \angle 6$  F
- 11.  $\angle 1 \cong \angle 6$  T corresp.  $\angle$ 's

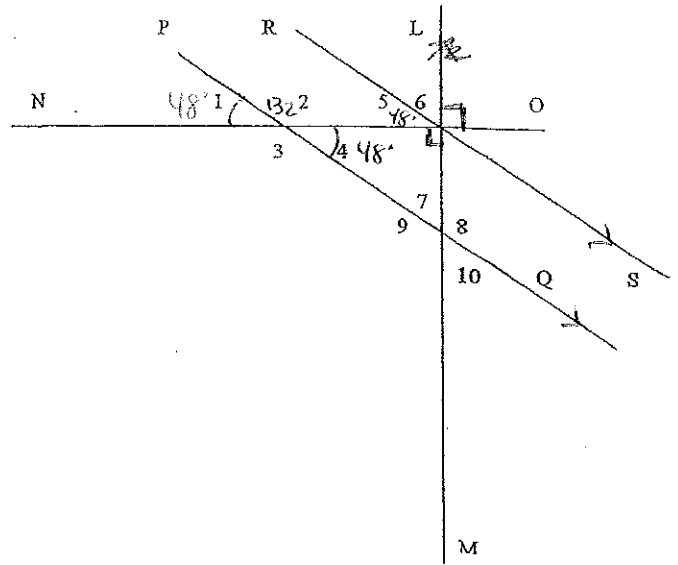
Always, Sometimes, Never

- 12. Alternate Interior Angles formed by two parallel lines cut by a transversal are always congruent.
- 13. Corresponding Angles formed by two parallel lines cut by a transversal are ~~never~~ sometimes supplementary. (if  $\perp$ )
- 14. Alternate Interior Angles are sometimes congruent.
- 15. Same Side Interior Angles formed by two parallel lines cut by a transversal are sometimes congruent. (if  $\perp$ )
- 16. Alternate Interior Angles formed by two parallel lines cut by a transversal are sometimes complementary.

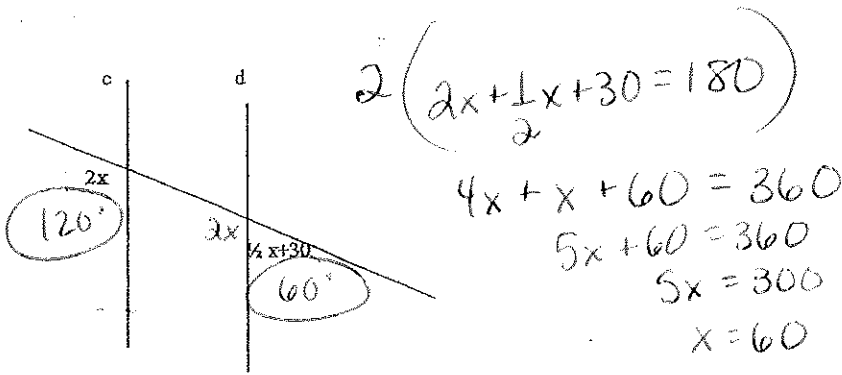


Use the information to find the measure of the labeled angles.  $\overline{PQ} \parallel \overline{RS}$ ,  $\overline{LM} \perp \overline{NO}$ ,  $m\angle 1 = 48$

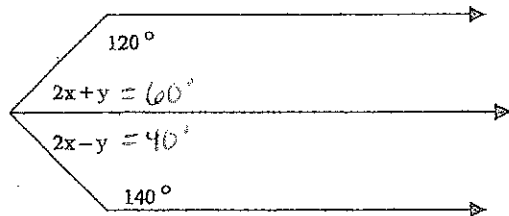
- 29.  $m\angle 2 = 132^\circ$
- 30.  $m\angle 3 = 132^\circ$
- 31.  $m\angle 4 = 48^\circ$
- 32.  $m\angle 5 = 48^\circ$
- 33.  $m\angle 6 = 42^\circ$
- 34.  $m\angle 7 = 42^\circ$
- 35.  $m\angle 8 = 138^\circ$
- 36.  $m\angle 9 = 138^\circ$
- 37.  $m\angle 10 = 42^\circ$



38. Use the diagram  $c \parallel d$ . Find  $m\angle 2$



39. Find the values of  $x$  and  $y$  in the diagram. Horizontal lines are parallel to each other.

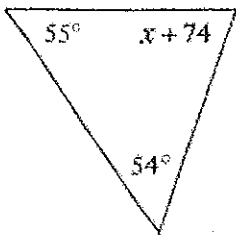


$$\begin{array}{r}
 2x + y = 60 \\
 + \quad 2x - y = 40 \\
 \hline
 4x = 100 \\
 \hline
 x = 25
 \end{array}$$

$$\begin{array}{r}
 2(25) + y = 60 \\
 50 + y = 60 \\
 \hline
 y = 10
 \end{array}$$

Solve for x:

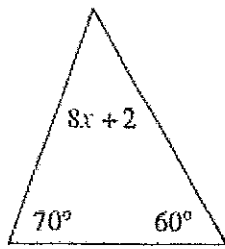
13.



$$55 + 54 + x + 74 = 180$$

$$x = -3$$

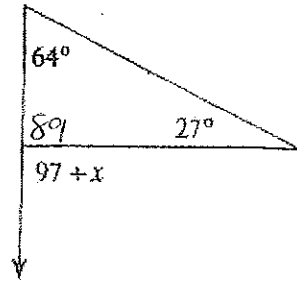
14.



$$8x + 2 + 70 + 60 = 180$$

$$x = 6$$

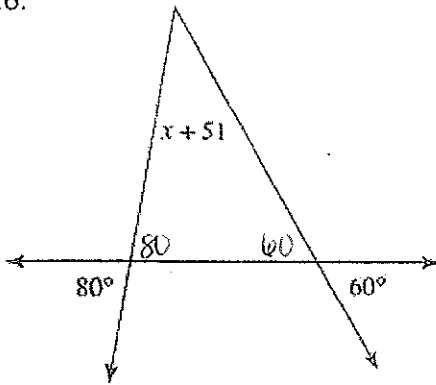
15.



$$89 + 97 + x = 180$$

$$x = -6$$

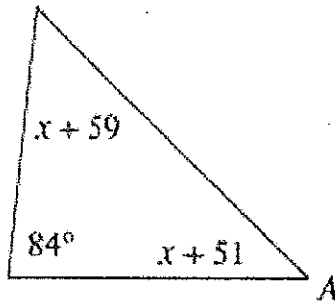
16.



$$140 + x + 51 = 180$$

$$x = -11$$

17.



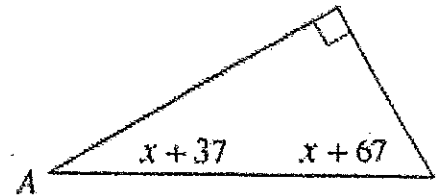
$$2x + 84 + 110 = 180$$

$$2x + 194 = 180$$

$$2x = -14$$

$$x = -7$$

18.



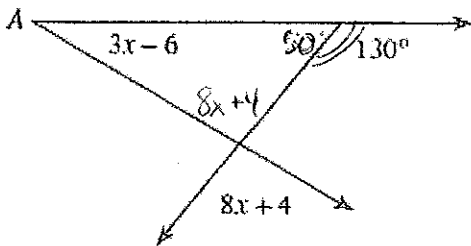
$$x + 37 + x + 67 + 90 = 180$$

$$2x + 104 = 90$$

$$2x = -14$$

$$x = -7$$

19.



~~$$3x - 6 + 8x + 4 + 50 = 180$$~~

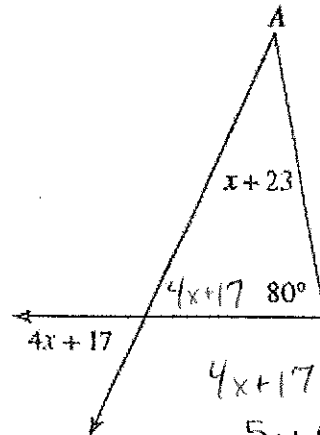
$$3x - 6 + 8x + 4 = 130$$

$$11x - 2 = 130$$

$$11x = 132$$

$$x = 12$$

20.



$$4x + 17 + x + 23 + 80 = 180$$

$$5x + 40 = 100$$

$$5x = 60$$

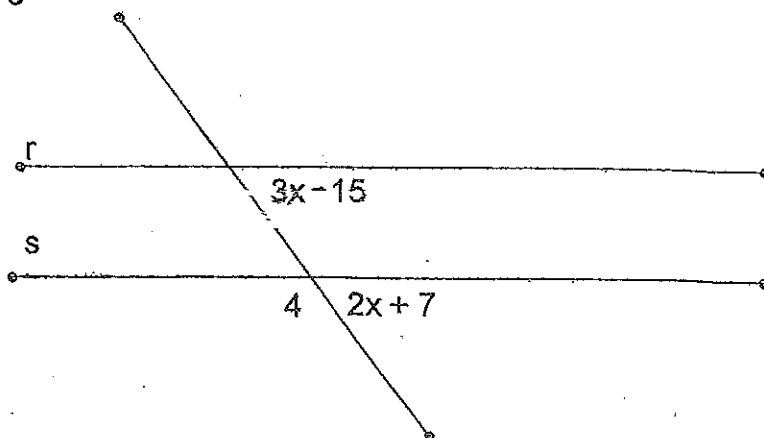
$$x = 12$$

For each of the following problems, use the given information to find the measure of each angle indicated.

1)  $r \parallel s$ ;  $m\angle 4 = ?$

$$3x - 15 = 2x + 7$$

$$x = 22$$



2)  $n \parallel p$ ;  $m\angle 5 = ?$

$$2x + y + x + 2y = 180 \Rightarrow 3x + 3y = 180$$

$$4x = 2x + y$$

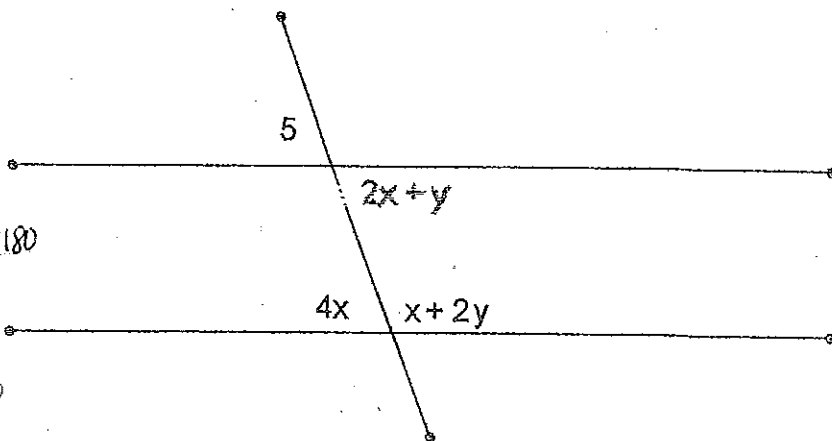
$$2x = y$$

$$x + y = 60$$

$$x + 2x = 60$$

$$3x = 60$$

$$x = 20$$



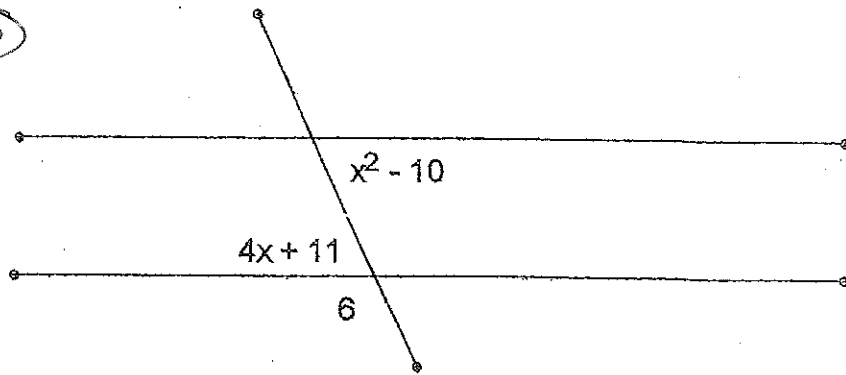
3)  $s \parallel t$ ;  $m\angle 6 = ?$

$$x^2 - 10 = 4x + 11$$

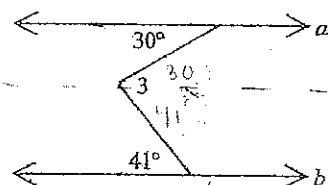
$$x^2 - 4x - 21 = 0$$

$$(x + 3)(x - 7) = 0$$

$$x = -3 \text{ or } 7$$



4) Given that  $a \parallel b$ , find the measure of  $\angle 3$ .

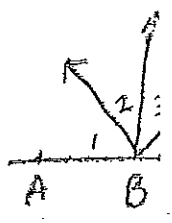


$$71^\circ$$

1)

Given:  $\overline{AC} \perp \overline{BD}$   
 $\angle 1 \cong \angle 4$

Prove:  $\angle 2 \cong \angle 3$

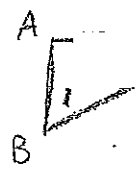


S	R
1) $\overline{AC} \perp \overline{BD}$	1) Given
2) $\angle ABD$ is a rt. $\angle$	2) Defn. $\perp$
3) $\angle CBD$ is a rt. $\angle$	3) <del>Defn.</del> Rt. $\angle$ & Thm
4) $\angle ABD \cong \angle CBD$	4) Defn. $\cong$
5) $m\angle 1 + m\angle 2 = m\angle ABD$ $m\angle 3 + m\angle 4 = m\angle CBD$	5) Angle Add. Post.
6) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	6) Substitution
7) $\angle 1 \cong \angle 4$	7) Given
8) $m\angle 1 = m\angle 4$	8) Defn. $\cong$
9) $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	9) Substitution
10) $m\angle 2 = m\angle 3$	10) Subtraction
12) $\angle 2 \cong \angle 3$	11) Defn. $\cong$

2)

Given:  $\overline{AB} \perp \overline{AE}$   
 $\overline{DE} \perp \overline{AE}$

Prove:  $\angle 1 \cong \angle 4$



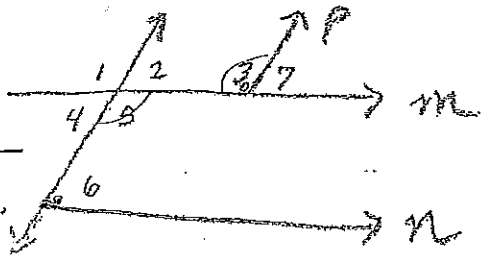
S	R
1) $\overline{AB} \perp \overline{AE}$ $\overline{DE} \perp \overline{AE}$	1) Given
2) $\overline{AB} \parallel \overline{DE}$	2) If 2 lines are $\perp$ to same line, then $\parallel$ to each other
3) $\angle 1 \cong \angle 4$	3) Alt. int. $\angle$ thm

3)

Given:  $\angle 6$  &  $\angle 3$  are supplem.

$l \parallel p$

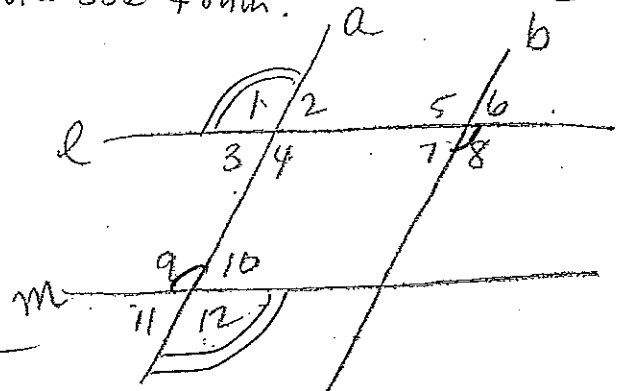
Prove:  $m \parallel n$



S:	R
1) $\angle 6$ & $\angle 3$ supplem	1) Given
2) $m\angle 6 + m\angle 3 = 180$	2) Defn. supplem.
3) $l \parallel p$	3) Given
4) $\angle 3 \cong \angle 5$	4) alt. int. $\angle$ thm
5) $m\angle 3 = m\angle 5$	5) Defn. $\cong$
6) $m\angle 6 + m\angle 5 = 180$	6) Substitution
7) $\angle 6$ & $\angle 5$ are supplem	7) Defn. supplem.
8) $m \parallel n$	8) Conv. of SSI $\angle$ thm.

Given:  $\angle 8 \cong \angle 9$   
 $\angle 1 \cong \angle 12$

Prove:  $a \parallel b$

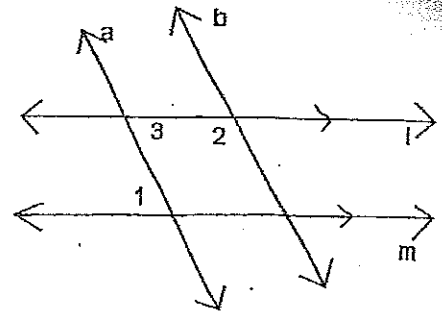


S	R
1) $\angle 8 \cong \angle 9$ ; $\angle 1 \cong \angle 12$	1) Given
2) $l \parallel m$	2) Conv. of alt. ext. $\angle$ thm
3) $\angle 12 \cong \angle 9$	3) Vert. $\angle$ Thm
4) $\angle 1 \cong \angle 8$	4) Transitive
5) $a \parallel b$	5) Conv. of alt. ext. $\angle$ thm

- ①  $\cong 12$
- ②  $\cong 9$
- ③  $\cong 8$
- ④  $\cong 8$

4. Given  $l \parallel m$ ;  $\angle 1$  and  $\angle 2$  are supplementary

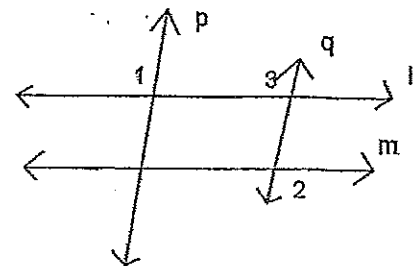
Prove:  $a \parallel b$



S	R
1) $l \parallel m$	1) Given
2) $\angle 1 \cong \angle 3$	2) alt. int. $\&$ thm
3) $m \angle 1 = m \angle 3$	3) defn. $\cong$
4) $\angle 1$ and $\angle 2$ are supplem.	4) given
5) $m \angle 1 + m \angle 2 = 180$	5) defn. $\cong$
6) $m \angle 3 + m \angle 2 = 180$	6) substitution
7) $\angle 3$ and $\angle 2$ are supple.	7) Defn. supple.
8) $a \parallel b$	8) CONV. of SSI $\&$ thm.

5. Given:  $p \parallel q$ ;  $\angle 1 \cong \angle 2$

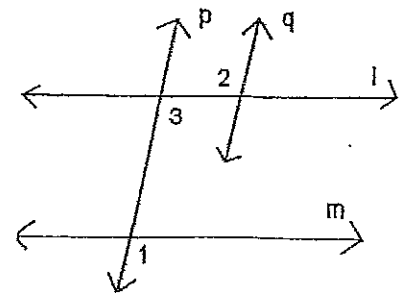
Prove:  $l \parallel m$



S	R
1) $p \parallel q$ ; $\angle 1 \cong \angle 2$	1) Given
2) $\angle 1 \cong \angle 3$	2) corresp. $\&$ thm
3) $\angle 2 \cong \angle 3$	3) Transitive
4) $l \parallel m$	4) conv. of alt. ext. $\&$ thm.

6. Given:  $l \parallel m$ ;  $p \parallel q$

Prove:  $\angle 1 \cong \angle 2$

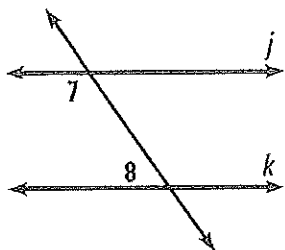


S	R
1) $l \parallel m$	1) given
2) $\angle 1 \cong \angle 3$	2) corresp. $\&$ thm
3) $p \parallel q$	3) given
4) $\angle 2 \cong \angle 3$	4) alt. int. $\&$ thm
5) $\angle 1 \cong \angle 2$	5) transitive



1) GIVEN  $\triangleright m\angle 7 = 125^\circ, m\angle 8 = 55^\circ$

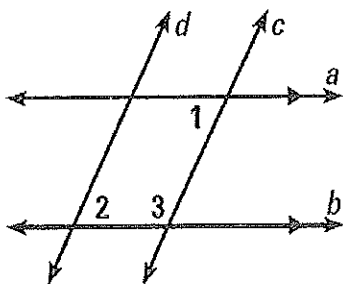
PROVE  $\triangleright j \parallel k$



S	R
1) $m\angle 7 = 125$ $m\angle 8 = 55$	1) Given
2) $m\angle 7 + m\angle 8 = 180$	2) simplify Add. POE
3) $\angle 7 \therefore \angle 8$ supplen	3) Defn. supp.
4) $j \parallel k$	4) Conv. of SSI & Thm

2) GIVEN  $\triangleright a \parallel b, \angle 1 \cong \angle 2$

PROVE  $\triangleright c \parallel d$

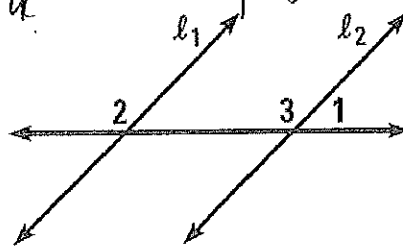


S	R
1) $a \parallel b$	1) Given
2) $\angle 1 \therefore \angle 3$ supplen.	2) SSI & Thm
3) $m\angle 1 + m\angle 3 = 180$	3) Defn. supp.
4) $\angle 1 \cong \angle 2$	4) given
5) $m\angle 1 = m\angle 2$	5) Defn. $\cong$
6) $m\angle 2 + m\angle 3 = 180$	6) Substitute
7) $\angle 2 \therefore \angle 3$ supplen.	7) Defn. supplen.
8) $c \parallel d$	8) Conv. of SSI & Thm

3) **PROOF** Complete the proof.

GIVEN  $\triangleright \angle 1$  and  $\angle 2$  are supplementary.

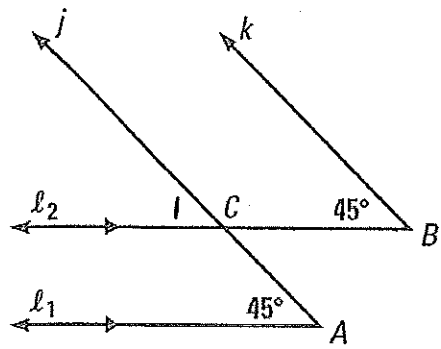
PROVE  $\triangleright l_1 \parallel l_2$



S	R
1) $\angle 1 \therefore \angle 2$ supplen.	1) given
2) $\angle 1 \therefore \angle 3$ lin. pr.	2) Defn. Lin. pr.
3) $\angle 1 \therefore \angle 3$ supplen.	3) Lin. pr. post.
4) $m\angle 2 \cong m\angle 3$	4) $\cong$ supplen. thm
5) $l_1 \parallel l_2$	5) Conv. of corresp. & thm

4) GIVEN  $\triangleright l_1 \parallel l_2, m\angle A = m\angle B = 45^\circ$

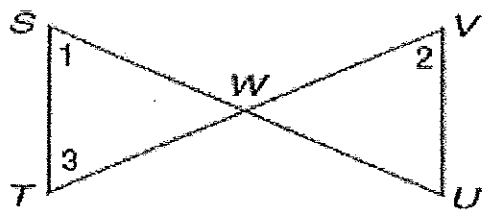
PROVE  $\triangleright j \parallel k$



S	R
1) $l_1 \parallel l_2; m\angle A = m\angle B = 45^\circ$	1) Given
2) $\angle 1 \cong \angle A$	2) corresp. $\angle$ thm
3) $m\angle 1 = m\angle A$	3) Defn $\cong$
4) $m\angle 1 = 45$	4) substitution
5) $m\angle 1 = m\angle B$	5) substitution
6) $\angle 1 \cong \angle B$	6) Defn $\cong$
7) $j \parallel k$	7) conv. of corresp. $\angle$ thm

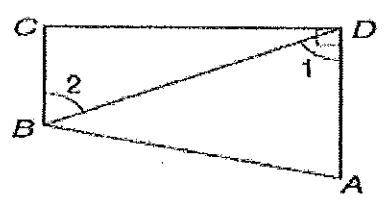
5) Given:  $\angle 2 \cong \angle 1$   
 $\angle 1 \cong \angle 3$

Prove:  $\overline{ST} \parallel \overline{UV}$



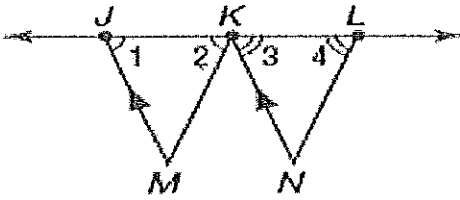
S	R
1) $\angle 2 \cong \angle 1$ $\angle 1 \cong \angle 3$	1) Given
2) $\angle 2 \cong \angle 3$	2) Transitive
3) $\overline{ST} \parallel \overline{UV}$	3) Conv. of Alt. Int. $\angle$ thm

6) Given:  $\overline{AD} \perp \overline{CD}$   
 $\angle 1 \cong \angle 2$   
 Prove:  $\overline{BC} \perp \overline{CD}$



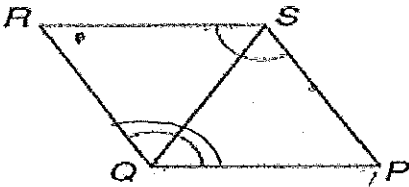
S	R
1) $\overline{AD} \perp \overline{CD}; \angle 1 \cong \angle 2$	1) Given
2) $\overline{AD} \parallel \overline{CB}$	2) Conv. of Alt. Int. $\angle$ thm
3) $\overline{CB} \perp \overline{CD}$	3) If a line is $\perp$ to one of 2 $\parallel$ lines, then $\perp$ to other line.

7) Given:  $\overline{JM} \parallel \overline{KN}$   
 $\angle 1 \cong \angle 2$   
 $\angle 3 \cong \angle 4$   
 Prove:  $\overline{KM} \parallel \overline{LN}$



S	R
1) $\overline{JM} \parallel \overline{KN}$ $\angle 1 \cong \angle 2$ $\angle 3 \cong \angle 4$	1) given
2) $\angle 1 \cong \angle 3$	2) Corresp. $\angle$ thm
3) $\angle 2 \cong \angle 4$	3) Transitive
4) $\overline{KM} \parallel \overline{LN}$	4) Conv. of corresp. $\angle$ thm.

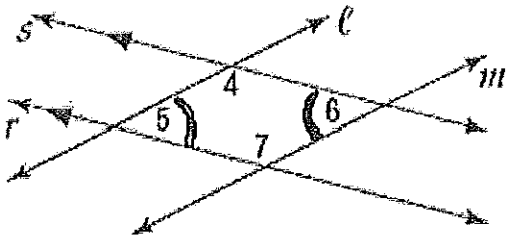
8) Given:  $\angle RSP \cong \angle PQR$   
 $\angle QRS$  and  $\angle PQR$  are supplementary.  
 Prove:  $\overline{PS} \parallel \overline{QR}$



S	R
1) $\angle RSP \cong \angle PQR$ $\angle QRS$ & $\angle PQR$ are supp.	1) Given
<del>2) <math>\overline{PS} \parallel \overline{QR}</math></del>	<del>2) Conv. SSI <math>\angle</math> thm</del>
2) $m\angle QRS + m\angle PQR = 180$	2) Defn. supp.
3) $m\angle RSP = m\angle PQR$	3) Defn. $\cong$
4) $m\angle QRS + m\angle RSP = 180$	4) Substitution
5) $\angle QRS$ & $\angle RSP$ supp	5) Defn. supp.
6) $\overline{PS} \parallel \overline{QR}$	6) Conv. SSI $\angle$ thm

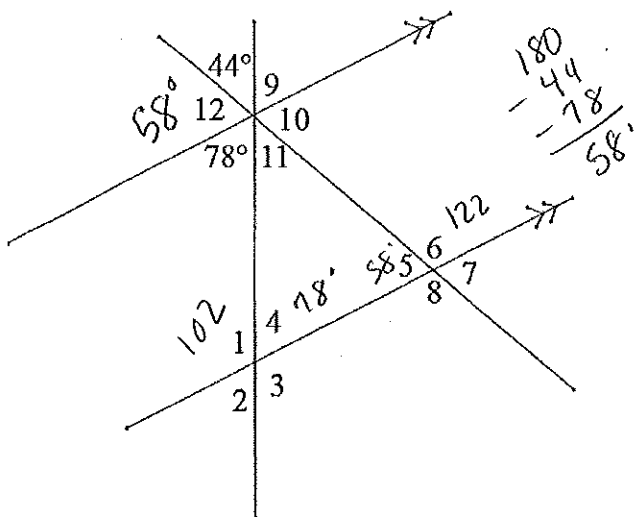
9) Given:  $r \parallel s$   
 $\angle 5 \cong \angle 6$

Prove:  $l \parallel m$



S	R
1) $r \parallel s$ ; $\angle 5 \cong \angle 6$	1) Given
2) $\angle 4 \cong \angle 5$ suppl.	2) SSI $\angle$ thm
3) $m\angle 4 + m\angle 5 = 180$	3) Defn. supp.
4) $m\angle 5 = m\angle 6$	4) Defn. $\cong$
5) $m\angle 4 + m\angle 6 = 180$	5) Substitution
6) $\angle 4$ & $\angle 6$ suppl.	6) Defn. supp.
7) $l \parallel m$	7) Conv. of SSI $\angle$ thm

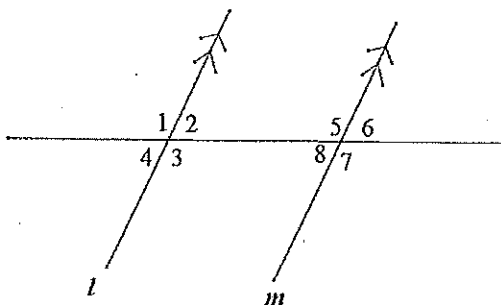
9. In the diagram below, find the missing angle measures.



$$\begin{aligned}
 m\angle 1 &= 102^\circ \\
 m\angle 2 &= 78^\circ \\
 m\angle 3 &= 102^\circ \\
 m\angle 4 &= 78^\circ \\
 m\angle 5 &= 58^\circ \\
 m\angle 6 &= 122^\circ
 \end{aligned}$$

$$\begin{aligned}
 m\angle 7 &= 58^\circ \\
 m\angle 8 &= 122^\circ \\
 m\angle 9 &= 78^\circ \\
 m\angle 10 &= 58^\circ \\
 m\angle 11 &= 44^\circ \\
 m\angle 12 &= 58^\circ
 \end{aligned}$$

10. Given that  $l \parallel m$ , find the value(s) of  $x$  and each angle. Be sure to check for extraneous solutions.



a.  $m\angle 3 = (x^2 + 112)^\circ$   
 $m\angle 8 = (16x + 131)^\circ$

$$\begin{aligned}
 x^2 + 112 + 16x + 131 &= 180 \\
 x^2 + 16x + 243 &= 180 \\
 x^2 + 16x + 63 &= 0 \\
 (x + 7)(x + 9) &= 0 \\
 x &= -7, -9
 \end{aligned}$$

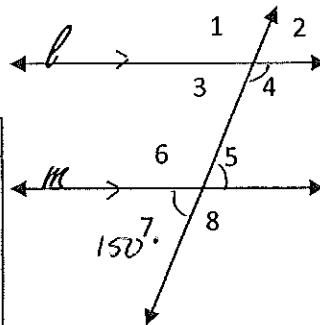
$19^\circ$   
 $161^\circ$

b.  $m\angle 1 = (x^2 - 7x)^\circ$   
 $m\angle 7 = (-x + 7)^\circ$

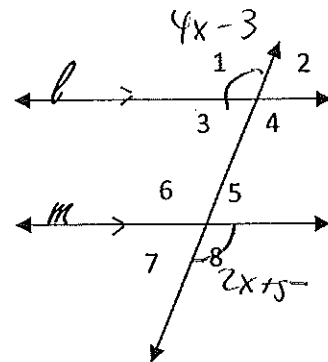
$$\begin{aligned}
 x^2 - 7x &= -x + 7 \\
 x^2 - 6x - 7 &= 0 \\
 (x + 1)(x - 7) &= 0 \\
 x &= -1, 7
 \end{aligned}$$

## Geometry 21: Practice With Parallel Lines Proofs

Complete the following proofs:

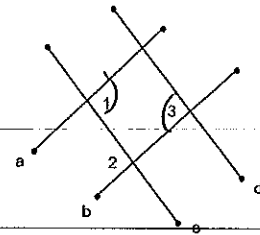
1. Given:  $l \parallel m$  ;  $m \angle 7 = 150^\circ$      Prove:  $m \angle 4 = 30$ 

Statements	Reasons
1) $l \parallel m$ ; $m \angle 7 = 150^\circ$	1) Given
2) $m \angle 7 \cong \angle 5$	2) Vert. $\angle$ 's Thm
3) $m \angle 7 = m \angle 5$	3) Defn. $\cong$
4) $\angle 4$ and $\angle 5$ supplm.	4) SSI $\angle$ 's Thm
5) $m \angle 4 + m \angle 5 = 180$	5) Defn. supplm.
6) $m \angle 4 + m \angle 7 = 180$	6) Substitution
7) $m \angle 4 + 150 = 180$	7) Substitution
8) $m \angle 4 = 30$	8) Subtraction

2. Given:  $l \parallel m$  ;  $m \angle 8 = (2x+5)^\circ$  ;  $m \angle 1 = (4x-3)^\circ$      Prove:  $x = 4$ 

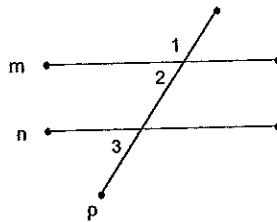
Statements	Reasons
1) $l \parallel m$ ; $m \angle 8 = 2x+5$ $m \angle 1 = 4x-3$	1) Given
2) $\angle 1 \cong \angle 8$	2) Alt-ext. $\angle$ 's Thm
3) $m \angle 1 = m \angle 8$	3) Defn. $\cong$
4) $4x-3 = 2x+5$	4) Substitution
5) $2x = 8 = 5$	5) Subtraction
6) $2x = 8$	6) Addition
7) $x = 4$	7) Division

3. Given:  $a \parallel b$ ,  $\angle 1 \cong \angle 3$   
 Prove:  $c \parallel d$



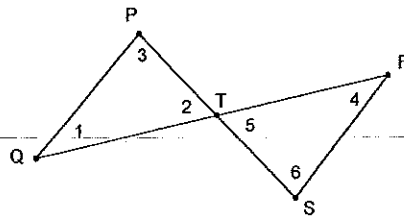
Statements	Reasons
1) $a \parallel b$ ; $\angle 1 \cong \angle 3$	1) Given
2) $\angle 1 \cong \angle 2$	2) Alt. int $\angle$ Thm
3) $\angle 2 \cong \angle 3$	3) Transitive
4) $c \parallel d$	4) Conv. of corresp $\angle$ thm

4. Given:  $\angle 1$  and  $\angle 3$  are supplementary  
 Prove:  $m \parallel n$



Statements	Reasons
1) $\angle 1$ ; $\angle 3$ supp	1) Given
2) $\angle 1$ ; $\angle 2$ supp	2) Lin. pr. post.
3) $\angle 2 \cong \angle 3$	3) $\cong$ supp. thm
4) $m \parallel n$	4) Conv. of corresp $\angle$ thm

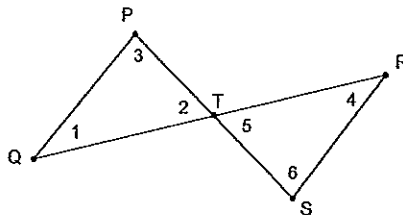
5. Given:  $\overline{PQ} \parallel \overline{RS}$   
 Prove:  $\angle 3 \cong \angle 6$



Statements	Reasons
1) $\overline{PQ} \parallel \overline{RS}$	1) Given
2) $\angle 3 \cong \angle 6$	2) alt. int. $\angle$ thm

2 steps

6. Given:  $\angle 1 \cong \angle 2$ ,  $\angle 5 \cong \angle 4$   
 Prove:  $\overline{PQ} \parallel \overline{RS}$



Statements	Reasons
1) $\angle 1 \cong \angle 2$ , $\angle$	1) Given
2) $\angle 2 \cong \angle 5$	2) Vert. $\angle$ thm
3) $\angle 5 \cong \angle 4$	3) Given
4) $\angle 1 \cong \angle 4$	4) Transitive
5) $\overline{PQ} \parallel \overline{RS}$	5) Conv. of alt. int $\angle$ thm

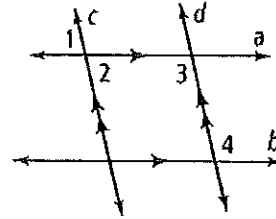
# 3-2 Practice (continued) Form K

## Properties of Parallel Lines

9. **Developing Proof** Supply the missing reasons in the two-column proof.

Given:  $a \parallel b, c \parallel d$

Prove:  $\angle 1$  and  $\angle 4$  are supplementary.



*Bad proof!*

Statements	Reasons
1) $\angle 1 \cong \angle 2$ $m\angle 1 = m\angle 2$	1) <u>Vert. <math>\angle</math> Thm</u>
2) $c \parallel d$	2) Given
3) $\angle 2$ and $\angle 3$ are supplementary. $m\angle 2 + m\angle 3 = 180$	3) <u>same side int. <math>\angle</math> thm</u>
4) $a \parallel b$	4) Given
5) $\angle 3 \cong \angle 4$ $m\angle 3 = m\angle 4$ $m\angle 1 + m\angle 4 = 180$ Subst.	5) <u>alt. int. <math>\angle</math> thm</u>
6) $\angle 1$ and $\angle 4$ are supplementary.	6) <u>Dedu. supp.</u>

**Algebra** Find the value of  $x$ . Then find the measure of each labeled angle.

10.  $2x + 24 + x = 180$   
 $3x = 156$   
 $x = 52$

11.  $3x + 6 = 4x - 12$   
 $18 = x$

**Algebra** Find the values of the variables.

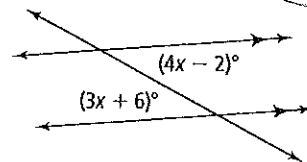
12.  $4x + 12 = 90$   
 $4x = 78$   
 $x = 19.5$

13.  $2x + 12 + 3x + 23 = 180$   
 $5x + 35 = 180$   
 $5x = 145$   
 $x = 29$   
 $y = 110$

14. **Error Analysis** Which solution for the value of  $x$  in the figure at the right is incorrect? Explain.

A.  $4x - 2 = 3x + 6$   
 $x = 8$

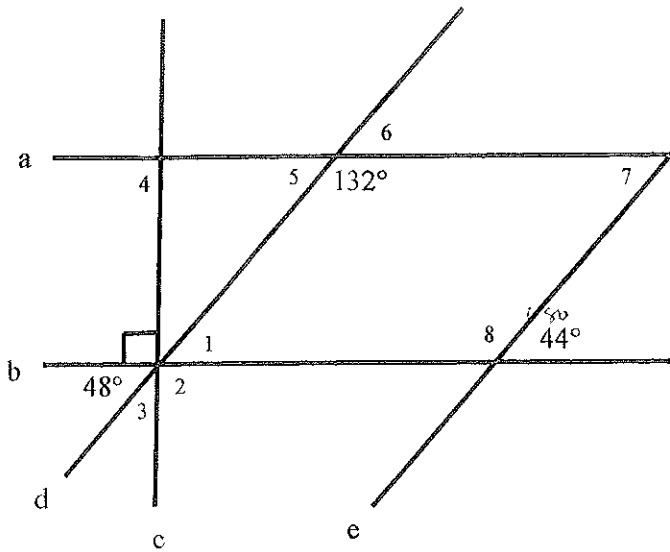
B.  $4x - 2 + 3x - 6 = 180$   
 $7x - 8 = 180$   
 $x = 24.6$



*is correct b/c alt. int  $\angle$  are  $\cong$  not supplm.*



1 Find the measures of each of the numbered angles in the figure below.

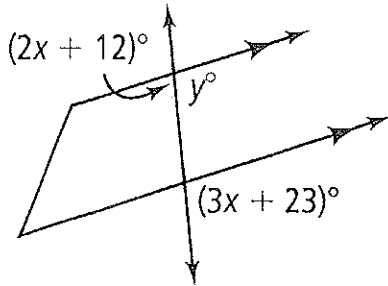


1 = 48° VAT 2 = 90° VAT  
 3 = 42° comp. 4 = 90  
 5 = 48° LP 6 = 48° VAT  
 7 = 44° AIA 8 = 136°

Is a || b? yes Is d || e? No

Find the value of the variables in each diagram. Show all work!

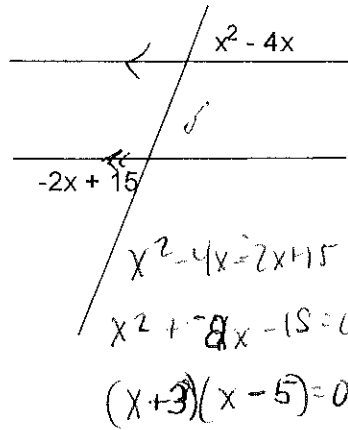
2.



$2x + 12 + y = 180$   $y = 3x + 23$   
 $2x + 12 + 3x + 23 = 180$   $y = 3(29) + 23$   
 $5x + 35 = 180$   $87 + 23$   
 $5x = 145$

2. x = 29 y = 110

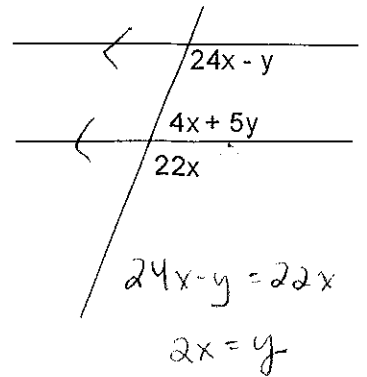
3.



$x^2 - 4x = -2x + 15$   
 $x^2 - 2x - 15 = 0$   
 $(x + 3)(x - 5) = 0$

3. x = -3, 5

4.

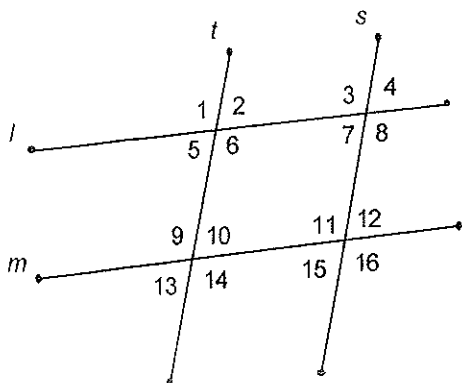


$24x - y = 22x$   
 $2x = y$   
 $4x + 5y + 22x = 180$   
 $26x + 5y = 180$   
 $36x = 180$   
 $x = 5$

4. x = 5 y = 10

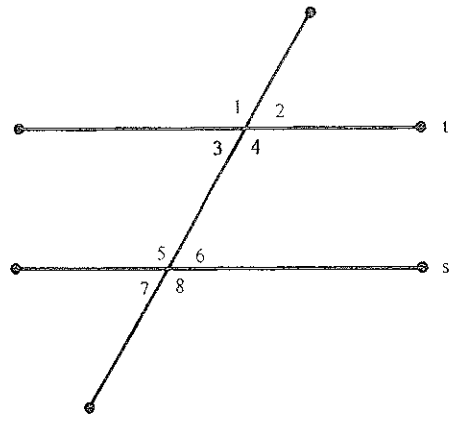
Classify each pair of angles as;

a) alternate interior b) alternate exterior c) same side interior d) corresponding or e) none of these x = 5



- 5) <1 & <9 d
- 6) <3 & <12 e
- 7) <4 & <15 b
- 8) <6 & <11 e
- 9) <8 & <14 e
- 10) <12 & <10 a
- 11) <7 & <12 a
- 12) <6 & <7 c
- 13) <13 & <4 e
- 14) <16 & <3 b
- 15) <5 & <4 b
- 16) <11 & <14 a

Write the 2 column proofs below;



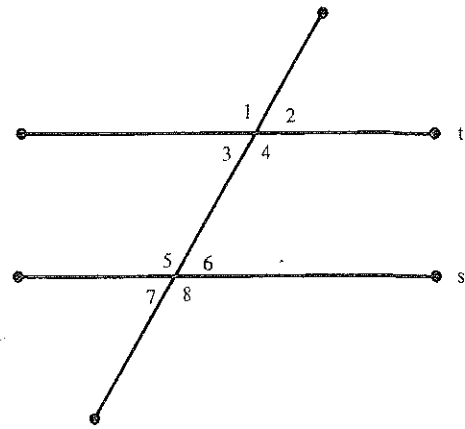
17. Given:  $t \parallel s$

Prove:  $m\angle 5 + m\angle 2 = 180^\circ$

Statements	Reasons
1) $t \parallel s$	1) Given
2) $\angle 5 \cong \angle 4$	2) Alt. int. $\angle$ Thm.
3) $\angle 2$ and $\angle 4$ are Supplem.	3) Lin. Pr. Post.
4) $m\angle 4 + m\angle 2 = 180$	4) Defn. Supplem.
5) $m\angle 5 = m\angle 4$	5) Defn. $\cong$
6) $m\angle 5 + m\angle 2 = 180$	6) Substitution

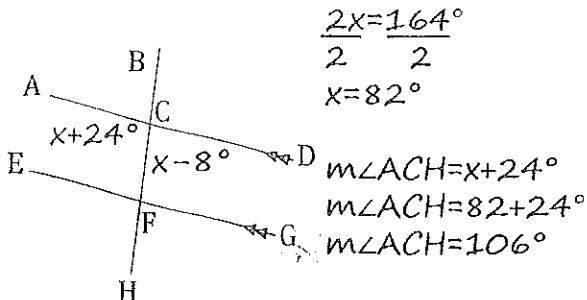
18. Given:  $t \parallel s$ ;  $m\angle 7 = 81^\circ$

Prove:  $m\angle 1 = 99^\circ$



S	R
1) $t \parallel s$	1) given
2) $\angle 1 \cong \angle 8$	2) alt. ext. $\angle$ Thm
3) $m\angle 1 = m\angle 8$	3) defn. $\cong$
4) $\angle 7$ & $\angle 8$ are supple.	4) Lin. Pr. Post.
5) $m\angle 7 + m\angle 8 = 180$	5) Defn. supplem.
6) $m\angle 7 + m\angle 1 = 180$	6) Substitution
7) $m\angle 7 = 81$	7) given
8) $81 + m\angle 1 = 180$	8) substitution
9) $m\angle 1 = 99$	9) Subtraction

37.  $m\angle ACH = X + 24^\circ$ ,  $m\angle DCH = X - 8^\circ$ . Find  $m\angle ACH$ .

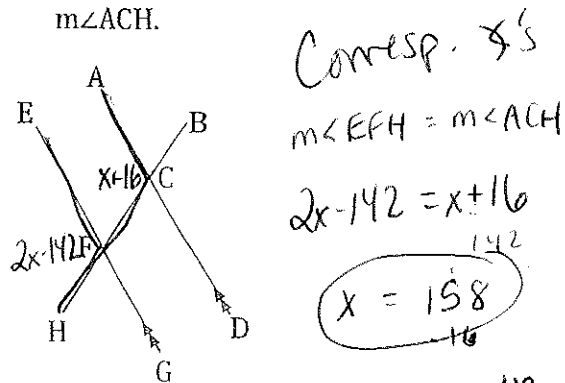


$$\begin{aligned} 2x &= 164^\circ \\ \frac{2x}{2} &= \frac{164^\circ}{2} \\ x &= 82^\circ \\ m\angle ACH &= x + 24^\circ \\ m\angle ACH &= 82 + 24^\circ \\ m\angle ACH &= 106^\circ \end{aligned}$$

Linear Pair

$$\begin{aligned} m\angle ACH + m\angle DCH &= 180^\circ \\ (x + 24^\circ) + (x - 8^\circ) &= 180^\circ \\ x + 24^\circ + x - 8^\circ &= 180^\circ \\ 2x + 16^\circ &= 180^\circ \\ -16^\circ \quad -16^\circ & \end{aligned}$$

39.  $m\angle EFH = 2X - 142^\circ$ ,  $m\angle ACH = X + 16^\circ$ . Find  $m\angle ACH$ .



Corresp.  $\angle$ 's  
 $m\angle EFH = m\angle ACH$

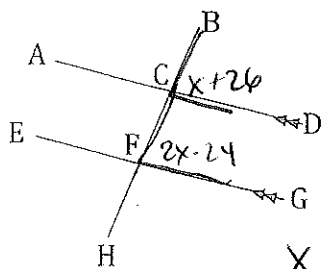
$$2x - 142 = x + 16$$

$$\begin{aligned} 2x - 142 &= x + 16 \\ -x &= 158 \\ x &= 158 \end{aligned}$$

$$m\angle ACH = 174^\circ$$

$$158 + 16 = 174^\circ$$

41.  $m\angle BCD = X + 26^\circ$ ,  $m\angle BFG = 2X - 24^\circ$ . Find  $m\angle BFG$ .



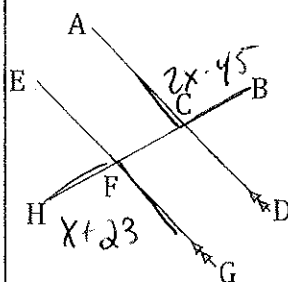
Corresp.

$$x + 26 = 2x - 24$$

$$50 = x$$

$$m\angle BFG = \frac{2(50) - 24}{36 - 14} = 76^\circ$$

38.  $m\angle ACB = 2X - 45^\circ$ ,  $m\angle HFG = X + 23^\circ$ . Find  $m\angle HFG$ .



Alt. ext.  $\angle$ 's

$$m\angle ACB = m\angle HFG$$

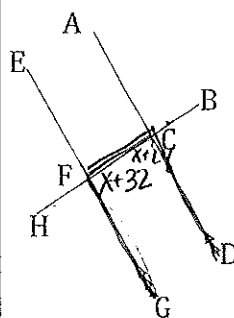
$$2x - 45 = x + 23$$

$$x = 68$$

$$m\angle HFG = 68 + 23$$

$$91^\circ$$

40.  $m\angle GFB = x + 32^\circ$ ,  $m\angle DCH = X + 24^\circ$ . Find  $m\angle DCH$ .



Same Side Int.

$$x + 32 + x + 24 = 180$$

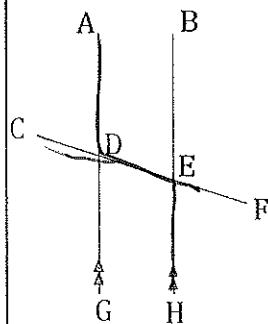
$$2x + 56 = 180$$

$$2x = 124$$

$$x = 62$$

$$m\angle DCH = 62 + 24 = 86^\circ$$

42.  $m\angle ADF = 2X + 4^\circ$ ,  $m\angle HEC = 4X - 14^\circ$ . Find  $m\angle HEC$ .



Alt, Int.  $\angle$ 's

$$m\angle ADF = m\angle HEC$$

$$2x + 4 = 4x - 14$$

$$m\angle HEC = \frac{4(9) - 14}{36 - 14} = 22^\circ$$

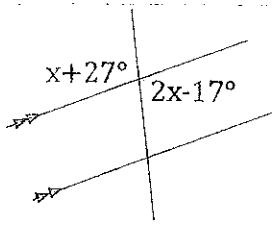
$$18 = 2x$$

$$x = 9$$

Bubble all the correct answers from above. Don't bubble incorrect answers.

- 76°  
  110°  
 91°  
 94°  
 106°  
 97°  
 22°  
 165°  
 86°  
 92°

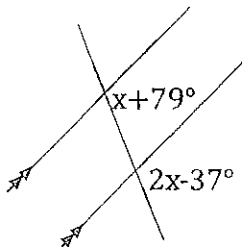
23.



Vert.  $\angle$ 's  
 $x+27 = 2x-17$

$44 = x$

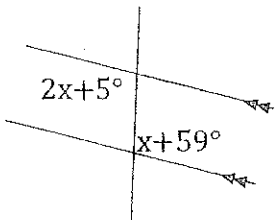
25.



Corresp.  
 $x+79 = 2x-37$

$116 = x$

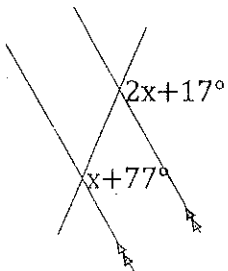
27.



Alt. Int.  $\angle$   
 $2x+5 = x+59$

$x = 54$

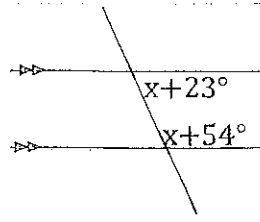
29.



Corresp.  
 $2x+17 = x+77$

$x = 60$

24.



SSI

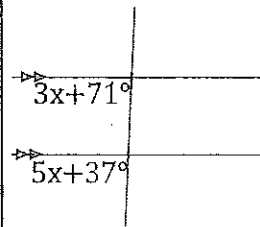
$x+23 + x+54 = 180$

$2x+77 = 180$

$2x = 103$

$x = 51.5$

26.



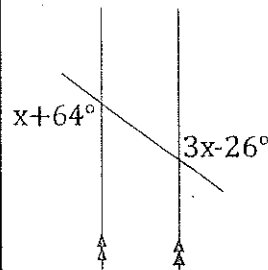
Corresp.

$3x+71 = 5x+37$

$34 = 2x$

$x = 17$

28.



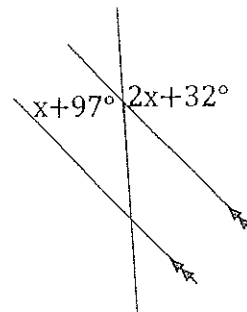
Alt. Ext.  $\angle$

$x+64 = 3x-26$

$90 = 2x$

$x = 45$

30.



Vert.  $\angle$

$x+97 = 2x+32$

$65 = x$

Bubble all the correct answers from above. Don't bubble incorrect answers.

- 31° 
  116° 
  20° 
  17° 
  54° 
  98° 
  51.5° 
  45° 
  60° 
  72.5° 
  65° 
  44° 
  30.5° 
  24°

G21

# Parallel Lines Proof Worksheet

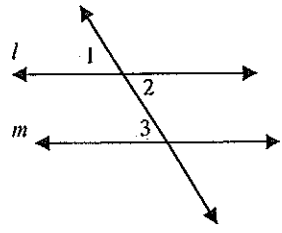
ANSWERS

Name \_\_\_\_\_ (Do NOT write your proofs in this format!)

Write a 2 column or flow proof on your own paper.

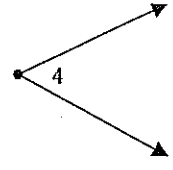
1. Given:  $l \parallel m$ ;  $\angle 2 \cong \angle 4$   
 Prove:  $\angle 4 \cong \angle 3$

$l \parallel m$   
 $\angle 2 \cong \angle 4$  Given  
 $\angle 4 \cong \angle 2$  Symmetric  
 $\angle 2 \cong \angle 3$  alt. int.  $\&$  Thm  
 $\angle 4 \cong \angle 3$  Trans.



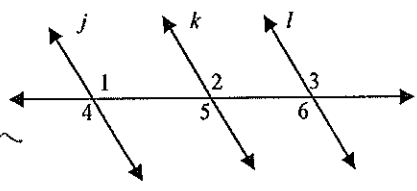
2. Given:  $l \parallel m$ ;  $\angle 1 \cong \angle 4$   
 Prove:  $\angle 3 \cong \angle 4$

$l \parallel m$  Given  
 $\angle 3 \cong \angle 1$  Corresp.  $\&$  Post.  
 $\angle 1 \cong \angle 4$  Given  
 $\angle 3 \cong \angle 4$  Transitive



3. Given:  $j \parallel k$ ,  $k \parallel l$   
 Prove:  $\angle 1 \cong \angle 3$

$j \parallel k$ ,  $k \parallel l$  Given  
 $\angle 1 \cong \angle 2$  Corresp.  $\&$  Thm  
 $\angle 2 \cong \angle 3$  Corresp.  $\&$  Thm  
 $\angle 1 \cong \angle 3$  Trans.

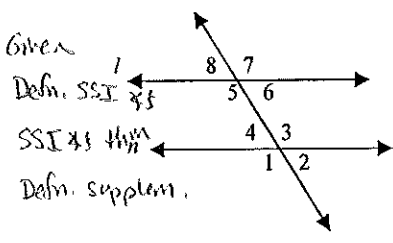


4. Given:  $j \parallel k$ ,  $k \parallel l$   
 Prove:  $\angle 1 \cong \angle 6$

$j \parallel k$ ;  $k \parallel l$  Given  
 $\angle 1 \cong \angle 2$  Corresp.  $\&$  Post.  
 $\angle 2 \cong \angle 6$  alt. int.  $\&$  Thm  
 $\angle 1 \cong \angle 6$  Transitive

5. Given:  $l \parallel n$   
 Prove:  $m\angle 3 + m\angle 6 = 180^\circ$

$l \parallel n$  Given  
 $\angle 3$  &  $\angle 6$  are SSI  $\&$   $\&$  Defn. SSI  $\&$  Thm  
 $\angle 3$  &  $\angle 6$  are suppln. SSI  $\&$  Thm  
 $m\angle 3 + m\angle 6 = 180$  Defn. suppln.



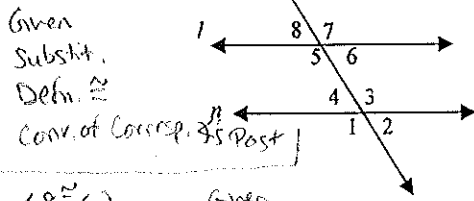
6. Given:  $l \parallel n$   
 Prove:  $m\angle 2 + m\angle 7 = 180^\circ$

$l \parallel n$  Given  
 $\angle 6$  &  $\angle 7$  suppln. LPP  
 $m\angle 6 + m\angle 7 = 180$  Defn. suppln.  
 $\angle 2 \cong \angle 6$  Corresp.  $\&$  Post.  
 $m\angle 2 = m\angle 6$  Defn.  $\cong$   
 $m\angle 2 + m\angle 7 = 180$  Substitution

3.4

7. Given:  $m\angle 1 = 101^\circ$ ,  $m\angle 5 = 101^\circ$   
 Prove:  $l \parallel n$

$m\angle 1 = 101$ ;  $m\angle 5 = 101$   
 $m\angle 1 = m\angle 5$   
 $\angle 1 \cong \angle 5$   
 $l \parallel n$



8. Given:  $m\angle 3 = 105^\circ$ ,  $m\angle 6 = 75^\circ$   
 Prove:  $l \parallel n$

$m\angle 3 = 105$ ,  $m\angle 6 = 75$   
 $m\angle 3 + m\angle 6 = 105 + 75$   
 $m\angle 3 + m\angle 6 = 180$   
 $\angle 3$  &  $\angle 6$  are suppln.  
 $l \parallel n$

Use for #7-#10

9. Given:  $\angle 8 \cong \angle 2$   
 Prove:  $l \parallel n$

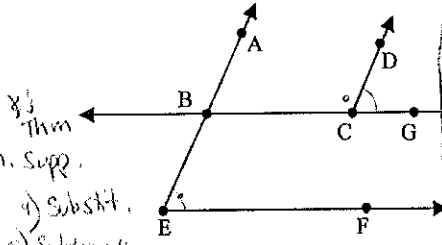
$\angle 8 \cong \angle 2$  Given  
 $l \parallel n$  Conv. of alt. ext.  $\&$  thm

10. Given:  $\angle 7$  is supplementary to  $\angle 2$   
 Prove:  $l \parallel n$

$\angle 7$  is suppln. to  $\angle 2$  Given  
 $\angle 7$  suppln. to  $\angle 6$  LPP  
 $m\angle 7 + m\angle 2 = 180$  Defn. of suppln.  
 $m\angle 7 + m\angle 6 = 180$   
 $l \parallel n$

11. Given:  $m\angle BCD + m\angle BEF = 180^\circ$ ,  $\overline{AB} \parallel \overline{DC}$   
 Prove:  $\overline{BC} \parallel \overline{EF}$

1)  $m\angle BCD + m\angle BEF = 180$  Given  
 $\overline{AB} \parallel \overline{DC}$   
 2)  $\angle BED$  and  $\angle ABC$  suppln. SSI  $\&$  Thm  
 3)  $m\angle BCD + m\angle ABC = 180$  Defn. Suppln.  
 4)  $m\angle BCD + m\angle BEF = m\angle BCD + m\angle ABC$  Substit.  
 5)  $m\angle BEF = m\angle ABC$  Subtraction  
 6)  $\angle BEF \cong \angle ABC$  Defn.  $\cong$   
 7)  $\overline{BC} \parallel \overline{EF}$  Conv. of Corresp.  $\&$  Post.



12. Given:  $\overline{BC} \parallel \overline{EF}$ ,  $\angle BEF \cong \angle DCG$   
 Prove:  $\overline{AB} \parallel \overline{DC}$

1)  $\overline{BC} \parallel \overline{EF}$  Given  
 2)  $\angle ABC \cong \angle BEF$  Corresp.  $\&$  Post.  
 3)  $\angle BEF \cong \angle DCG$  Given  
 4)  $\angle ABC \cong \angle DCG$  Transitive  
 5)  $\overline{AB} \parallel \overline{DC}$  Conv. of Corresp.  $\&$  Post.

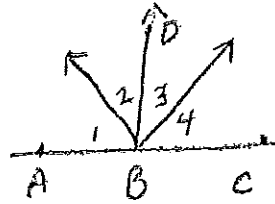
4)  $m\angle 7 + m\angle 2 = m\angle 7 + m\angle 6$  Subst.  
 5)  $m\angle 2 = m\angle 6$  Subtraction  
 6)  $\angle 2 \cong \angle 6$  Defn.  $\cong$   
 7)  $l \parallel n$  Conv. of Corresp.  $\&$  Thm.

Parallel Proofs Practice: Do these on a separate sheet of paper.

3.1

Given:  $\overline{AC} \perp \overline{BD}$   
 $\angle 1 \cong \angle 4$

Prove:  $\angle 2 \cong \angle 3$

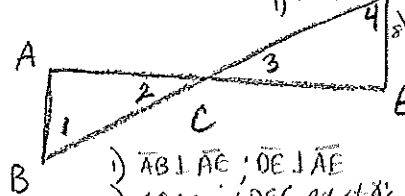


- 1)  $\overline{AC} \perp \overline{BD}; \angle 1 \cong \angle 4$  1) Given
- 2)  $\angle ABD = \angle CBD$  are rt.  $\angle$ s 2) Defn.  $\perp$
- 3)  $m\angle ABD = 90$  3) Defn. rt.  $\angle$
- 4)  $m\angle 1 + m\angle 2 = m\angle ABD$  4) angle add. post.
- 5)  $m\angle 3 + m\angle 4 = m\angle CBD$  5) Substit.
- 6)  $m\angle ABD = m\angle CBD$  6) Substit.
- 7)  $m\angle 1 = m\angle 4$  7) Defn.  $\cong$
- 8)  $m\angle 2 = m\angle 3$  8) Subtraction
- 9)  $\angle 2 \cong \angle 3$  9) Defn.  $\cong$

3.2

Given:  $\overline{AB} \perp \overline{AE}$   
 $\overline{DE} \perp \overline{AE}$

Prove:  $\angle 1 \cong \angle 4$



- 1)  $\overline{AB} \perp \overline{AE}; \overline{DE} \perp \overline{AE}$  1) Given
- 2)  $\angle BAC = \angle DEC$  are rt.  $\angle$ s 2) Defn.  $\perp$
- 3)  $m\angle BAC = 90, m\angle DEC = 90$  3) Defn. rt.  $\angle$
- 4)  $m\angle BAC = m\angle DEC$  4) Substit.
- 5)  $\angle BAC \cong \angle DEC$  5) Defn.  $\cong$
- 6)  $\overline{AB} \parallel \overline{DE}$  6) Converse of A.A. Thm
- 7)  $\angle 1 \cong \angle 4$  7) Alt. int.  $\angle$  Thm.

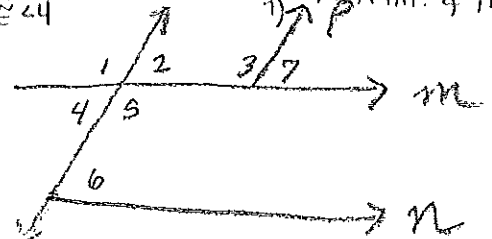
3.4

3) Given:  $\angle 6$  &  $\angle 3$  are supplm.

$l \parallel p$

Prove:  $m \parallel n$

- 1)  $l \parallel p$  1) Given
- 2)  $\angle 2$  &  $\angle 3$  suppl. 2) SSF  $\&$  Thm
- 3)  $m\angle 2 + m\angle 3 = 180$  3) Defn. suppl.
- 4)  $\angle 6$  &  $\angle 3$  suppl. 4) Given
- 5)  $m\angle 6 + m\angle 3 = 180$  5) Defn. suppl.
- 6)  $m\angle 2 + m\angle 3 = m\angle 6 + m\angle 3$  6) Substitution
- 7)  $m\angle 2 = m\angle 6$  7) Subtraction
- 8)  $\angle 2 \cong \angle 6$  8) Defn.  $\cong$
- 9)  $m \parallel n$  9) Conv. of Corresp.  $\&$  Post.



Given:  $\angle 8 \cong \angle 9$   
 $\angle 1 \cong \angle 2$

Prove:  $a \parallel b$

- 1)  $\angle 8 \cong \angle 9$  1) Given
- 2)  $\angle 12 \cong \angle 9$  2) VAT
- 3)  $\angle 9 \cong \angle 8$  3) Symmetric
- 4)  $\angle 1 \cong \angle 8$  4) Transitive
- 5)  $a \parallel b$  5) Conv. of Alt. ext.  $\&$  Thm

