

Name:
Period:

Note to student: This packet should be used as practice for the Geometry 22 final exam. This should not be the only tool that you use to prepare yourself for the exam. You must go through your notes, re-do homework problems, class work problems, formative assessment problems, and questions from your tests and quizzes throughout the year thus far.

Section 1

1) Classify each statement as true or false, and explain your reasoning in each false case.

a) Two planes intersect in only one point. false; two planes intersect at a line

b) A ray starts at one point on a line and goes on forever. true

c) The intersection of 2 planes is one line true

d) Any four points are collinear. false; any two points are coplanar

2) Describe the difference and similarities of skew and parallel lines.
Both skew lines & parallel lines do not/will never intersect. Parallel lines are coplanar, while skew lines are not.

3) Use the figure below for #6-14. Note that \overleftrightarrow{RN} pierces the plane at N. It is not coplanar with V.

a) Name two segments shown in the figure. \overline{CN} , \overline{AN} , \overline{NM}

b) What is the intersection of \overline{CM} and \overleftrightarrow{RN} ? N

c) Name three collinear points. A, N, X OR C, N, M

d) What are two other ways to name plane V? Plane AMX, Plane ANC, Plane CNX

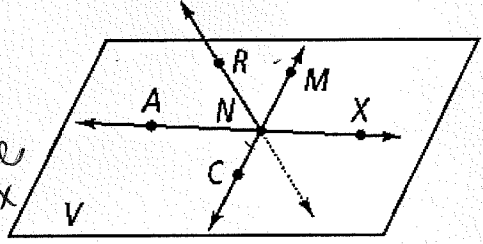
e) Are points R, N, M, and X coplanar? NO

f) Name two rays shown in the figure. \overrightarrow{NR} , \overrightarrow{NM} , \overrightarrow{AN}

g) Name the pair of opposite rays with endpoint N. ① \overrightarrow{NM} , \overrightarrow{NR} ② \overrightarrow{AN} , \overrightarrow{NX}

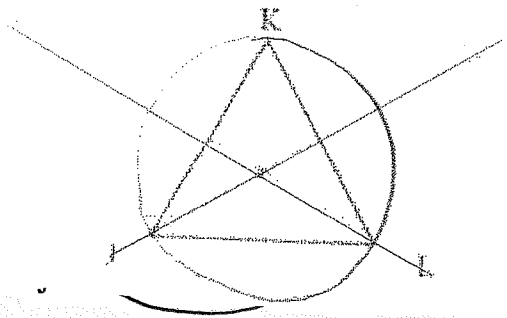
h) \overline{AN} is the same as \overline{NA} . True or False? true

i) \overline{ANX} names a plane. True or False? false; collinear points cannot name a plane

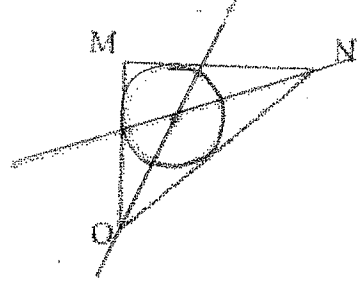


4) Construct and label the following:

a) The circumscribed circle of $\triangle KLI$



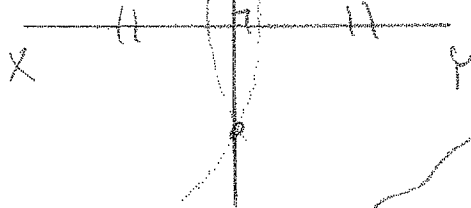
b) The inscribed circle of $\triangle MNO$



4a):

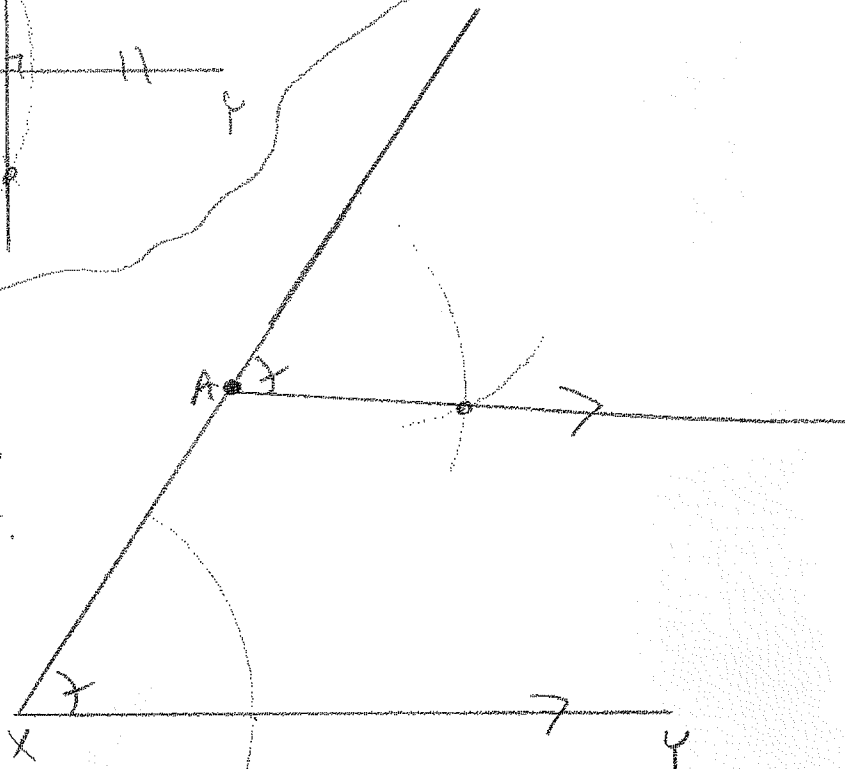


4b)

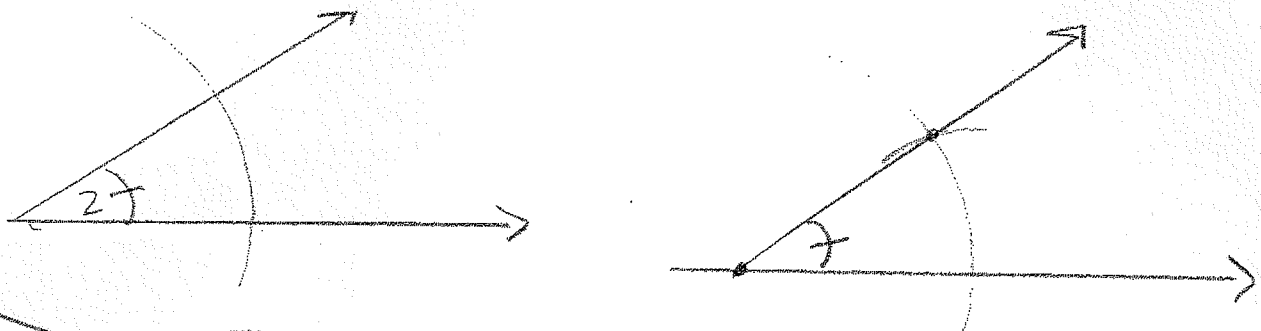


4c)

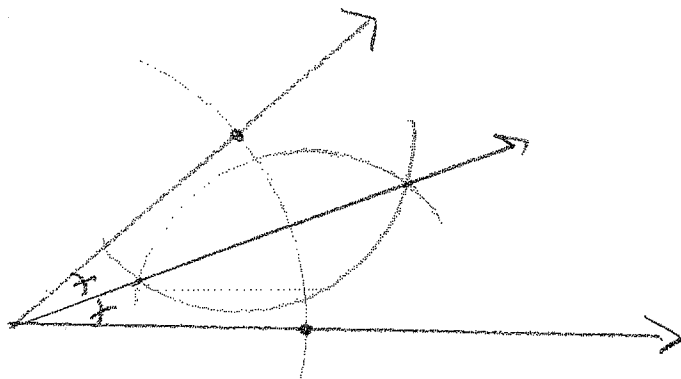
$\cong \Delta s$, therefore
by conv. of corr.
 Δs Post. the
lines are ||



4d)



4e)



5) For the following exercises, do the construction using the figures below.

a) Construct \overline{AB} congruent to \overline{XY} .



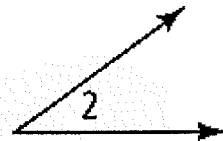
b) Construct the perpendicular bisector of \overline{XY} .



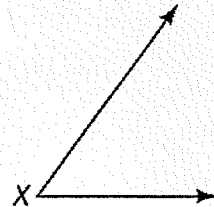
c) Construct a segment parallel to \overline{XY} .



d) Construct an angle congruent to $\angle 2$

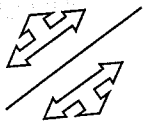


e) Construct the angle bisector of $\angle X$



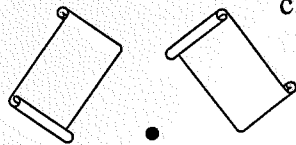
6) Below each figure write the name of the kind of rigid transformation shown.

a.



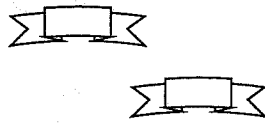
reflection

b.



rotation

c.



translation

Section 2

Complete the following statements:

1) $\angle ABC$ and $\angle BCD$ are complementary. $m\angle ABC = 6x^\circ$ and $m\angle BCD = 12x^\circ$. Find x .

$$6x + 12x = 90$$

$$18x = 90$$

$$\boxed{x = 5}$$

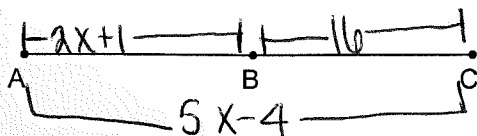
2) $\angle ABC$ and $\angle BCD$ are supplementary. $m\angle ABC = 40x^\circ$ and $m\angle BCD = 20^\circ$. Find x .

$$40x + 20 = 180$$

$$40x = 160$$

$$\boxed{x = 4}$$

3) $AB = 2x + 1$, $BC = 16$ inches, $AC = 5x - 4$. Use the diagram to solve for x :



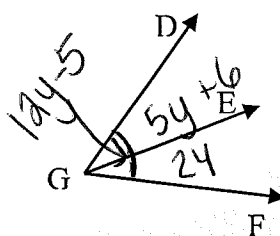
$$2x + 1 + 16 = 5x - 4$$

$$2x + 17 = 5x - 4$$

$$21 = 3x$$

$$\boxed{x = 7}$$

4) Solve for y : $m\angle DGF = 12y - 5$, $m\angle EGF = 24^\circ$, $m\angle DGE = 5y + 6$



$$5y + 6 + 24 = 12y - 5$$

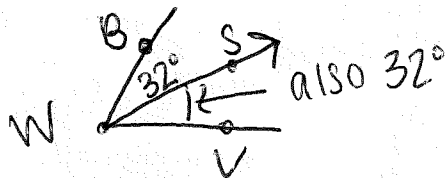
$$5y + 30 = 12y - 5$$

$$30 = 7y$$

$$\boxed{y = 5}$$

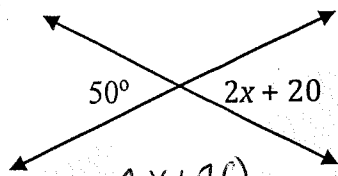
5) \overline{WS} bisects $\angle BWV$. $m\angle BWS = 32^\circ$. What is $m\angle BWV$?

64°



6) Determine the value of x :

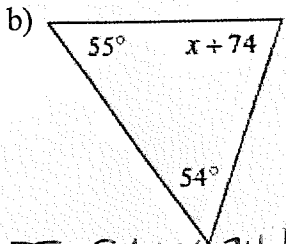
a)



$$50 = 2x + 20$$

$$30 = 2x$$

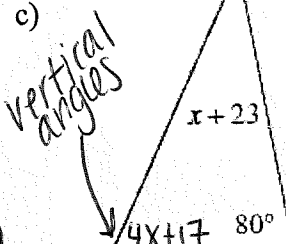
$$x = 15$$



$$55 + 54 + x + 74 = 180$$

$$x + 183 = 180$$

$$x = -3$$

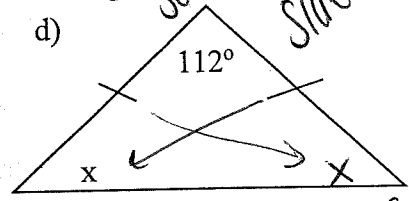


$$x + 23 + 4x + 17 + 80 = 180$$

$$5x + 120 = 180$$

$$5x = 60$$

$$x = 12$$



same side lengths =
same angle measures (opposite side length)

$$x + x + 112 = 180$$

$$2x + 112 = 180$$

$$2x = 68$$

$$x = 34^\circ$$

7) Use the following steps to determine whether the given statement is a definition.

Linear pairs are supplementary, adjacent angles.

a) Conditional statement

If two angles form a linear pair, then the angles are supplementary & adjacent.

b) Converse

If the two angles are supplementary & adjacent, then they are linear pairs.

c) Biconditional statement

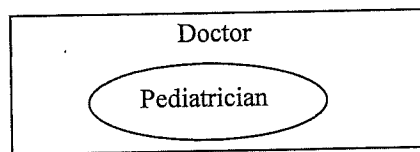
Two angles are linear pairs iff they are supplementary and adjacent.

d) Decide whether the statement is a definition. Explain your reasoning.

Yes \rightarrow both the conditional & converse are true.

8) Write the conditional statement that corresponds to the Venn diagram below:

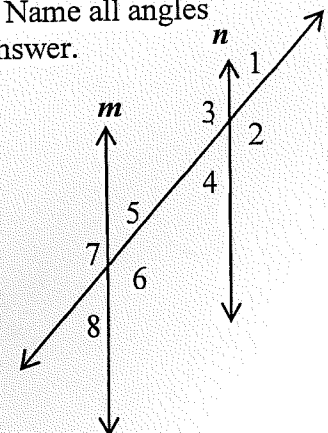
If you are a pediatrician, then you are a doctor.



Section 3

1) For the following exercises, refer to the diagram below. Lines m and n are parallel. Name all angles congruent to the given angle and give the theorems or postulates that justify your answer.

- a) $\angle 6, \angle 7, \angle 3, \angle 2$ \leftarrow alt. thm, int. thm, vertical, corresponding
- b) $\angle 8, \angle 5, \angle 1, \angle 4$ \leftarrow alt. thm, ext. theorem, corresponding
- c) $\angle 5, \angle 8, \angle 4, \angle 1$ \leftarrow alt. int. thm, corresponding
- d) $\angle 7, \angle 6, \angle 2, \angle 3$ \leftarrow alt. ext. theorem, corresponding
- Vertical \angle theorem



2) For the figure to the right $m\angle BCD = 160 - 3x^\circ$, and

$m\angle CFH = 35^\circ$. What is x ?

What Theorem or Postulate supports your answer?

$$160 - 3x + 35 = 180$$

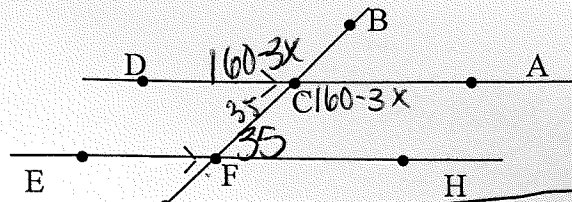
$$-3x + 195 = 180$$

$$-3x = -15$$

$$\boxed{x = 5}$$

① vertical angle theorem

② S.S. interior angle theorem



* also alt. int theorem + linear pair postulate

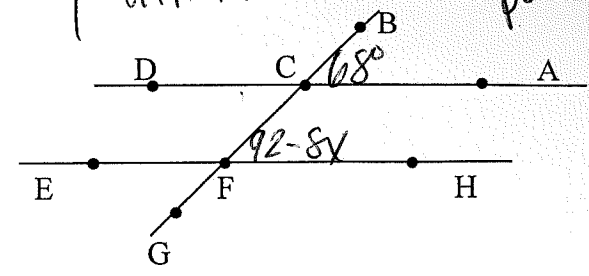
3) For the figure to the right $m\angle BCA = 68^\circ$, and $m\angle CFH = 92 - 8x^\circ$. What value of x makes $\overline{AD} \parallel \overline{EH}$? What Theorem or Postulate supports your answer?

$$92 - 8x = 68$$

$$-8x = -24$$

$$\boxed{x = 3}$$

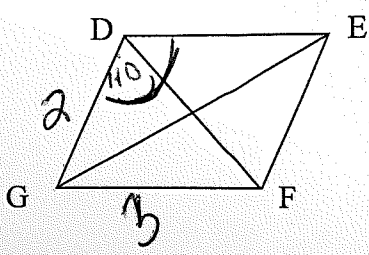
converse of corresponding angles theorem



4) Fill in the blanks so that the sentences are true.

- a) The sum of angles in any quadrilateral is 360° .
- b) In a parallelogram diagonals bisect each other and opposite angles are
- c) congruent
- d) A kite and a rhombus (square) have perpendicular diagonals.
- e) A trapezoid is a quadrilateral with only one pair of parallel sides.
- f) A square is a quadrilateral with 4 congruent sides and 4 right angles.
- g) A rhombus is a parallelogram with four congruent sides.
- h) A parallelogram is a quadrilateral with 2 pairs of parallel sides.
- i) Any four-sided polygon is a quadrilateral.
- j) A rectangle is a quadrilateral with 4 right angles, congruent diagonals, 2 pairs of parallel/cong. sides that bisect each other.

5) Polygon DEFG is a parallelogram. $GF = 3$ in, $DG = 2$ in, $m\angle GDE = 110^\circ$



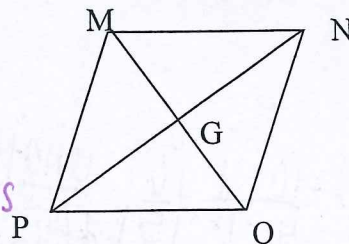
- a) $m\angle DGF = 70^\circ$ ← two consecutive angles in a parallelogram are supp.
- b) $m\angle GFE = 110^\circ$ ← opposite angles in a parallelogram are congruent
- c) $\overline{EF} = 2$ inches
- d) $\overline{DE} = 3$ inches

6) $MNOP$ is a rhombus. If $m\angle MNO = 88^\circ$, find each of the following:

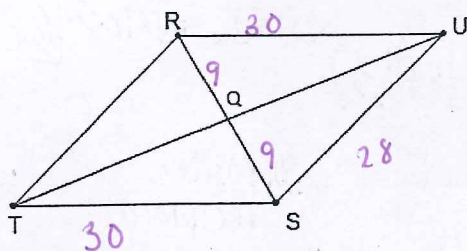
a) $m\angle NOP = 92$

b) $m\angle OPG = 44$

c) $m\angle OGN = 90$ ← diagonals are \perp in a rhombus



7) Parallelogram $RUST$

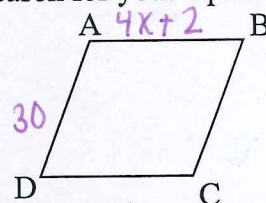


$m\angle RUS = 58^\circ$	$RU = 30\text{cm}$	$RQ = 9\text{cm}$
$m\angle UST = 122^\circ$	$US = 28\text{cm}$	$QS = 9\text{cm}$
$m\angle STR = 58^\circ$	$ST = 30\text{cm}$	$TQ = 25\text{cm}$
$m\angle TRU = 122^\circ$	$TR = 28\text{cm}$	$QU = 25\text{cm}$
	$RS = 18\text{cm}$	$UT = 50\text{cm}$

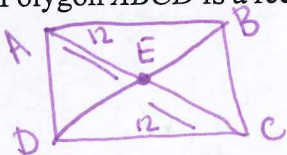
8) Polygon $ABCD$ is a rhombus. $AB = 4x + 2$ and $AD = 30$. What is x ? Give a reason for your equation.

$4x + 2 = 30$
 $4x = 28$
 $x = 7$

rhombi have all congruent sides



9) Polygon $ABCD$ is a rectangle. \overline{AC} and \overline{BD} intersect to E . $AE = 12\text{ ft}$. What is BD ?

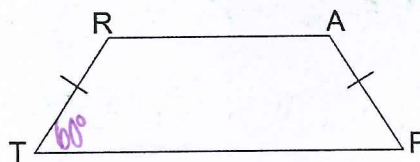


24ft; diagonals are congruent in rectangles.

10) Use trapezoid $TRAP$ to the right to answer the following:

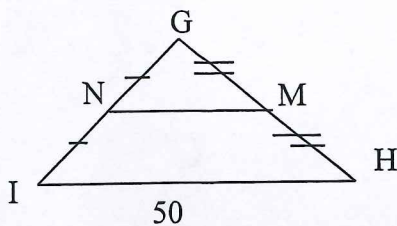
If $m\angle T = 60^\circ$ find the measures of the other angles.

$m\angle R = 120^\circ$ $m\angle A = 120^\circ$ $m\angle P = 60^\circ$

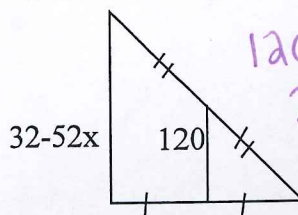


11) Find the following.

a) $NM = 25$



b) $x = -4$



$120 \times 2 = 32 - 52x$
 $240 = 32 - 52x$
 $208 = -52x$
 $x = -4$

c) What is \overline{NM} called? midsegment

12) Find the **slope, midpoint, and length** of each of the following segments whose endpoints are given.

a) $(-1, 4)$ and $(4, 10)$

$m(\text{slope}) = \frac{10-4}{4-(-1)} = \frac{6}{5}$
 $\text{length} = \sqrt{(4-(-1))^2 + (10-4)^2} = \sqrt{25+36} = \sqrt{61} \approx 7.81$
 $\text{midpoint} = \left(\frac{-1+4}{2}, \frac{10+4}{2} \right) = \left(\frac{3}{2}, 7 \right)$

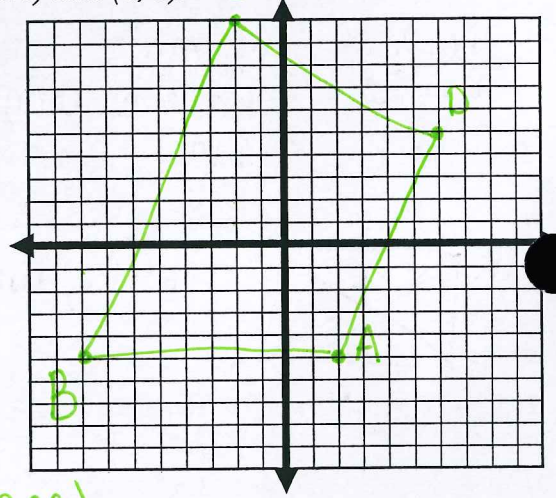
b) $(8, 0)$ and $(10, 6)$

$\text{slope} = \frac{6-0}{10-8} = \frac{6}{2} = 3$
 $\text{midpoint} = \left(\frac{8+10}{2}, \frac{0+6}{2} \right) = \left(\frac{18}{2}, \frac{6}{2} \right) = (9, 3)$
 $\text{Distance} = \sqrt{(8-10)^2 + (0-6)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{4+36} = \sqrt{40} \approx 6.3$

13) Lines that are parallel have the same slopes and lines that are perpendicular have opposite reciprocal slopes.

14) For the following, a quadrilateral has vertices $A(2, -5)$, $B(-8, -5)$, $C(-2, 10)$ and $D(6, 5)$.

a) Graph the quadrilateral on the grid provided.



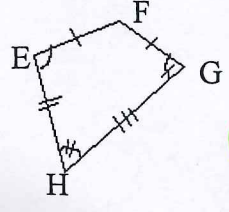
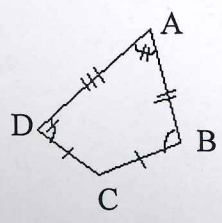
b) What type of quadrilateral is this? Show ALL work necessary to justify your answer.

$m_{AB} \rightarrow \frac{-5-(-5)}{-8-2} = \frac{0}{-10} = 0$
 $m_{BC} \rightarrow \frac{10-(-5)}{-2-(-8)} = \frac{15}{6} = \frac{5}{2}$
 $m_{AD} \rightarrow \frac{5-(-5)}{6-2} = \frac{10}{4} = \frac{5}{2}$
 $m_{CD} \rightarrow \frac{5-10}{6-(-2)} = \frac{-5}{8}$

One pair of slopes the same (parallel lines)
Trapezoid.

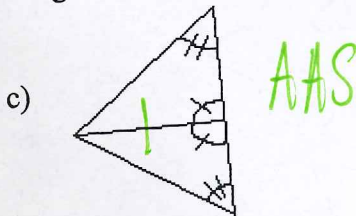
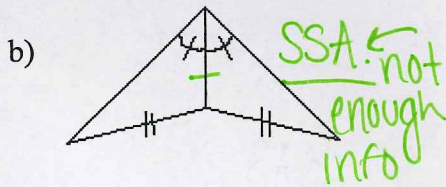
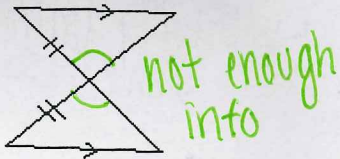
Section 4

1) Write a congruency statement for the following polygons. Why are they congruent?

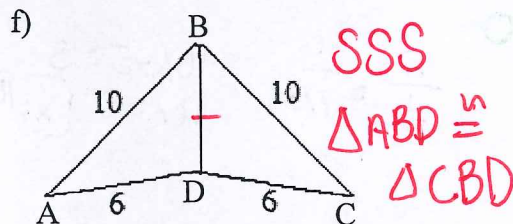
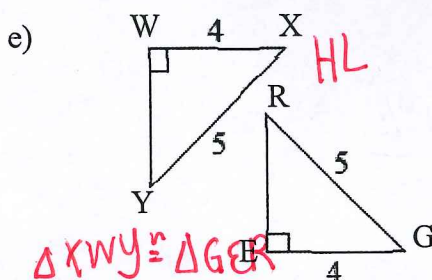
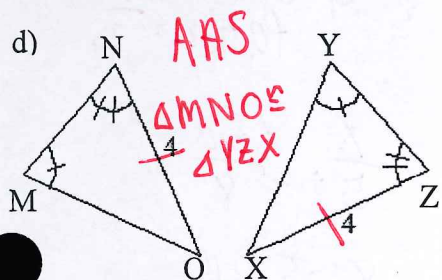
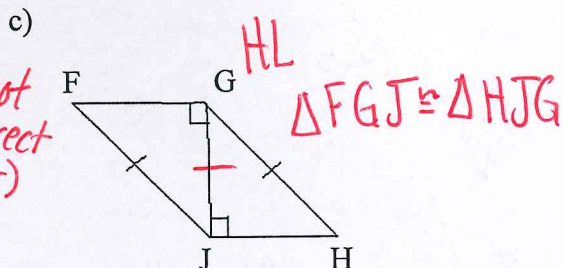
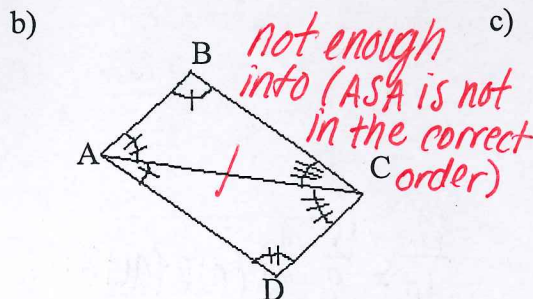
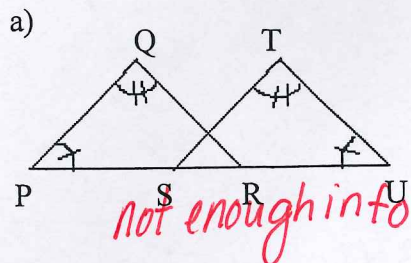


$ABCD \cong HEFG$
 Corresponding angles/segments are congruent.
 (CPCTC) ^{not} triangles... but same idea

2) Determine whether each pair of triangles can be proven **congruent** by using the SSS, SAS, ASA or AAS congruence postulates. If so, identify what postulate is used.

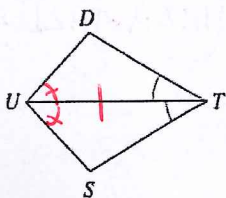


3) Determine whether each pair of triangles can be proven congruent by using the SSS, SAS, ASA, AAS or HL congruence postulates. If so, identify what postulate is used and write a congruency statement.

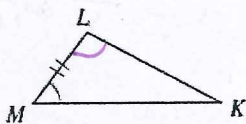


4) Label and state what additional information is required in order to know that the triangles are **congruent** for the reason given.

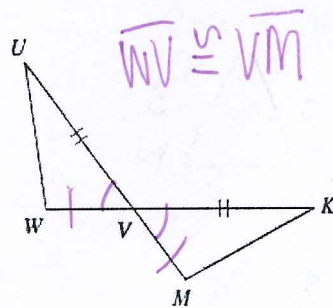
a) ASA



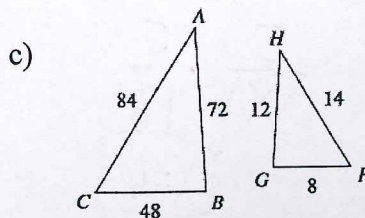
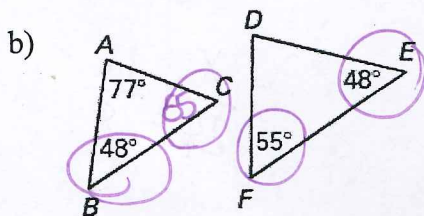
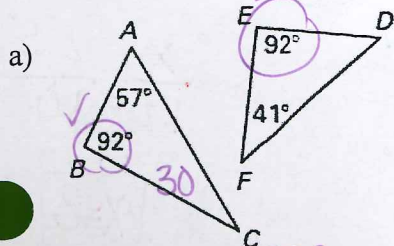
b) ASA



c) SAS

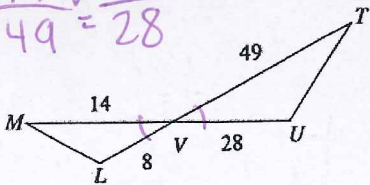


5) Determine whether or not the triangles below are **similar** (you may need to do a little work to figure it out) by AA, SSS, or SAS, or none of them. If they are similar, complete the similarity statement.



$\frac{48}{8} = \frac{72}{12}$
 $b \cong b$

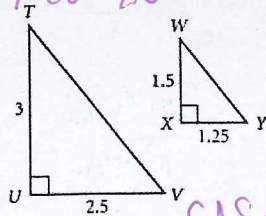
d) $\frac{14}{49} \sqrt{\frac{8}{28}}$



SAS
VVT

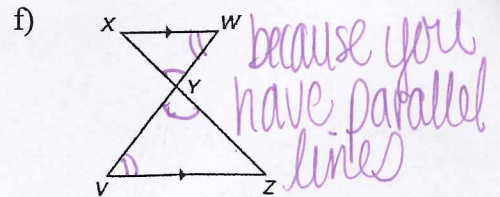
$\triangle LVM \sim$

e) $\frac{2.5}{1.25} = \frac{3}{1.5} \rightarrow 2 \neq 2$



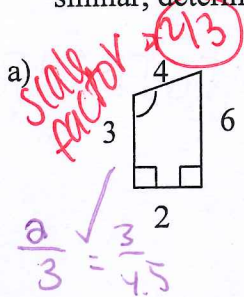
SAS
WXY

$\triangle TUV \sim$

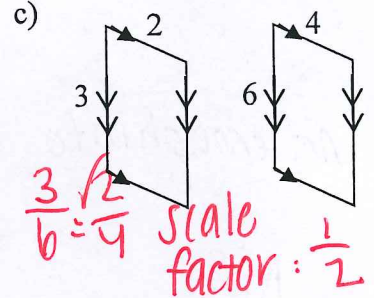
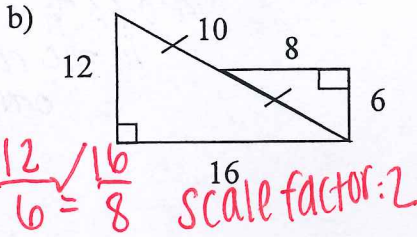


$\triangle WXY \sim \triangle ZYV$

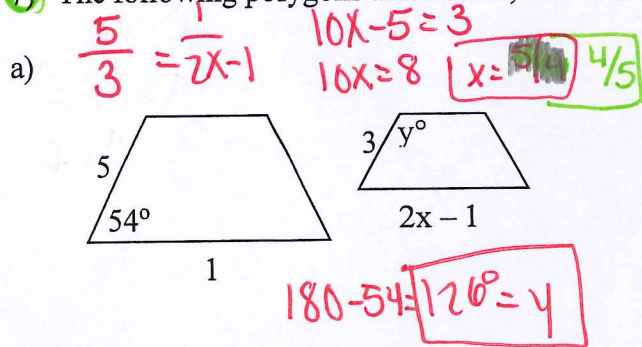
6) Determine whether the polygons are similar, not similar, or not enough information given. If they are similar, determine the scale factor comparing the first to second figure.



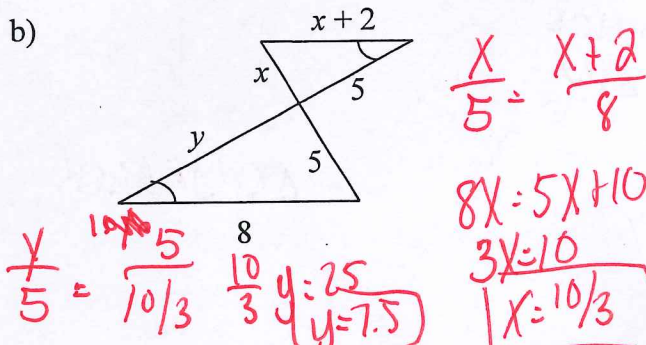
$\frac{2}{3} \sqrt{\frac{3}{4.5}}$ similar



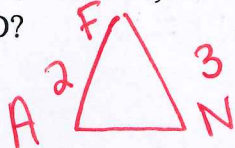
7) The following polygons are similar; find x and y.



$180 - 54 = 126 = y$



8) $\triangle AFN \sim \triangle DPG$, $AF = 2$ cm., $FN = 3$ cm., $DG = 10$ cm., and $PD = 8$ cm. Find AN . If $m\angle A = 36^\circ$, what is $m\angle D$?



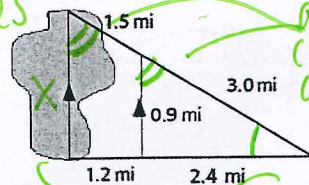
$\frac{2}{8} = \frac{x}{10}$ $20 = 8x$ $x = 2.5$ $m\angle A = m\angle D = 36^\circ$

9) Use the following image to explain why the two triangles are similar, then estimate the length of the lake.

$\frac{0.9}{2.4} = \frac{x}{3.6}$
 $3.24 = 2.4x$

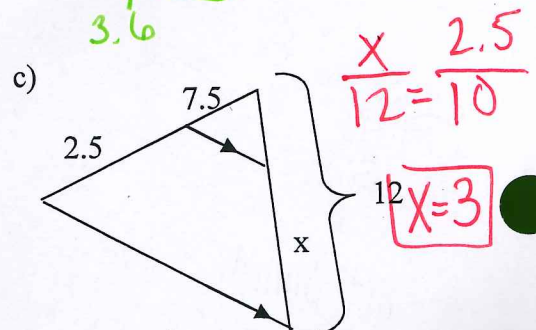
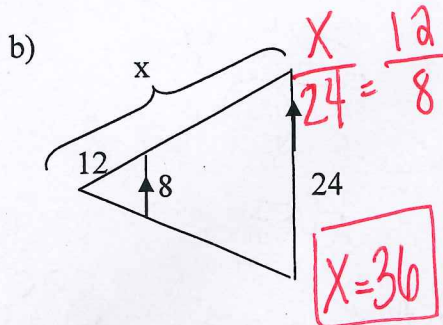
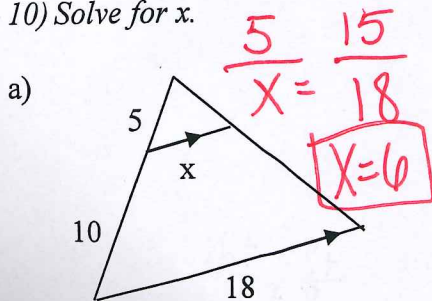
$x = 1.35$ miles

the two triangles are similar by the AA post.

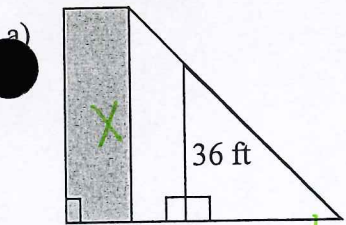


corresponding angles postulate

10) Solve for x.

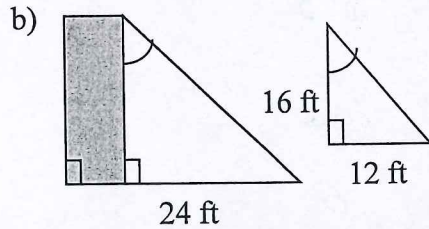


11) Use the diagram to find the height of each building.



$$\frac{70}{X} = \frac{40}{36} \quad 40X = 2520$$

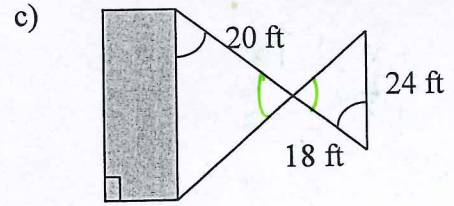
$$X = 63 \text{ ft}$$



$$\frac{24}{12} = \frac{X}{16}$$

$$12X = 384$$

$$X = 32 \text{ ft}$$



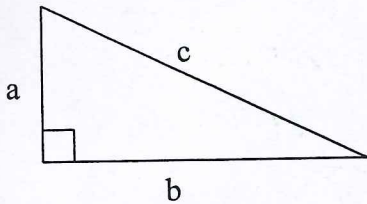
$$\frac{18}{20} = \frac{24}{X}$$

$$18X = 480$$

$$X = 26.7 \text{ ft}$$

Section 5

1) For # 1-3 two lengths of the right triangle are given. Find the missing length.



a) $a = 13$ $b = \sqrt{27} \approx 5.2$ $c = 14$

$$13^2 + b^2 = 14^2$$

$$b^2 = 196 - 169$$

$$b^2 = 27 \quad b = \sqrt{27}$$

b) $a = 12$ $b = 16$ $c = 20$

$$12^2 + 16^2 = c^2$$

$$144 + 256 = c^2$$

$$c^2 = 400$$

$$c = 20$$

c) $a = \sqrt{120} \approx 10.95$ $b = 7$ $c = 13$

$$a^2 + 7^2 = 13^2$$

$$a^2 + 49 = 169$$

$$a^2 = 120$$

$$a = \sqrt{120}$$

2) A triangle has side lengths given below. Determine what type of triangle each set is (acute, obtuse, or right). Show work to support your answer.

a. 24, 40, and 32

$$24^2 + 32^2 = 40^2$$

$$1600 = 1600$$

right

b. 30, 24, and 19

$$19^2 + 24^2 = 30^2$$

$$361 + 576 = 900$$

$$937 > 900$$

acute

c. 6, 14, and 11

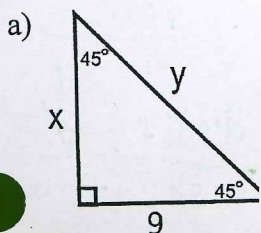
$$6^2 + 11^2 = 14^2$$

$$36 + 121 = 196$$

$$157 < 196$$

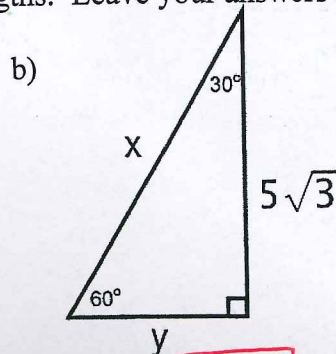
obtuse

3) Find the missing side lengths. Leave your answers in radical form.

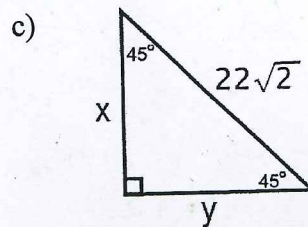


$$X = 9$$

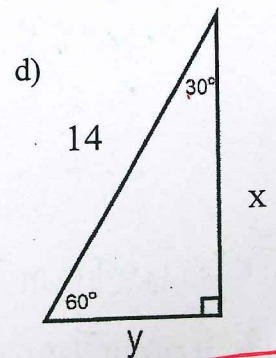
$$y = 9\sqrt{2}$$



$$y = 5 \quad X = 10$$

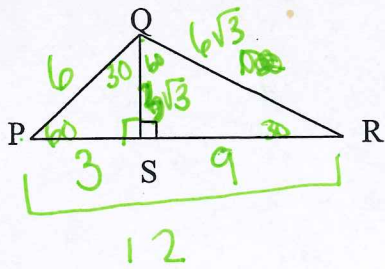


$$X = y = 22$$



$$y = 7 \quad X = 7\sqrt{3}$$

4) For the following, $\triangle PQR$, $m\angle PQR = 90^\circ$, $PQ = 6$, $m\angle QPS = 60^\circ$, and $PR = 12$.



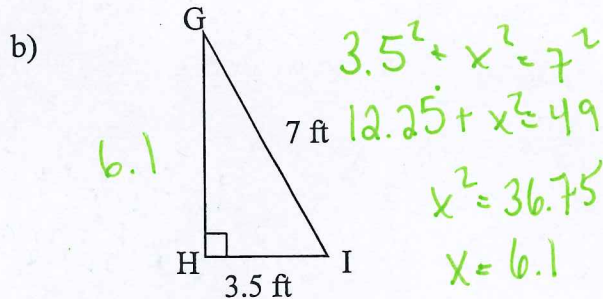
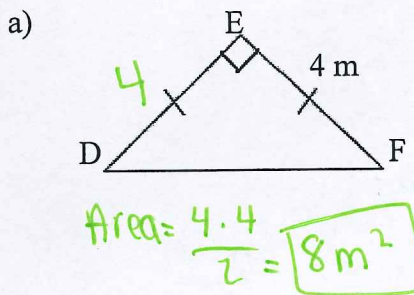
a) Find $QR = 6\sqrt{3}$

b) Find $QS = 3\sqrt{3}$

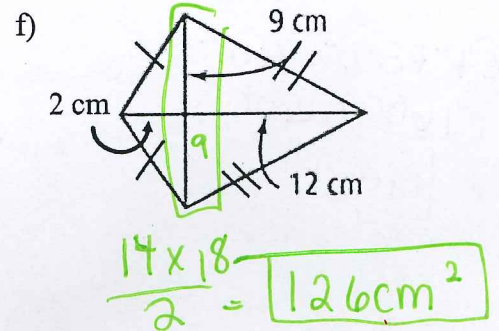
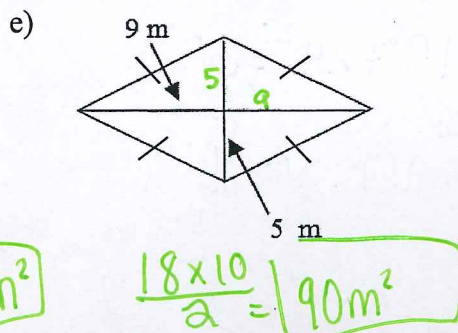
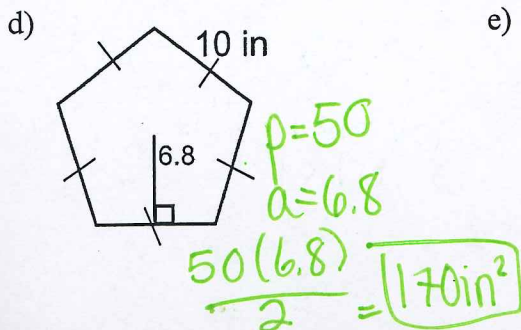
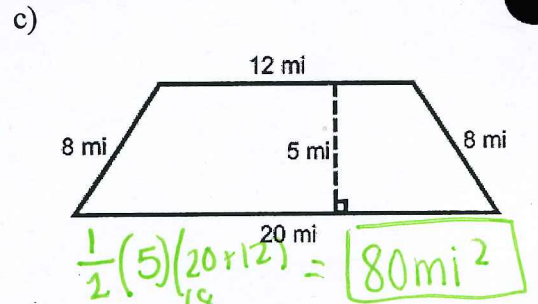
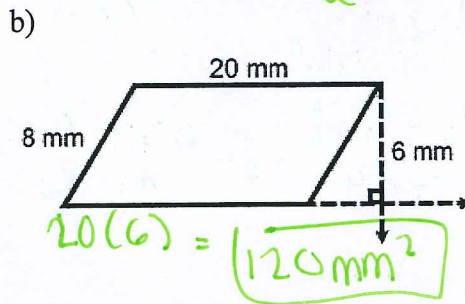
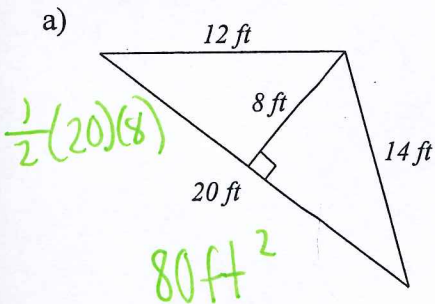
c) Find $SR = 9$

d) Find the area of $\triangle PQR = \frac{1}{2}bh = \frac{1}{2}(12)(3\sqrt{3}) = 18\sqrt{3}$

5) Find the area of each figure. Round your answers to the nearest tenth.



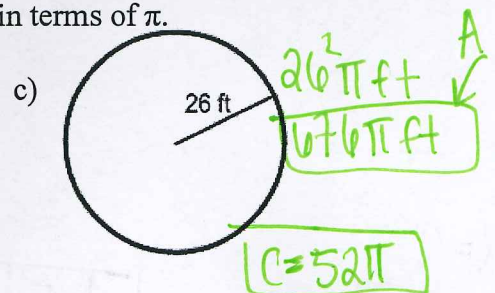
6) Find the area of the following figures.



7) Find the circumference AND area of each figure. Leave your answer in terms of π .

a) $r = 8 \text{ mm}$
 $\pi r^2 = 64\pi \text{ mm}^2$
 $2(8)\pi = 16\pi \text{ mm}$

b) $d = 26 \text{ cm}$
 $r = 13$
 $C = 26\pi \text{ cm}$
 $A = 169\pi \text{ cm}^2$



8) Round your answers to ^{7a)} 15a) to the nearest hundredth.

$C = \underline{50.27 \text{ mm}}$ $A = \underline{201.06 \text{ mm}^2}$

9) Find the radius of each circle from the given information. Round to the nearest tenth if necessary.

a) Area = $256\pi \text{ in}^2$
 $A = \pi r^2$
 $\frac{256\pi}{\pi} = \frac{\pi r^2}{\pi}$
 $\sqrt{256} = \sqrt{r^2}$ $r = 16 \text{ in}$

b) Circumference = 120 ft
 $C = 2\pi r$
 $\frac{120}{2} = \frac{2\pi r}{2}$
 $60 = \pi r$
 $r \approx 19.1 \text{ ft}$

10) If the area of a parallelogram is 100 cm^2 and the length of the base is 25 cm, what is the height?

$A = bh$
 $\frac{100}{25} = \frac{25h}{25}$
 $h = 4 \text{ cm}$

11) If the area of a parallelogram is 45 ft^2 and the height is 3 ft, what is the length of the base?

$A = bh$
 $\frac{45}{3} = \frac{b(3)}{3}$
 $b = 15 \text{ ft}$

12) If the area of a trapezoid is 250 in^2 , the lengths of the bases are 23 in and 27 in, what is the height?

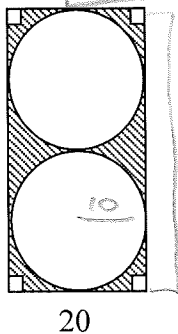
$A = h \frac{(b_1 + b_2)}{2}$
 $250 = h \frac{(23 + 27)}{2}$
 $250 = h \frac{(50)}{2}$
 $250 = 25h$
 $h = 10 \text{ in}$

13) If the area of a triangle is 343 u^2 and the height is 14 u, what is the length of the base?

$A = \frac{bh}{2}$
 $343 = \frac{b(14)}{2}$
 $343 = 7b$
 $b = 49 \text{ u}$

14) Find the area of the shaded region.

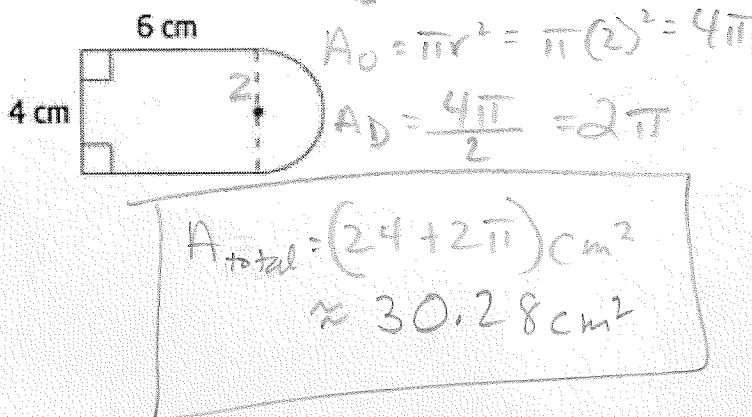
$A_{\square} = bh = (20)(40) = 800$
 $A_{\circ} = \pi r^2 = \pi (10)^2 = 100\pi$
 $A_{\text{no } \circ} = 2(100\pi) = 200\pi$



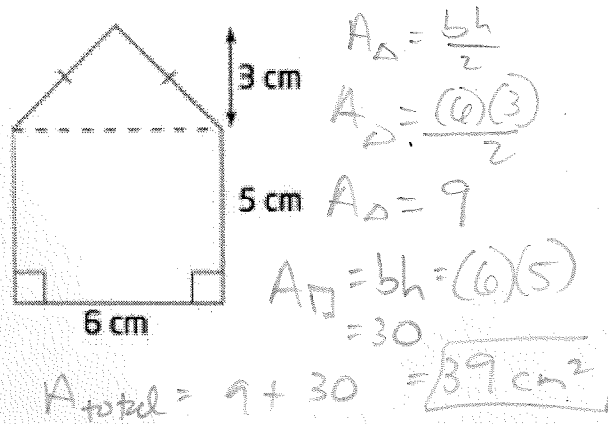
$A_{\square} - A_{\text{no } \circ} = A_{\text{shaded}}$
 $(800 - 200\pi) \text{ u}^2$
 $\approx 171.68 \text{ u}^2$

15) Find the area of the composite figures below.

a)

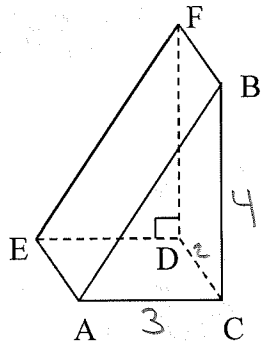


b)



Section 6

1) For the following, refer to the solid below.



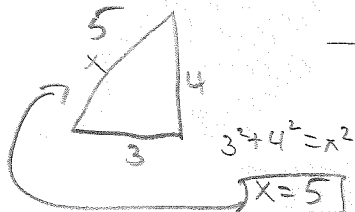
- a) Name the solid. Triangular prism
- b) Name a pair of parallel planes. ABC // EFD
- c) Name two segments skew to \overline{BF} . \overline{AC} and \overline{ED}
- d) Name two segments \perp to plane BFD. \overline{AC} and \overline{ED}
- e) What is the volume of the solid if $BC = 4$, $AC = 3$, and $DC = 2$.

$$B = \frac{bh}{2} = \frac{(3)(4)}{2} = 6$$

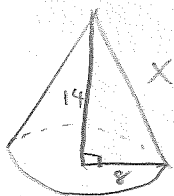
$$V = BH = (6)(2) = \boxed{12u^3}$$

~~$$P = 3+4+5 = 12$$~~

$$H = 2$$



2) What is the slant height of a right cone with a radius of 8 in. and a height of 14 in. $\sqrt{260} = 2\sqrt{65} \approx 16.12$



$$14^2 + 8^2 = x^2$$

$$196 + 64 = x^2$$

$$260 = x^2$$

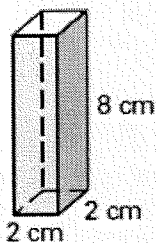
$$x = \sqrt{260} = 2\sqrt{65} \approx 16.12$$

Find the Surface Area and Volume of each right prism. Round to the hundredth if necessary.

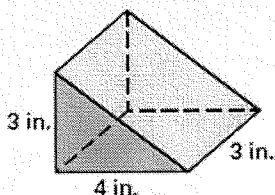
** see attached answers*

3) Find the Surface Area, Lateral Area, and Volume for the following solids. Give an exact answer.

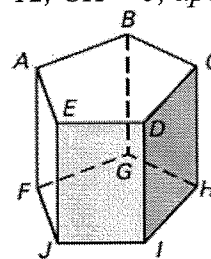
a.



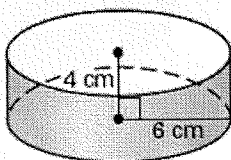
b.



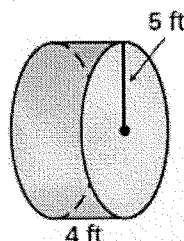
c. $BC = 12$, $CH = 5$, apothem = 6.2



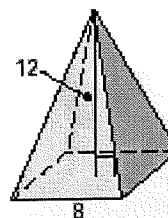
d.



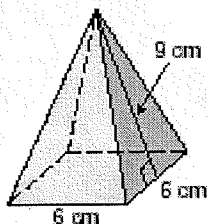
e.



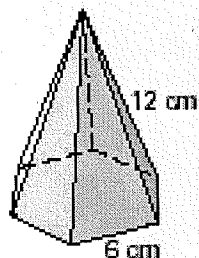
f.



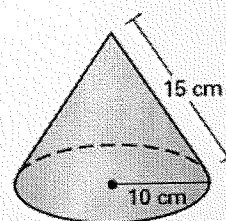
g.



h. apothem = 5.2 cm



i.



#3

a) Rectangular prism

$$A = bh = (2)(2) = 4$$

$$P = a + a + a + a = 8$$

$$LA = HP = (8)(8) = 64 \text{ cm}^2$$

$$SA = HP + 2B = (8)(8) + 2(4) = 72 \text{ cm}^2$$

$$V = BH = (4)(8) = 32 \text{ cm}^3$$

b) Triangular prism

$$B = \frac{1}{2}bh = \frac{1}{2}(3)(4) = 6$$

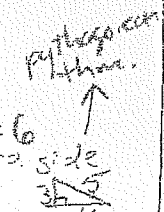
$$H = 3$$

$$P = 3 + 4 + 5 = 12$$

$$LA = HP = (3)(12) = 36 \text{ in}^2$$

$$SA = HP + 2B = (3)(12) + 2(6) = 48 \text{ in}^2$$

$$V = BH = (6)(3) = 18 \text{ in}^3$$



c) Pentagonal prism

$$B = \frac{1}{2}ap = \frac{1}{2}(6.2)(60) = 186$$

$$H = 5$$

$$P = 60$$

$$LA = HP = (5)(60) = 300 \text{ u}$$

$$SA = HP + 2B = (5)(60) + 2(186) = 672 \text{ u}^2$$

$$V = BH = (186)(5) = 930 \text{ u}^3$$

d) cylinder

$$r = 6$$

$$H = 4$$

$$LA = 2\pi rH = 2\pi(6)(4) = 48\pi \text{ cm}^2$$

$$SA = 2\pi rH + 2\pi r^2 = 2\pi(6)(4) + 2\pi(6)^2 = 120\pi \text{ cm}^2$$

$$V = \pi r^2H = \pi(6)^2(4) = 144\pi \text{ cm}^3$$

e) cylinder

$$r = 5$$

$$H = 4$$

$$LA = 2\pi rH = 2\pi(5)(4) = 40\pi \text{ ft}^2$$

$$SA = 2\pi rH + 2\pi r^2 = 2\pi(5)(4) + 2\pi(5)^2 = 90\pi \text{ ft}^2$$

$$V = \pi r^2H = \pi(5)^2(4) = 100\pi \text{ ft}^3$$

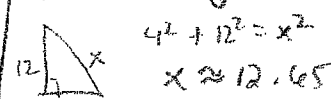
f) square base pyramid

$$B = s^2 = (8)(8) = 64$$

$$H = 12$$

$$P = 32$$

$$l = 12.65$$



$$LA = \frac{1}{2}Pl = \frac{1}{2}(32)(12.65) = 202.4 \text{ u}^2$$

$$SA = \frac{1}{2}Pl + B = \frac{1}{2}(32)(12.65) + 64 = 266.4 \text{ u}^2$$

$$V = \frac{1}{3}BH = \frac{1}{3}(64)(12) = 256 \text{ u}^3$$

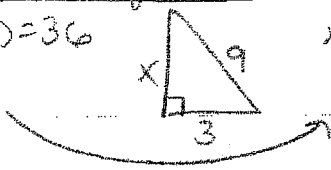
g) square base pyramid

$$B = (6)(6) = 36$$

$$H = 8.49$$

$$P = 24$$

$$l = 9$$



$$LA = \frac{1}{2}Pl = \frac{1}{2}(24)(9) = 108 \text{ cm}^2$$

$$SA = \frac{1}{2}Pl + B = \frac{1}{2}(24)(9) + 36 = 144 \text{ cm}^2$$

$$V = \frac{1}{3}BH = \frac{1}{3}(36)(8.49) = 101.88 \text{ cm}^3$$

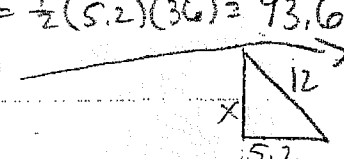
h) hexagonal pyramid

$$B = \frac{1}{2}ap = \frac{1}{2}(5.2)(36) = 93.6$$

$$H = 10.81$$

$$P = 36$$

$$l = 12$$



$$LA = \frac{1}{2}Pl = \frac{1}{2}(36)(12) = 216 \text{ cm}^2$$

$$SA = \frac{1}{2}Pl + B = \frac{1}{2}(36)(12) + 93.6 = 309.6 \text{ cm}^2$$

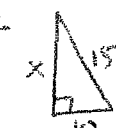
$$V = \frac{1}{3}BH = \frac{1}{3}(93.6)(10.81) = 337.27 \text{ cm}^3$$

i) cone

$$r = 10$$

$$l = 15$$

$$H = 11.18$$

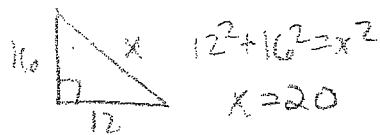


$$LA = \pi r l = \pi(10)(15) = 150\pi \text{ cm}$$

$$SA = \pi r l + \pi r^2 = \pi(10)(15) + \pi(10)^2 = 250\pi \text{ cm}^2$$

$$V = \frac{1}{3}\pi r^2 H = \frac{1}{3}\pi(10)^2(11.18) = 372.67\pi \text{ cm}^3$$

j) Cone
 $r = 12$
 $d = 20$
 $h = 16$



$$LA = \pi r l = \pi (12)(20) = \boxed{240\pi \text{ in}^2}$$

$$SA = \pi r l + \pi r^2 = \pi (12)(20) + \pi (12)^2 = \boxed{384\pi \text{ in}^2}$$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (12)^2 (16) = \boxed{768\pi \text{ in}^3}$$

j) Composites

SA

Rect prism

$B = 36$
 $H = 8$
 $P = 24$

only 1 base

$$SA = HP + \cancel{B} = (8)(24) + 36 = \boxed{228}$$

Pyramid

$B = 36$
 $P = 24$
 $l = 5$

base covered

$$SA = \frac{1}{2} Pl + \cancel{B} = \frac{1}{2} (24)(5) = \boxed{60}$$

$$\text{Total SA} = 228 + 60 = \boxed{348 \text{ cm}^2}$$

V

Rect prism

$B = 36$
 $H = 8$

$$V = BH = (36)(8) = \boxed{288}$$

Pyramid

$B = 36$
 $H = 4$



$$V = \frac{1}{3} BH = \frac{1}{3} (36)(4) = \boxed{48}$$

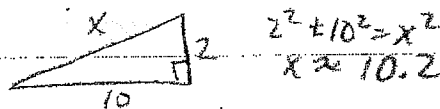
$$\text{Total volume} = 288 + 48 = \boxed{336 \text{ cm}^3}$$

k)

SA

Cone

$r = 2$
 $l = 10.2$



base covered

$$SA = \pi r l + \cancel{r^2} = \pi (2)(10.2) = \boxed{20.4\pi}$$

Sphere

$r = 2$

$\frac{1}{2}$ sphere

$$SA = \frac{1}{2} (4\pi r^2)$$

$$= \frac{1}{2} \cdot 4\pi (2)^2 = \boxed{8\pi}$$

$$\text{Total SA} = 20.4\pi + 8\pi = \boxed{28.4\pi \text{ m}^2}$$

Cone

$r = 2$
 $h = 10$

$$V = \frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (2)^2 (10) = \boxed{\frac{40\pi}{3}}$$

Sphere

$r = 2$

$\frac{1}{2}$ sphere

$$V = \frac{4\pi r^3}{3} / 2$$

$$= \frac{4\pi (2)^3}{3} = \frac{16\pi}{3}$$

$$\text{Total volume} = \frac{40\pi}{3} + \frac{16\pi}{3} = \boxed{\frac{56\pi}{3} \text{ m}^3}$$

k) SA

cylinder

$$r = 3$$

$$H = 5.1$$

$$SA = 2\pi rH + \pi r^2$$

$$2\pi(3)(5.1) + \pi(3)^2$$

$$= 39.6\pi$$

$$\text{Total SA} = 39.6\pi + 25.2\pi = 64.8\pi \text{ cm}^2$$

only 1 base

cone

$$r = 3$$

$$l = 8.4$$

$$SA = \pi r l + \pi r^2$$

$$= \pi(3)(8.4) + \pi(3)^2 = 25.2\pi$$

base is covered

V cylinder

$$r = 3$$

$$H = 5.1$$

$$V = \pi r^2 H$$

$$= \pi(3)^2(5.1)$$

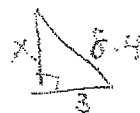
$$= 45.9\pi$$

$$\text{Total V} = 45.9\pi + 23.55\pi = 69.45\pi \text{ cm}^3$$

cone

$$r = 3$$

$$H = 7.85$$



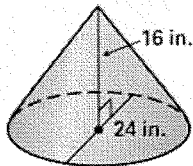
$$r^2 + 3^2 = 8.4^2$$

$$\times \times 7.85$$

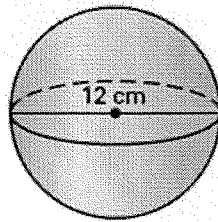
$$V = \frac{1}{3}\pi r^2 H = \frac{1}{3}\pi(3)^2(7.85)$$

$$= 23.55\pi$$

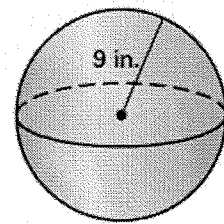
j.



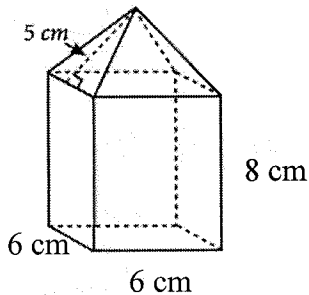
k.



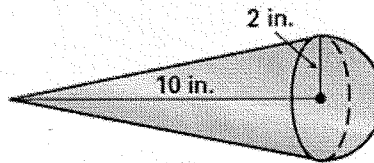
l.



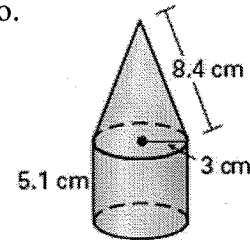
m.



n.



o.



- 4) The surface area of a square pyramid is given by 540 cm^2 and the side of the square is 10 cm. Find the slant height of the square pyramid.

$$B = bh = (10)(10) = 100$$

$$P = 4(10) = 40$$

$$SA = \frac{1}{2}Pl + B$$

$$540 = \frac{1}{2}(40)l + 100$$

$$540 = 20l + 100$$

$$540 = 20l + 100$$

$$440 = 20l$$

$$l = 22 \text{ cm}$$

- 5) The volume of a cylinder is 960π cubic inches. The height of the cylinder is 15 inches. Find the radius.

$$V = \pi r^2 h$$

$$960\pi = \pi r^2 (15)$$

$$\frac{960}{15} = \frac{15r^2}{15} \rightarrow \sqrt{r^2} = \sqrt{64}$$

$$r = 8 \text{ in}$$

- 6) If a cylinder has surface area of 128π sq ft, and the height of the cylinder is 12 feet, find the radius and the volume.

$$SA = 2\pi r h + 2\pi r^2$$

$$128\pi = 2\pi r(12) + 2\pi r^2$$

$$64 = 12r + r^2$$

$$0 = r^2 + 12r - 64$$

$$0 = (r + 16)(r - 4)$$

$$r + 16 = 0 \quad r - 4 = 0$$

$$r = -16 \quad r = 4 \text{ ft}$$

$$V = \pi r^2 h$$

$$V = \pi (4)^2 (12)$$

$$V = 192\pi \text{ ft}^3$$

- 7) The volume of a spherical ball is $5,000\pi$ cm^3 . What is the radius of the ball?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{5000\pi}{\pi} = \frac{4}{3}\pi r^3$$

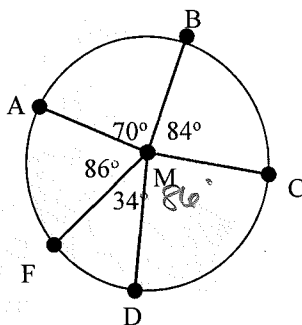
$$\frac{3 \cdot 5000}{4} = \frac{4}{3}\pi r^3 \cdot \frac{3}{4}$$

$$\sqrt[3]{3750} = \sqrt[3]{r^3}$$

$$r \approx 15.54 \text{ cm}$$

Section 7

- 1) Find the degree measures of each arc or angle by using the central angle measures given in $\odot M$



a) $m\widehat{AC}$ 154°

b) $m\widehat{FA}$ 86°

c) $m\widehat{CBF}$ 240°

d) $m\widehat{DB}$ 170°

e) $m\widehat{ADC}$ 206°

f) $m\widehat{DCA}$ 240°

g) $m\angle DMC$ 86°

2) Determine arc with length L of a circle with radius 8.5 in and degree measure 240° .

$$\frac{L}{2\pi r} = \frac{\mu}{360}$$

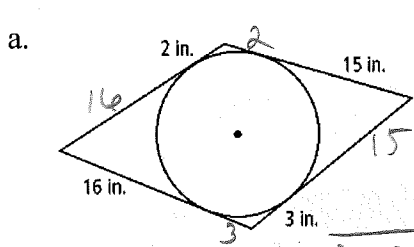
$$\frac{L}{2\pi(8.5)} = \frac{240}{360}$$

$$\frac{L}{17\pi} \approx \frac{2}{3}$$

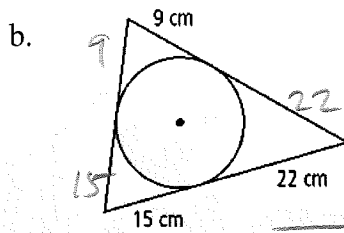
$$3L = 34\pi$$

$$L = \frac{34\pi}{3} \approx 35.60 \text{ in}$$

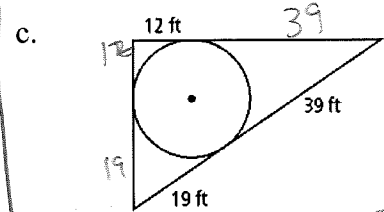
3) Each polygon circumscribes a circle. What is the perimeter of each polygon?



$$4 + 30 + 6 + 32 = 72 \text{ in}$$



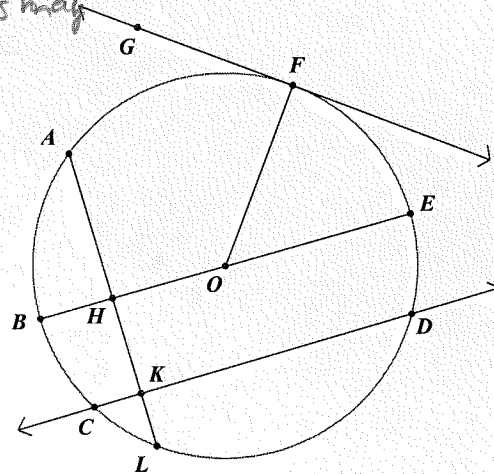
$$18 + 44 + 30 = 92 \text{ cm}$$



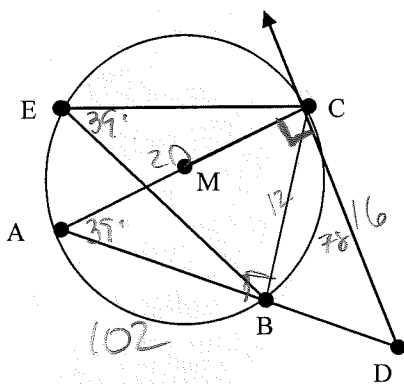
$$24 + 38 + 78 = 140 \text{ ft}$$

4) Using circle O below, name the following: *★ answers may vary*

- a. Diameter \overline{BO}
- ★ b. Central Angle $\angle BOF$, $\angle FOE$
- ★ c. Minor Arc \overline{FE} , \overline{AC}
- ★ d. Major Arc \overline{LFD} , \overline{ADE}
- ★ e. Semicircle \overline{BPE} , \overline{BLE}
- ★ f. Radius \overline{OE} , \overline{OF} , \overline{OB}
- g. Tangent \overline{GF}
- h. Point of Tangency F



5) For the following, in $\odot M$, \overline{AC} is the diameter, \overline{DC} is tangent to the circle at point C , and $m\widehat{BC} = 78^\circ$.

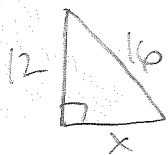


- a) $m\angle BAC$ 39°
- b) $m\angle BEC$ 39°
- c) $m\widehat{AB}$ 102°
- d) $m\angle ACB$ 51°
- e) $m\angle ABC$ 90°
- f) $m\angle ACD$ 90°

- g) \widehat{AE} is a minor arc, \widehat{ACB} is a major arc
- h) \overline{MC} is a radius, \overline{AC} is a diameter
- i) \overline{CD} is a tangent line

In $\odot M$, if $BC = 12$, $CD = 16$, and $AC = 20$, find the following:

- j) $BD = \sqrt{112} = 4\sqrt{7} \approx 10.58$
- k) $AD = \sqrt{656} = 4\sqrt{41} \approx 25.61$

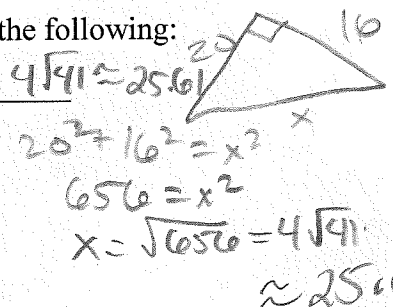


$$12^2 + x^2 = 16^2$$

$$144 + x^2 = 256$$

$$x^2 = 112$$

$$x = \sqrt{112} = 4\sqrt{7} \approx 10.58$$



$$20^2 + 16^2 = x^2$$

$$656 = x^2$$

$$x = \sqrt{656} = 4\sqrt{41}$$

$$\approx 25.61$$

★ answers may vary

Find the measure of the arc or angle in $\odot M$.

7. $m\angle QMP = 60^\circ$

8. $m\angle NMO = 110^\circ$

9. $m\angle PNO = 35^\circ$

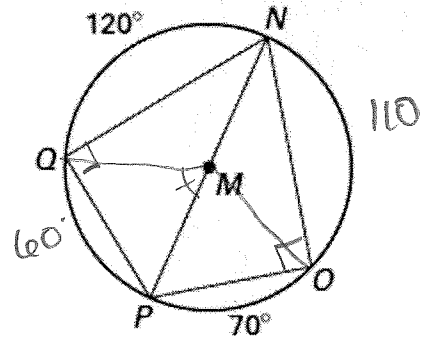
10. $m\angle QNP = 30^\circ$

11. $m\widehat{QO} = 130^\circ$

12. $m\widehat{NOP} = 180^\circ$

13. $m\widehat{PQ} = 60^\circ$

14. $m\widehat{OQN} = 250^\circ$



15) What is the value of x ? Lines that appear to be tangent are tangent. Round to the nearest hundredth if necessary.

a)
$$90 + 40 + 90 + x = 360$$

$$220 + x = 360$$

$$x = 140$$

b)
$$x + 70 + 90 = 180$$

$$x + 160 = 180$$

$$x = 20$$

c)
$$x^2 + 9^2 = (x+6)^2$$

$$x^2 + 81 = x^2 + 12x + 36$$

$$81 = 12x + 36$$

$$45 = 12x$$

$$x = 3.75$$

d)
$$x^2 + 20^2 = (x+12)^2$$

$$x^2 + 400 = x^2 + 24x + 144$$

$$400 = 24x + 144$$

$$256 = 24x$$

$$x = \frac{32}{3} \approx 10.67$$

16) Write the equation for the circle with center $(2, 4)$ and radius = 7 in

$$(x-2)^2 + (y-4)^2 = 49$$

17) Write the equation for the circle with center $(-3, 1)$ and diameter = 18 in $r = 9$

$$(x+3)^2 + (y-1)^2 = 81$$

18) Find the center and radius of the circle: $(x-7)^2 + (y+12)^2 = 144$

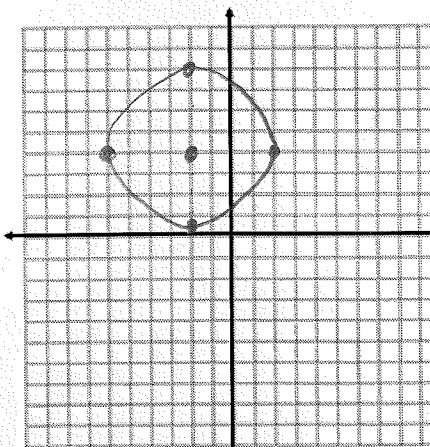
C: $(7, -12)$ $r = 12$

19) Find the center and radius of the circle: $(x+5)^2 + (y+8)^2 = 225$

C: $(-5, -8)$ $r = 15$

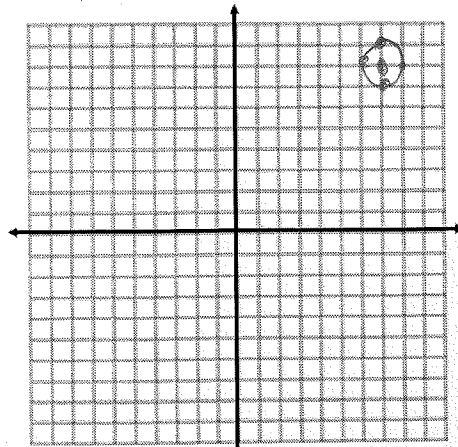
20) Graph the circle on the coordinate plane.

a. $(x+2)^2 + (y-4)^2 = 16$



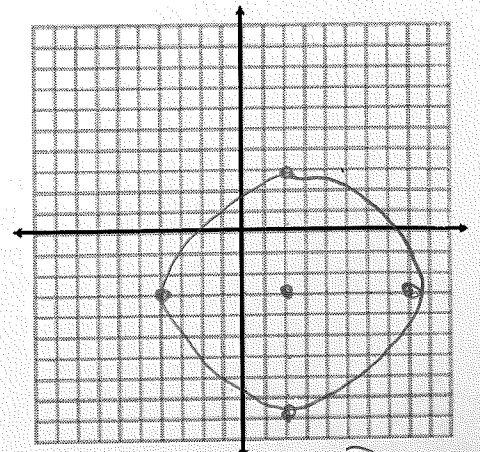
C: $(-2, 4)$
 $r = 4$

b. $(x-7)^2 + (y-8)^2 = 1$



C: $(7, 8)$
 $r = 1$

c. $(x-2)^2 + (y+3)^2 = 36$



C: $(2, -3)$
 $r = 6$

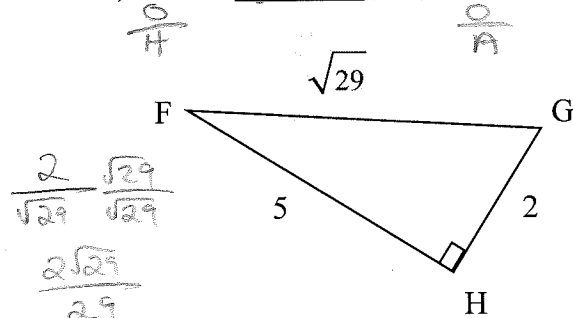
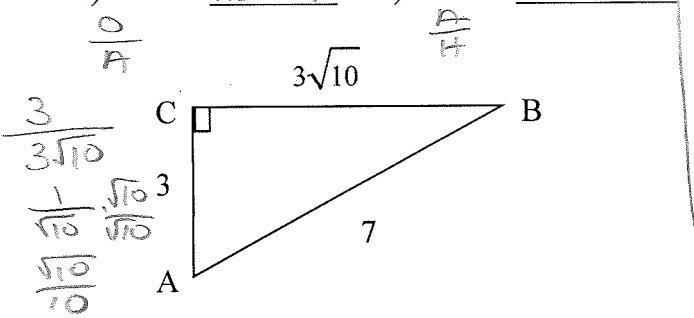
Section 8

SOH - CAH - TOA

1) Using the triangles below, determine the trigonometric ratio. Leave your answers as simplified fractions.

a) $\tan B = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$ b) $\cos A = \frac{3}{7}$

c) $\sin F = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$ d) $\tan G = \frac{5}{2}$



2) Find the marked side of each of the following triangles.

a) $\cos(58^\circ) = \frac{y}{32.3}$
 $y = 32.3 \cdot \cos(58^\circ) = 17.12$

b) $\cos(20^\circ) = \frac{w}{17}$
 $w = 17 \cdot \cos(20^\circ) = 16.09$

c) $\sin(47^\circ) = \frac{x}{29}$
 $x = 29 \cdot \sin(47^\circ) = 21.21$

d) $\tan(35^\circ) = \frac{b}{0.39}$
 $b = 0.39 \cdot \tan(35^\circ) = 0.27$

3) Find the value for each of the marked angles.

a) $\tan(\theta) = \frac{3.6}{6.2}$
 $\theta = \tan^{-1}(3.6/6.2) = 30^\circ$

b) $\cos(\theta) = \frac{56}{72.9}$
 $\theta = \cos^{-1}(56/72.9) = 40^\circ$

c) $\sin(\theta) = \frac{4.9}{7.8}$
 $\theta = \sin^{-1}(4.9/7.8) = 39^\circ$

d) $\tan(x) = \frac{97}{128}$
 $x = \tan^{-1}(97/128) = 37^\circ$

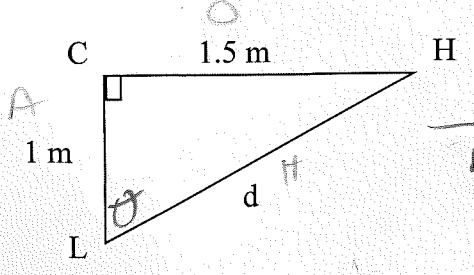
4) A skateboarding ramp is 12 in. high and rises at an angle of 17°. How long is the base of the ramp? What is the length of the ramp? Round your answer to the nearest inch.

$\tan(17^\circ) = \frac{12}{y}$
 $y = 39 \text{ in} \leftarrow \text{base}$

$\sin(17^\circ) = \frac{12}{x}$
 $x = 41 \text{ in} \leftarrow \text{length}$

5) Joey is walking home from the library. He can walk for 1 mile along the street, then turn right and walk 1.5 miles along another street; or he can cut across a large field straight to his house. At what angle, θ , should he head off from the library, and how far, d , should he cut across the field?

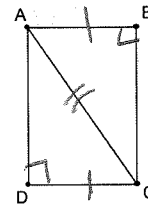
$\theta = 56^\circ$
 $d = 1.8 \text{ mi}$



$\tan(\theta) = \frac{1.5}{1}$
 $\theta = 56^\circ$
 $1^2 + 1.5^2 = d^2$
 $3.25 = d^2$
 $d = \sqrt{3.25} \approx 1.8$

Proofs

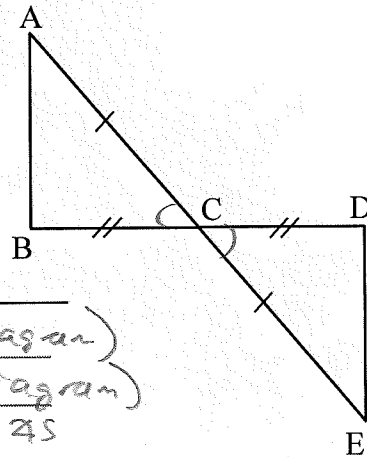
- 1) Given: $\angle B$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{CD}$
 Prove: $\angle DAC \cong \angle BCA$



Statements	Reasons
1. $\angle B$ and $\angle D$ are right angles, $\overline{AB} \cong \overline{CD}$	1. Given
2. $\triangle ADC$ and $\triangle CBA$ are right triangles	2. Def. right triangles
3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Prop. of \cong
4. $\triangle ADC \cong \triangle CBA$	4. Hypotenuse Leg Thm
5. $\angle DAC \cong \angle BCA$	5. Corresponding parts of \cong triangles are \cong

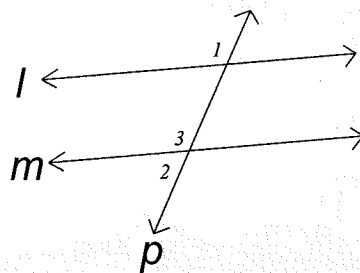
2)

Fill in the blanks in the table below to prove $\angle CBA \cong \angle CDE$



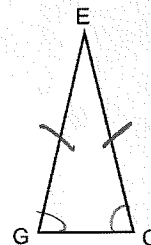
Statement	Reason
$\overline{CB} \cong \overline{DC}$	Given (labeled in diagram)
$\overline{CA} \cong \overline{CE}$	Given (labeled in diagram)
$\angle BCA$ & $\angle DCE$ are vertical angles	Def. of vertical \angle s
$\angle BCA \cong \angle DCE$	Vertical Angles Thm.
$\triangle BCA \cong \triangle DCE$	SAS thm
$\angle CBA \cong \angle CDE$	Corresponding Parts of Congruent \triangle s are \cong

- 3) Given: $m\angle 1 = 100^\circ$, $m\angle 2 = 80^\circ$
 Prove: $l \parallel m$



Statements	Reasons
1. $m\angle 1 = 100^\circ$, $m\angle 2 = 80^\circ$	1. Given
2. $\angle 2$ and $\angle 3$ form a linear pair	2. Def. of linear pair
3. $\angle 2$ and $\angle 3$ are supplementary	3. Linear Pair Property
4. $m\angle 2 + m\angle 3 = 180^\circ$	4. Def. of Supplementary \angle s
5. $80^\circ + m\angle 3 = 180^\circ$	5. Substitution Prop. of =
6. $m\angle 3 = 100^\circ$	6. Subtraction Prop. of =
7. $m\angle 1 = m\angle 3$	7. Substitution Prop. of =
8. $\angle 1 \cong \angle 3$	8. Def. of Angle Congruence
9. $\angle 1$ and $\angle 3$ are Corresponding Angles	9. Def. of Corr. \angle s
10. $l \parallel m$	10. <u>Converse</u> of Corr. \angle s Thm

- 4) Given: $\overline{GE} \cong \overline{OE}$; $m\angle E = 38^\circ$
 Prove: $m\angle G = 71^\circ$



Statements	Reasons
1. $\overline{GE} \cong \overline{OE}$	1. Given
2. $\angle G \cong \angle O$	2. Isosceles Triangle Thm
3. $m\angle G = m\angle O$	3. Def. Angle Congruence
4. $m\angle E = 38^\circ$	4. Given
5. $m\angle G + m\angle E + m\angle O = 180^\circ$	5. Triangle Sum Thm
6. $m\angle G + 38^\circ + m\angle G = 180^\circ$	6. Substitution Prop. =
7. $2 \cdot m\angle G + 38^\circ = 180^\circ$	7. Combine Like Terms
8. $2 \cdot m\angle G = 142^\circ$	8. Subtraction Prop. =
9. $m\angle G = 71^\circ$	9. Division Prop. =