

Name: Answers Per: _____ Date: _____

Coordinate Geometry 6.7 Polygons in the Coordinate Plane

Objective: The students will be able to classify polygons in the coordinate plane using the formulas for slope, distance, and midpoint.

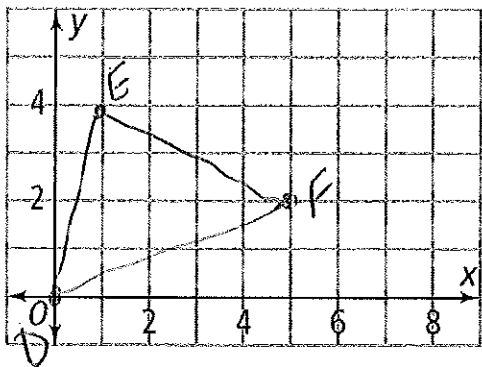
	Distance Formula	Midpoint Formula	Slope Formula
Formula	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
When to Use It	To determine whether <ul style="list-style-type: none"> sides are congruent diagonals are congruent 	To determine <ul style="list-style-type: none"> the coordinates of the midpoint of a side whether diagonals bisect each other 	To determine whether <ul style="list-style-type: none"> opposite sides are parallel diagonals are perpendicular sides are perpendicular

For each of the following, decide which formula to use, *Distance*, *Midpoint*, or *Slope Formula*:

- You want to know whether diagonals bisect each other. midpoint
- You want to find whether opposite sides of a quadrilateral are parallel. slope
- You want to know whether sides of a polygon are congruent. distance

CLASSIFYING A TRIANGLE

4. $\triangle DEF$ has vertices $D(0, 0)$, $E(1, 4)$, and $F(5, 2)$. Is $\triangle DEF$ scalene, isosceles, or equilateral? Is it a right triangle? Explain and show supporting work.



THINK: What formula(s) should you use?

distance: $DE = \sqrt{(1-0)^2 + (4-0)^2}$
 $\sqrt{1^2 + 4^2} = \sqrt{17}$

$EF = \sqrt{(2-4)^2 + (5-1)^2}$
 $\sqrt{(-2)^2 + (4)^2} = \sqrt{20}$

$DF = \sqrt{(5-0)^2 + (2-0)^2}$
 $\sqrt{25 + 4} = \sqrt{29}$

all diff. so scalene

Right Δ ? slope

$DE = \frac{4-0}{1-0} = \frac{4}{1}$

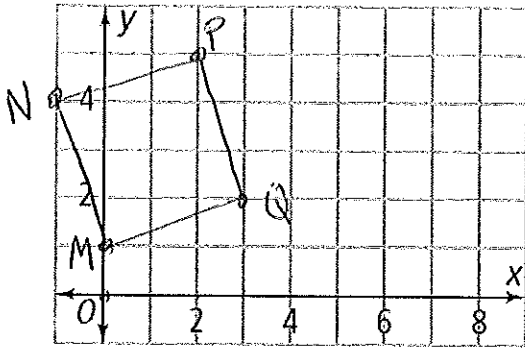
$EF = \frac{2-4}{5-1} = \frac{-2}{4} = -\frac{1}{2}$

$DF = \frac{2-0}{5-0} = \frac{2}{5}$

NOT a right Δ

CLASSIFYING A PARALLELOGRAM

5. $\square MNPQ$ has vertices $M(0, 1)$, $N(-1, 4)$, $P(2, 5)$, and $Q(3, 2)$. Is $\square MNPQ$ a rectangle? Is it a square? Explain and show supporting work.



THINK: How can you show that it is a rectangle?

How can you show that it is a square?

What formula(s) should you use?

Distances:

$$MN = \sqrt{(-1-0)^2 + (4-1)^2} = \sqrt{1+9} = \sqrt{10}$$

$$NP = \sqrt{(2+1)^2 + (5-4)^2} = \sqrt{9+1} = \sqrt{10}$$

$$PQ = \sqrt{(3-2)^2 + (2-5)^2} = \sqrt{1+9} = \sqrt{10}$$

$$QM = \sqrt{(3-0)^2 + (2-1)^2} = \sqrt{9+1} = \sqrt{10}$$

Diagonals

$$NQ = \sqrt{(3+1)^2 + (2-4)^2} = \sqrt{16+4} = \sqrt{20}$$

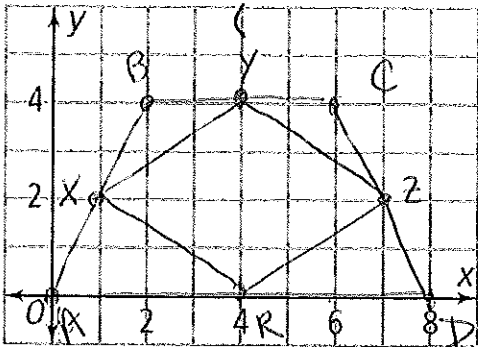
$$PM = \sqrt{(2-0)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20}$$

Rectangle

Rhombus w/
 \cong diag.

CLASSIFYING A QUADRILATERAL

6. An isosceles trapezoid has vertices $A(0, 0)$, $B(2, 4)$, $C(6, 4)$, and $D(8, 0)$. What special quadrilateral is formed by connecting the midpoints of the sides of $ABCD$?



THINK: Judging by appearance, what type of quadrilateral is formed?

looks like a rhombus

How can you show that it is what you think it is?

What formula(s) should you use?

Slopes \downarrow OR distance

- $X(1, 2)$
- $Y(4, 4)$
- $Z(7, 2)$
- $R(4, 0)$

Slopes

$$\overline{XY} = \frac{4-2}{4-1} = \frac{2}{3}$$

$$\overline{YZ} = \frac{2-4}{7-4} = \frac{-2}{3}$$

$$\overline{ZR} = \frac{2-0}{7-4} = \frac{2}{3}$$

$$\overline{RX} = \frac{2-0}{1-4} = \frac{2}{-3}$$

parallel

parallel

Slopes of diagonals

$$\overline{YR} = \frac{4-0}{4-4} = \text{undef.}$$

$$\overline{XZ} = \frac{2-2}{1-7} = 0$$

Rhombus

\parallel gm w/ \perp diag.

How can you use the formulas to determine if a polygon on a coordinate plane is a(n) ...

(NOTE: There may be more than one way!)

scalene triangle	Distance ~ sides <u>NOT</u> \cong
isoseles triangle	Distance ~ 2 sides \cong
equilateral triangle	Distance ~ 3 sides \cong
parallelogram	slopes \rightarrow ^{2 prs.} opp. sides \parallel . (same slope) <u>OR</u> Midpoint \rightarrow diag. bisect each other <u>OR</u> Distance \rightarrow ^{2 prs.} opp. sides \cong
rhombus	Distance \rightarrow 4 \cong sides
rectangle	Distance \rightarrow opp. sides \cong AND then Distance \rightarrow diag. \cong <u>OR</u> slopes \rightarrow 4 sides all \perp
square	Distance \rightarrow Diagonals \cong <u>AND</u> slope \rightarrow Diagonals \perp } Both <u>OR</u> Distance \rightarrow 4 \cong sides <u>AND</u> dist. \rightarrow diag. \cong
trapezoid	use slopes to show 2 sides \parallel and other 2 <u>NOT</u> \parallel ,
isosceles trapezoid	use slopes to find show 2 sides \parallel ; other 2 <u>NOT</u> \parallel . then Distance formula to show 2 sides \cong
kite	Distance formula for all sides to show 2 pairs of consec. sides \cong ,

YOU TRY!

1. Is $\triangle ABC$ scalene, isosceles, or equilateral? Show work to support your answer.

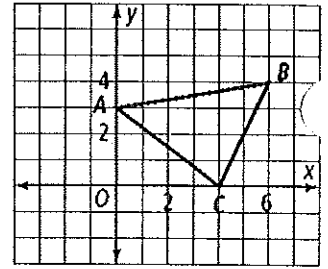
$A(0, 3)$
 $B(6, 4)$
 $C(4, 0)$

$$AB = \sqrt{(6-0)^2 + (4-3)^2} = \sqrt{36+1} = \sqrt{37}$$

$$BC = \sqrt{(6-4)^2 + (4-0)^2} = \sqrt{4+16} = \sqrt{20}$$

$$AC = \sqrt{(4-0)^2 + (0-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Scalene



2. Is quadrilateral $GHIJ$ a parallelogram? Show work to support your answer.

$G(-3, 3)$
 $H(0, 4)$
 $I(4, -1)$
 $J(1, -2)$

Slopes

$$\overline{GH} = \frac{4-3}{0-3} = \frac{1}{-3} = -\frac{1}{3}$$

$$\overline{IJ} = \frac{-2-(-1)}{1-4} = \frac{-1}{-3} = \frac{1}{3}$$

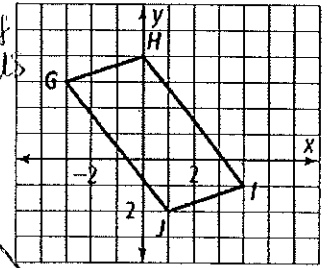
$$\overline{HI} = \frac{4-(-1)}{0-4} = \frac{5}{-4} = -\frac{5}{4}$$

$$\overline{GJ} = \frac{-2-3}{1-(-3)} = \frac{-5}{-4} = \frac{5}{4}$$

yes parallelogram

OR Midpts of Diagonals

$$\overline{HJ} = (\frac{1}{2}, 1)$$

$$\overline{GI} = (\frac{1}{2}, 1)$$


same, diag. bis. each other

3. Is parallelogram $ABCD$ a rhombus? Show work to support your answer.

$A(-3, -4)$
 $B(5, 2)$ } Diag. \overline{AB}

$C(-2, 3)$
 $D(4, -5)$ } Diag. \overline{CD}

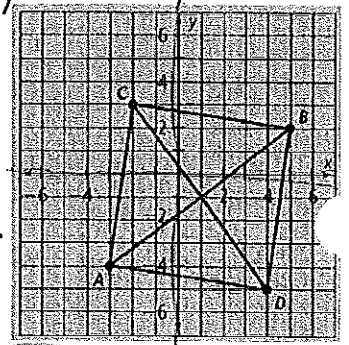
Slopes

$$\frac{2+4}{5-3} = \frac{6}{2} = 3$$

$$\frac{3+5}{-2-4} = \frac{8}{-6} = -\frac{4}{3}$$

app. recip. so \perp

yes rhombus



4. $\triangle KLM$ has vertices at $J(-2, 4)$, $K(1, 6)$, and $L(4, 4)$.

a. Determine whether $\triangle KLM$ is scalene, isosceles, or equilateral. Explain.

$$JK = \sqrt{(1+2)^2 + (6-4)^2} = \sqrt{9+4} = \sqrt{13}$$

$$KL = \sqrt{(4-1)^2 + (4-6)^2} = \sqrt{9+4} = \sqrt{13}$$

$$JK = \sqrt{(4+2)^2 + (4-4)^2} = \sqrt{36+0} = \sqrt{36} = 6$$

isosceles

b. Determine whether $\triangle KLM$ is a right triangle. Explain.

Slopes

$$\overline{JK} = \frac{6-4}{1+2} = \frac{2}{3}$$

$$\overline{KL} = \frac{6-4}{1-4} = \frac{2}{-3}$$

$$\overline{JL} = \frac{4-4}{4+2} = \frac{0}{6} = 0$$

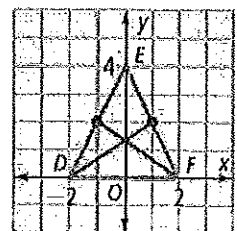
NOT a right Δ , b/c no \perp slopes

5. Trapezoid $ABCD$ has vertices at $A(2, 1)$, $B(12, 1)$, $C(9, 4)$, and $D(5, 4)$. Which formula would help you find out if this trapezoid is isosceles? Explain.

Use distance formula to show whether 2 sides are \cong .

6. For $\triangle DEF$, describe how you would determine whether the lengths of the medians from base angles D and F are congruent.

Use midpt formula to find midpts of ED ; EF
 then use Distance Formula to find lengths of the 2 medians



6-7

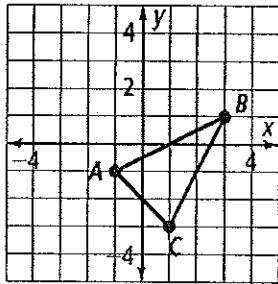
Practice

REVISED
Form K

Polygons in the Coordinate Plane

Determine whether $\triangle ABC$ is scalene, isosceles, or equilateral. Then determine if it is a right triangle. Explain.

1.



To start, determine the vertices of the triangle. Then use the Distance Formula to find the length of each side.

$A(-1, -1), B(3, 1), C(1, -3)$
 $AB = \sqrt{(3+1)^2 + (1+1)^2}$
 $16 + 4 = \sqrt{20}$

$BC = \sqrt{(1-3)^2 + (-3-1)^2}$
 $4 + 16 = \sqrt{20}$

$AC = \sqrt{(1+1)^2 + (-3+1)^2}$
 $4 + 4 = \sqrt{8}$

isosceles

ABC is is NOT and is/is not (circle one) a rt. \triangle

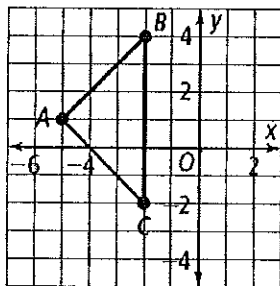
slope $AB = \frac{1+1}{3+1} = \frac{2}{4} = \frac{1}{2}$

$BC = \frac{-3-1}{1-3} = \frac{-4}{-2} = 2$

$AC = \frac{-3+1}{1+1} = \frac{-2}{2} = -1$

2.

~~AB~~
 $A(-5, 1)$
 $B(-2, 4)$
 $C(-2, -2)$



$AB = \sqrt{(4-1)^2 + (-2+5)^2}$
 $9 + 9 = \sqrt{18}$

$BC = \sqrt{(4+2)^2 + (-2+2)^2}$
 $\sqrt{36 + 0} = 6$

$AC = \sqrt{(1+2)^2 + (-5+2)^2}$
 $9 + 9 = \sqrt{18}$

isosceles

ABC is IS and is/is not (circle one) a rt. \triangle

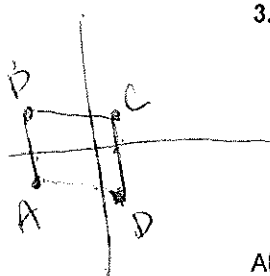
$AB = \frac{4-1}{-2+5} = \frac{3}{3} = 1$

$BC = \frac{4+2}{-2+2} = \frac{6}{0}$ undef.

$AC = \frac{1+2}{-5+2} = \frac{3}{-3} = -1$

Determine whether the parallelogram is a rhombus, rectangle, square, or none. Use formulas, show work and Explain.

3. $A(-3, -1), B(-3, 2), C(1, 1), D(1, -2)$



Diag. $\overline{BD} = \frac{-2-2}{1+3} = \frac{-4}{4} = -1$

$\overline{AC} = \frac{1+1}{-3+3} = \frac{2}{0} = \frac{1}{2}$

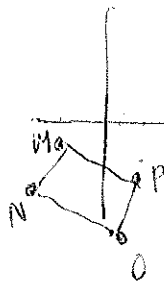
NOT \perp
 so NOT rhombus or sq.

$\perp?$ slopes $AB = \frac{2+1}{-3+3} = \frac{3}{0}$ undef

$BC = \frac{1-2}{1+3} = \frac{-1}{4}$ so NOT rect. or sq.

ABCD is a parallelogram because NOT rhombus, rect, or sq.

4. $M(-2, -1), N(-3, -3), O(1, -5), P(2, -3)$



Diag. slopes = $MO = \frac{-5+1}{1+2} = \frac{-4}{3}$

$NP = \frac{-3+3}{-3-2} = \frac{0}{-5} = 0$ so not rhombus or sq.

sides? slope $MN = \frac{-3+1}{-3+2} = \frac{-2}{-1} = 2$

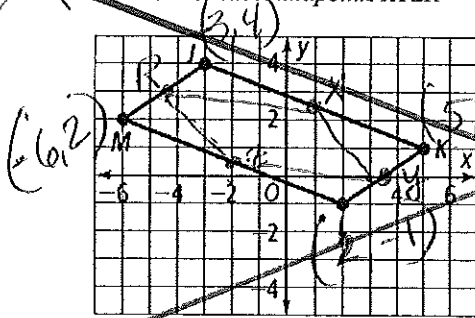
$\overline{NO} = \frac{-5+3}{1+3} = \frac{-2}{4} = -\frac{1}{2}$

rt. \angle so rectangle

MNOP is a rectangle because a //gm w/ rt \angle is a rectangle

What is the most precise classification of the quadrilateral formed by connecting in order the midpoints of each figure below?

5. $\square JKLM$ label midpoints $XYZR$



midpoints of

$JK = X(3, 2.5)$ $KL = Y(3, 0)$

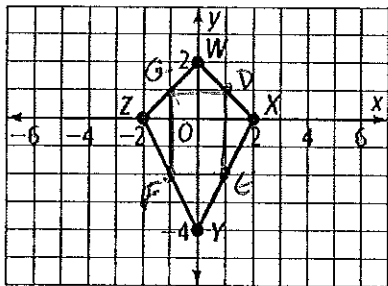
$LM = Z(-2, 1)$ $MJ = R(-9, 3)$

Slopes
 $\overline{XY} \Rightarrow \frac{0 - 2.5}{3 - 3} = \frac{-2.5}{0} = \text{undef.}$
 $\overline{YZ} \Rightarrow \frac{1 - 0}{-2 - 3} = \frac{1}{-5} = -\frac{1}{5}$

$\overline{ZR} = \frac{3 - 1}{-9 - (-2)} = \frac{2}{-7} = -\frac{2}{7}$

$XYZR$ is a _____ because _____

6. kite $WXYZ$ label midpoints $DEFG$



midpoints of

$WX = D(1.5, 1.5)$ $XY = E(1.5, 0.5)$

$YZ = F(-0.5, 0.5)$ $ZW = G(-0.5, 1.5)$

Slopes
 $\overline{GD} = \frac{1.5 - 1.5}{-0.5 - 1.5} = 0$
 $\overline{DE} = \frac{0.5 - 1.5}{1.5 - 1.5} = \frac{-1}{0} = \text{undef.}$

$DEFG$ is a rectangle because consec. sides are \perp ; opp. sides \parallel

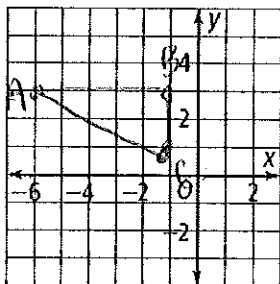
7. Graph and label the triangle. Determine whether it is **8. Graph the quadril.**

\rightarrow scalene, isosceles, or equilateral?

\rightarrow classify it precisely

\rightarrow is a right triangle?

7. $A(-6, 3)$, $B(-1, 3)$, and $C(-1, 1)$

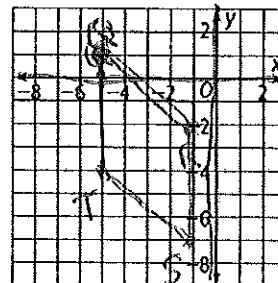


$AB = \sqrt{(5)^2 + (0)^2} = 5$
 $BC = \sqrt{(0)^2 + (2)^2} = 2$
 $AC = \sqrt{(-1+6)^2 + (-2)^2} = \sqrt{25+4} = \sqrt{29}$

Scalene, right

Slopes $AB = \frac{3-3}{-1+6} = 0$
 $BC = \frac{1-3}{-1+1} = \frac{-2}{0} = \text{undef.}$

8. $Q(-5, 1)$, $R(-1, -2)$, $S(-1, -7)$, $T(-5, -4)$



Slopes
 $\overline{QR} = \frac{-2-1}{-1+5} = \frac{-3}{4}$
 $\overline{RS} = \frac{-7+2}{-1+1} = \frac{-5}{0} = \text{undef.}$
 $\overline{ST} = \frac{-4+7}{-5+1} = \frac{3}{-4} = -\frac{3}{4}$
 $\overline{TQ} = \frac{-4-1}{-5+5} = \frac{-5}{0} = \text{undef.}$

Diag. \perp ?
 slopes $\overline{RT} = \frac{-4+2}{-5+1} = \frac{-2}{-4} = \frac{1}{2}$
 $\overline{QS} = \frac{-7-1}{-1+5} = \frac{-8}{4} = -2$

Rhombus